THE MOTION OF ELECTRONS IN HYDROGEN AND HELIUM

By H. B. Wahlin

Abstract

Mobilities of electrons in H_2 and He at 760 mm pressure have been measured by the Rutherford alternating potential method for fields ranging from 0.5 to 46 volts/cm. The limiting mobilities for zero field are calculated to be, for H_2, 6,700 cm/sec per volt/cm and for He, 10,840 cm/sec per volt/cm.

Energy loss of an electron on impact in H_2 and He.—*K. T. Compton's theory of electronic mobilities* is shown to fit the observed variation of mobility with field strength provided it be assumed that the fraction of its energy an electron loses on impact is, in H_2, 2.16, and in He, 8.6 times what it would be were the collisions elastic.

Mean free path of electrons in H_2 and He.—From the limiting values of the mobilities at zero field it is calculated that the mean free path of electrons in thermal equilibrium with the gas is for H_2 at 1 mm pressure, 0.0408 cm, and for He at 1 mm pressure, 0.066 cm. These values are approximately half the kinetic theory values.

THE motion of electrons through a gas under the action of an electric field has been studied by Townsend and Bailey,[1] Loeb,[2] and the writer.[3] Using a method in which the velocity in the direction of the field was determined by a magnetic deflection and in which the velocity of agitation was determined from the lateral diffusion of the electron beam, Townsend and Bailey have succeeded in showing that the velocity of drift of the electrons per unit field intensity is not independent of the field as is the case with ions, but decreases with increasing voltage. This is due to two causes. First, the electrons are not in thermal equilibrium with the gas, but, because of energy gained in the field, reach a thermal velocity which exceeds that of the gas. Secondly, as has been shown by Ramsauer[4] the electron free path is not a constant, but decreases with increasing velocity of agitation for low velocities and increases again for higher velocities.

Townsend's method is not very well adapted to a study of the motion of electrons in low fields, and for that reason it was thought best to extend the measurements on the determination of electron free paths to such small fields that the electrons may be considered to be

[1] J. S. Townsend, J. Franklin Inst. 563 (Nov. 1925).
[2] L. B. Loeb, Phys. Rev. **20,** 397 (Nov. 1922); **23,** 157 (Feb. 1924).
[3] H. B. Wahlin, Phys. Rev. **23,** 169 (Feb. 1924).
[4] C. Ramsauer, Ann. d. Physik **64,** 513 (1921); **66,** 546 (1922).

in thermal equilibrium with the molecules. A second purpose of the present investigation was to see if a comparison could be made between the experimental results and a theoretical equation for electron mobilities derived by K. T. Compton.

The method used was the Rutherford alternating potential one with a Hartley oscillating circuit as a source of alternating potential. It differed only in detail from the one described in an earlier paper dealing with electron motion in nitrogen. The alternating potential was measured by a Kelvin multicellular voltmeter from 120 to 30 volts and by an electrometer with a phosphor bronze suspension below 30 volts.

The hydrogen was purified by passing it over hot copper, through a tube containing fused calcium chloride, through a second tube containing potassium hydroxide and slowly through a system of tubes 225 cm long containing phosphorus pentoxide.

In order to purify the helium (which originally contained about 2 percent oxygen and nitrogen) three traps each containing about 170 cc of finely divided charcoal were used. The charcoal had been activated with steam and was baked at a temperature of 300°C for 24 hours or more. The third trap was so arranged that it was connected permanently to the chamber holding the gas, and the helium circulated through this continuously by convection, while the readings were being taken. No change in the results could be noticed after circulating for a period of 24 hours so that it seems reasonable to assume that the gas was pure.

All determinations were made at a pressure of 760 mm. In obtaining the mobility curves, the frequency was kept constant and the alternating potential varied. From the intercepts of the mobility curves obtained by plotting the current against field intensity, approximate values of the mobility were computed from the equation

$$u=\pi nd/\sqrt{2x} \tag{1}$$

where n is the frequency, d the distance between the plates, and x the effective value of the alternating field.

Figs. 1 and 2 illustrate the types of curves obtained in H_2 and He respectively. It is to be noted that the intercepts are well defined so that any error arising from an ill-defined intercept was slight. In the case of curves obtained with low frequencies, i. e. where the intercept came in low fields, a number of curves were obtained without changing the frequency, and the mobility computed from the average value of the intercept thus obtained.

The mobilities computed from Eq. (1) are plotted as a function of the field intensity in Figs. 3 and 4, Fig. 3 for H_2 and Fig. 4 for He. As the mobility is not a constant, the values for higher fields must be

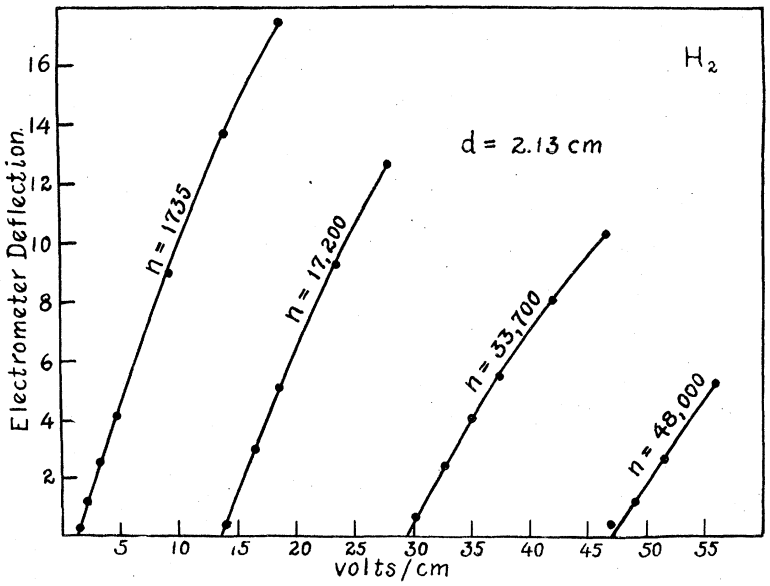

Fig. 1.

considered as approximate only, since in the derivation of Eq. (1) it is assumed that the mobility is independent of the field, and in

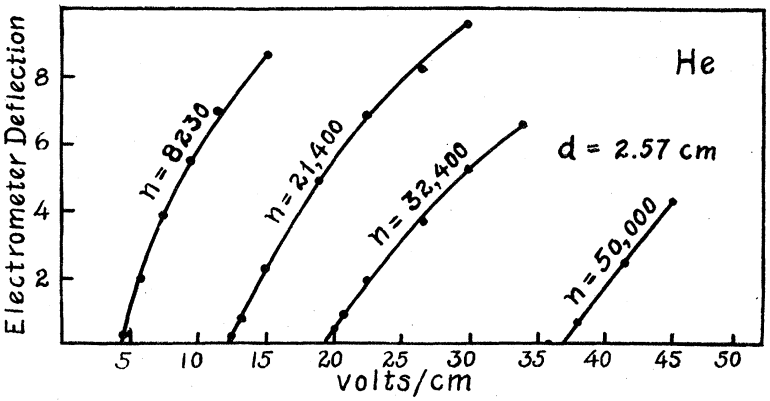

Fig. 2.

general this is far from being the case. However, an examination of these curves will show that for low fields the mobility is so nearly independent of the field that Eq. (1) is applicable and the true value of

the limiting mobility may be obtained directly from it. The values thus obtained are:

$$u_{H_2} = 6700 \text{ cm/sec per volt/cm},$$
$$u_{He} = 10{,}840 \text{ cm/sec per volt/cm}.$$

The use of the alternating potential method for determining mobilities has been criticized because the electrons do not move through the gas with the thermal velocity corresponding to the instantaneous values of the field. Compton has shown, however, that the distance the electrons must travel to gain even 99 percent of their terminal energy is small at a pressure of 760 mm, so that the error would be slight. There would be a phase lag between the terminal energy and the instantaneous value of the field driving the electrons from one plate to

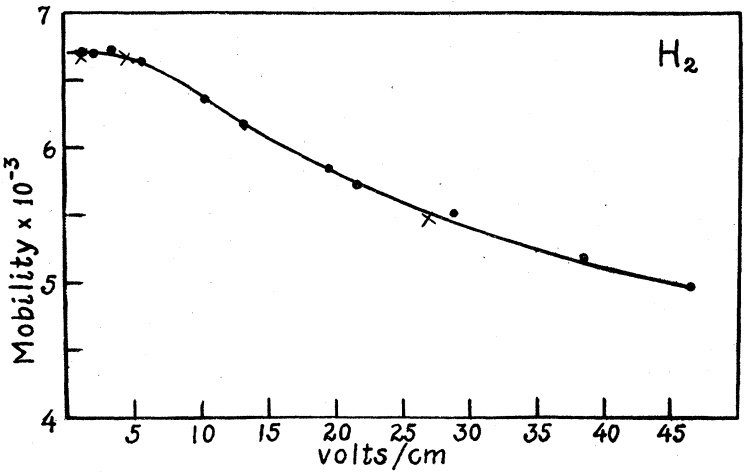

Fig. 3. Variation with field strength of electron mobilities in hydrogen.

the other. In other words, during that part of the half-cycle when the voltage is rising, the thermal energy is less than it should be, and during the part of the half-cycle when the voltage is decreasing it is greater. These two errors would tend to counterbalance each other.

In order to settle definitely any doubt concerning the question of terminal energy, the distance between the plates was varied from 2.13 cm to 4.0 cm in the case of H_2. The values represented by the dots (Fig. 3) were obtained with the former distance, and the values corresponding to the crosses with the latter. It will be noticed that they do not differ appreciably, so that any effect due to a non-terminal state is slight.

Test of Compton's Equation for Mobility

As stated above, Eq. (1) is applicable only over that range of field intensities for which the mobility may be considered as constant. When the field becomes so large that the thermal energy of the electrons differs appreciably from the molecular, the mobility will decrease because of the decreased time between impacts. In general, then, the mobility will be a function of the field. K. T. Compton[5] has derived this function and has shown that

$$u=\frac{.815\, e\lambda}{\sqrt{m}\left[\alpha T+(\alpha^2T^2+1.76\lambda^2e^2x^2/f)^{\frac{1}{2}}\right]^{\frac{1}{2}}}$$

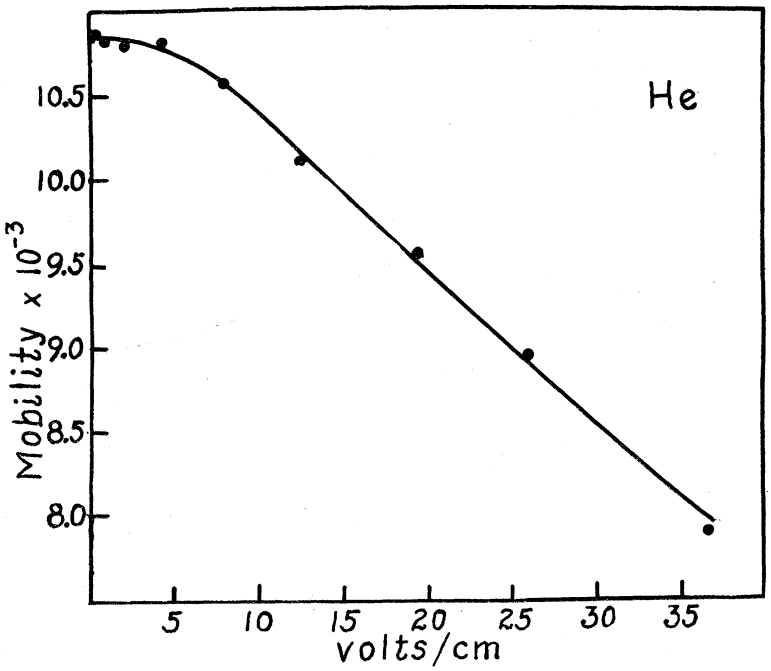

Fig. 4. Variation with field strength of electron mobilities in helium.

where e is the electronic charge, λ the mean free path, αT the thermal energy of the molecules, and f the fraction of its energy an electron loses on impact with a molecule.

This equation may be written in the form

$$u=\frac{a}{\left[1+(1+Bx^2)^{\frac{1}{2}}\right]^{\frac{1}{2}}}$$

if it is assumed that λ is independent of x.

[5] K. T. Compton, Phys. Rev. **22**, 333 (1923).

The equation of motion of the electrons would then be

$$\frac{dy}{dt}=\frac{ax}{[1+(1+Bx^2)^{\frac{1}{2}}]^{\frac{1}{2}}}$$

where x is the instantaneous value of the field, and dy/dt is the velocity in the direction of the field.

In the case under discussion since a sine field was employed, we get

$$\frac{dy}{dt}=\frac{aX\sin 2\pi nt}{[1+(1+BX^2\sin^2 2\pi nt)^{\frac{1}{2}}]^{\frac{1}{2}}} \tag{2}$$

Thus a direct check of Compton's equation is impossible in this case. However, it is possible to compare the experimentally determined frequency necessary to obtain an intercept with a given field with those obtained by a solution of this differential equation. Eq. (2) becomes for this case

$$\int_0^d dy=\int_0^{1/2n}\frac{aX\sin 2\pi nt\,dt}{[1+(1+BX^2\sin^2 2\pi nt)^{\frac{1}{2}}]^{\frac{1}{2}}}$$

Placing $2\pi nt=\theta$ and solving for n we get

$$n=\frac{aX}{\pi d}\int_0^{\pi/2}\frac{\sin\theta\,d\theta}{[1+(1+BX^2\sin^2\theta)^{\frac{1}{2}}]^{\frac{1}{2}}}$$

This is an elliptic integral and on reduction gives

$$n=\frac{a\sqrt{2r}}{\pi d\sqrt{B}}[2E(k,\varphi)-F(k,\varphi)] \tag{3}$$

in which

$$r=\sqrt{BX^2+1}\,,\qquad k=\sqrt{\frac{r+1}{2r}}\text{ and }\varphi=\sin^{-1}\sqrt{\frac{r-1}{r+1}}$$

The values of a may be determined immediately from the limiting mobility as the field approaches zero, since in this case Compton's equation reduces to the form $u=a/\sqrt{2}$. We thus get

$$a_{\mathrm{H}}=\ 9{,}490\text{ cm/sec per volt/cm}$$
$$a_{\mathrm{He}}=15{,}350\text{ cm/sec per volt/cm.}$$

The values of B which fit the experimental results most closely are

$$B_{\mathrm{H_2}}=\ .003$$
$$B_{\mathrm{He}}=\ .004$$

Tables I and II give the frequencies computed from Eq. (3), together with those experimentally determined.

TABLE I

Hydrogen

$a=9490$, Effective value of field	$B=.003$, n (calc.)	$d=2.13$ cm n (obs.)
1.23	1730	1735
2.09	2960	2959
3.52	4955	5000
5.73	8000	7990
10.40	14,000	14,000
13.2	17,400	17,200
19.5	24,300	24,000
21.7	26,600	26,200
28.9	33,300	33,700
38.5	41,200	42,200
46.0	46,700	48,000

TABLE II

Helium

$a=15,350$, Effective value of field	$B=.004$, n (calc.)	$d=2.57$ cm n (obs.)
.554	1055	1061
1.107	2120	2090
2.18	4160	4110
4.33	8220	8230
7.94	14,770	14,720
12.40	22,100	21,400
19.28	32,200	32,400
26.00	40,600	40,750
36.65	52,300	50,000

The agreement between the experimental and calculated values is quite close. For the higher frequency there is a slight variation which may be due to experimental errors.

CALCULATION OF MEAN FREE PATH

Since Compton's equation satisfied the experimental data so closely it must be concluded that over the range of voltages used in these experiments the mean free path is a constant and independent of the electron velocity. This is not the case in nitrogen, as shown in the article referred to above. The values for the mean free path may most readily be computed from the limiting mobility, using Townsend's equation, viz.

$$u=.75e\lambda/m\bar{v}$$

The values thus calculated for a pressure of 1 mm placing $\bar{v}=1.065\times10^7$ cm/sec. at 20°C, together with the kinetic theory values ($4\sqrt{2}$ times the molecular free path) are

	Experimental λ	Kinetic theory λ
H_2	.04080 cm	.0842 cm
He	.0660	.1313

The experimental values are thus seen to be lower than the kinetic theory values. This, however, is not in general the case. Since the experimental value of λ found for N_2 is .0996 cm at 1 mm pressure whereas according to kinetic theory reasoning it should be lower than both λ_{H_2} and λ_{He}.

Energy Loss of an Electron on Impact

In the discussion of Compton's equation above, we used it in the general form, simply assuming that the energy loss on impact is a fraction f of the electronic energy. From the experimentally determined value of B it is possible to calculate this fraction f. Since

$$B = 1.76\lambda^2 e^2 / f\alpha^2 T^2$$

we get, remembering that x is measured in volts,

$$f_{H_2} = 1.17 \times 10^{-3}$$
$$f_{He} = 2.32 \times 10^{-3}$$

If the impacts are elastic Compton has shown that

$$f = 2m/M$$

where m is the electronic and M the molecular mass. From this we get

$$f_{H_2} = .541 \times 10^{-3}$$
$$f_{He} = .270 \times 10^{-3}$$

The observed fraction is about 2.16 times what it should be on the basis of elastic impacts in H_2 and 8.6 times too large for He. The fraction lost is very small in either case and is independent of the energy of the electron at impact over the range of energies covered in these experiments. The fact that f for He is greater than that for H_2 may be due to the presence of some impurity not readily absorbed by the charcoal although it is difficult to imagine what it could be. No electronegative impurities were present, as shown by the high value obtained for the limiting mobility, and it seems reasonable therefore to conclude that other impurities were present in very small quantities.

Fortunately the mobility is independent of x and therefore also of f in low fields, so that the effect on the limiting mobility would be negligible, unless the molecular dimensions of the impurity differ enormously from those of helium.

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