ACOUSTIC TRANSMISSION WITH A HELMHOLTZ RESONATOR OR AN ORIFICE AS A BRANCH LINE

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Abstract

The author's theory of acoustic transmission in a conduit with a branch line is extended to the cases of a Helmholtz resonator and an orifice attached as branches. The effect of viscosity is included in an approximate form.

Helmholtz resonator. The components of the point impedance of the branch are found to be: $Z_1=(2\omega\rho\mu)^{1/2}(\pi R^3)^{-1}L$ and $Z_2=\rho\omega/c-\rho a^2/V\omega+(2\omega\rho\mu)^{1/2}L(\pi R^3)^{-1}$, wherein $\omega=2\pi\times$frequency, ρ is density, μ, viscosity, R, radius of orifice, L, its equivalent length, a, the velocity of sound, and V, the volume of the resonator. *Experimental verification of the theory is obtained* and in addition there is found the approximate value of a simple channel equivalent in viscosity effect to the orifice of the resonator. *The serious diminution of transmission* caused by the resonator is not limited to the neighborhood of the resonance frequency but *extends over a range of two octaves.*

Orifice. The components of the point impedance of the branch are found to be the same as those of the Helmholtz resonator, but with the term containing V omitted. Experimental verification is obtained with sizes of orifices from 7 percent to 42 percent the diameter of the conduit. *The radiation from such orifices is shown to be relatively small* and thus their impedances are practically imaginary. *Transmission* in the conduit *increases* with *frequency.* Action of *orifices* in *musical instruments* is explained by the theory. Relative values of radiation and transmission are shown by computation.

THIS is a continuation of the theory of acoustic transmission with branch lines and its purpose is a detailed investigation of the cases cited in the title.

Helmholtz resonator. In the previous article[1] was deduced the general expression for transmission,

$$(P'/P)^2=[(Z_1^2+Z_1\rho a/2S+Z_2^2)^2+(\rho a Z_2/2S)^2]\times[(Z_1+\rho a/2S)^2+Z_2^2]^{-2} \tag{1}$$

wherein P' and P refer to the actual pressure at the junction point of branch and conduit, Z_1 and Z_2 the components of the point impedance of the branch line, $Z=Z_1+iZ_2$, ρ is the density of the medium, a is the velocity of sound and S is the area of the conduit.

It was also shown that, neglecting viscosity, for the case of the Helmholtz resonator, $Z_1=0$ and $Z_2=\rho\omega/c-\rho a^2/V\omega$ in which ω is 2π times the frequency, c is the conductivity of the orifice and V is the volume of the chamber. The above general equation is then reduced to

$$|P'/P|^2=\{1+[4S^2(k/c-1/kV)^2]^{-1}\}^{-1} \tag{2}$$

[1] Stewart, Phys. Rev. **26**, 688 (1925).

The approximation will be made that the effect of the viscosity in the orifice is equivalent to the viscosity in a channel of length L. The equation of motion of the gas in such a channel in which the layer adhering to the wall is small in comparison with the diameter, is[2]

$$\rho \ddot{X} dx + (\tfrac{1}{2}\omega\rho\mu)^{\frac{1}{2}} \quad O/S(\dot{X} + 1/\omega \cdot \ddot{X}) dx = \rho a^2 \partial^2 X / \partial x^2 dx \tag{3}$$

Herein O is the perimeter, S is the area, μ is the coefficient of viscosity, and dx is the element of length of the tube. On the basis of assumption already stated the velocity at a distance from the walls may be regarded as $\dot{X}/S$. For a channel as short as here supposed, $\dot{X}$ may be regarded as the same throughout the length L at any instant, and consequently $\ddot{X}$ and $\partial^2 X/\partial x^2$, which are proportional to X in a tube of constant cross-section, may be also regarded as similarly constant. After integrating over L, dividing by S, and substituting the values of O and S in terms of the radius R, the following is obtained

$$\begin{aligned}[\rho L/S + (2\omega\rho\mu)^{1/2}(\pi R\omega)^{-1} L]\ddot{X} + (2\omega\rho\mu)^{1/2}(\pi R^3)^{-1} L\dot{X} \\ = \rho a^2 \partial^2 X/\partial x^2 \cdot L/S \ .\end{aligned} \tag{4}$$

From the fundamental equations of plane waves, $\rho a^2 \partial X/\partial x = -S\partial p/\partial x$, p being pressure, and consequently the right hand member is the difference in pressure acting on the mass in the channel. This is $P' - \rho a^2 X/V$, if X is the inward displacement into a Helmholtz resonator of volume V for the pressure exerted on the capacitance is $\rho a^2 X/V$. Since $\rho L/S$ is really the inertance of the channel, it should be replaced by ρ/c of the actual orifice. When these substitutions are made and the relationship $\ddot{X} = i\omega\dot{X} = -\omega^2 X$ is applied, there results,

$$\begin{aligned}\{(2\omega\rho\mu)^{1/2}(\pi R^3)^{-1} \cdot L + i[\rho\omega/c - \rho a^2/V\omega + (2\omega\rho\mu)^{1/2} \\ \times(\pi R^3)^{-1} \cdot L]\}\dot{X} = P'\end{aligned} \tag{5}$$

The coefficient of $\dot{X}$ is the impedance of the resonator and consequently we may write at once the values,

$$\begin{aligned} Z_1 &= {}^{/2}((2\omega\rho\mu)^1 \pi R^3)^{-1} \cdot L \ , \\ Z_2 &= \rho\omega/c - \rho a^2/V\omega + (2\omega\rho\mu)^{1/2} \cdot L(\pi R^3)^{-1} \ . \end{aligned} \tag{6}$$

If the values of Eq. (6) be now substituted in Eq. (1), the transmission can be computed for any frequency. Z_1, upon which dissipation obviously depends, vanishes with μ as would be anticipated. Z_2 depends upon the inertance and capacitance of the resonator, and the effect of viscosity is virtually an addition to the inertance or the subtraction from the capacitance of a function varying with ω and μ.

[2] Rayleigh, Theory of Sound, Vol. II, §347, Eq. (10).

The orifice. If there is an orifice without the resonator, the correct expression for the pressure acting on the mass in the channel is no longer $(P'-\rho a^2 X/V)$ but $(P'-\rho(k^2a/2\pi)\dot{X})$ where the second term in the latter[3] is the dissipative force at the opening. Making this substitution in (4) and proceeding as before, there is obtained

$$\{\rho k^2a/2\pi+(2\omega\rho\mu)^{1/2}(\pi R^3)^{-1}\cdot L+i[\rho\omega/c+(2\omega\rho\mu)^{1/2} \times(\pi R^3)^{-1}\cdot L]\}\dot{X}=P' . \tag{7}$$

Thus,

$$Z_1=\rho k^2a/2\pi+(2\omega\rho\mu)^{1/2}(\pi R^3)^{-1}\cdot L ,$$
$$Z_2=\rho\omega/c+(2\omega\rho\mu)^{1/2}(\pi R^3)^{-1}\cdot L . \tag{8}$$

Thus, dissipation depends upon both viscosity and radiation, while the inertance is affected by the presence of viscosity. The values of Eq. (8) may now be substituted in Eq. (1) and the transmission computed for any frequency.

Experimental and theoretical results. The circles on the plot of Fig. 1 show the experimental values of $|P'/P|^2$ as taken by an ear comparison method of intensity measurement[4] which is sufficiently accurate for the present purpose. In computing the theoretical results, the only uncertainty is the selection of the value c, for the conductivity of the orifice. Were it an orifice in an infinite plane wall, we have given the well-known theoretical[5] inferior and superior limits for the corrections to the length of one end of the channel; namely $.785R$ and $.85R$. But it is known that the correction without a flange is distinctly less, approximately $.6R$. The correct value for the present cases of an opening in a tube with or without a chamber, is not known and therefore must be selected. The writer has somewhat arbitrarily chosen the inferior theoretical flange value, $.785R$, making the total correction of length $2\times.785R$ or $\pi R/2$. The error will not be great and can be ascertained by later experimental refinements. The value of c is thus $\pi R^2/(L+\pi R/2)$.

The curve drawn in Fig. 1 has been computed in accord with Eq. (1) with the values of Z_1 and Z_2 given by Eqs. (6). It is found by trial that for any value of L comparable to the length of the orifice, the curve is not altered by assuming $\mu=0$ or by using (2). The dotted curve with ordinates designated on the right shows the computed response of the same Helmholtz resonator in the open with a source of sound producing

[3] Rayleigh, Theory of Sound, Vol. II, §311.

[4] The method used is really an improvement of that discussed in the Phys. Rev. **20,** 528 (1922), and will be described in a later article.

[5] Rayleigh, Theory of Sound, Vol. II, 183.

a pressure P at the resonator when the orifice is closed. If the internal variation of pressure is G, the ordinates represent[6] $|G/P|^2$ and indicate the sharpness of response of such a resonator as ordinarily used.

Fig. 2 is similarly prepared for the case of a small orifice, the chamber being reduced in size also so that the computed minimum transmission without viscosity is at approximately the same frequency as in Fig. 1. Curve a is the theoretical curve omitting viscosity and curve b the computed one using the values given in Eq. (6), assuming $\mu = .00018$ and $L = .09$ cm or six times the actual length of the orifice.

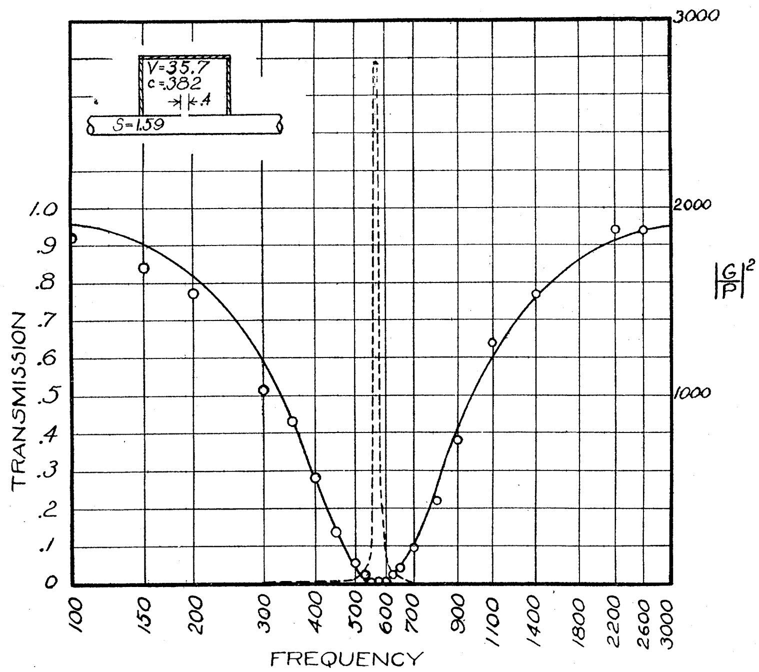

Fig. 1. Helmholtz resonator as a branch.

There are several points of interest in the comparisons. First, it is noticed that the resonator seriously influences transmission for more than an octave in either direction. This is hardly to be anticipated from the sharpness of the action of the resonator as ordinarily used and as indicated by the dotted curve in Fig. 1. Second, the comparison of experiment with theory justifies the assumption of an equivalent channel as a close approximation to the effect of viscosity in the region of the frequency of minimum transmission. This experiment is, in fact, an

[6] Rayleigh, Theory of Sound, Vol. II, §311.

$$|G/P|^2 = \left[(1-k^2V/c)^2 + (k^3V/2\pi)^2\right]^{-1}$$

approximate method of determining the equivalent length of this channel in this frequency region. Computation shows that a change of 10% in the assumed L, will produce a noticeable inferiority of agreement with experiment. Third, the comparison of theory and experiment gives entire confidence in the former. The variations from theory are probably caused by experimental inaccuracies.

Experiments with an orifice are perhaps more interesting because such cases occur frequently. Fig. 3 shows the dimensions of the orifices used, the square root of the observed transmission is indicated by marked points and the computations from theory are represented by a full line curve. The values last named are obtained by substituting the values

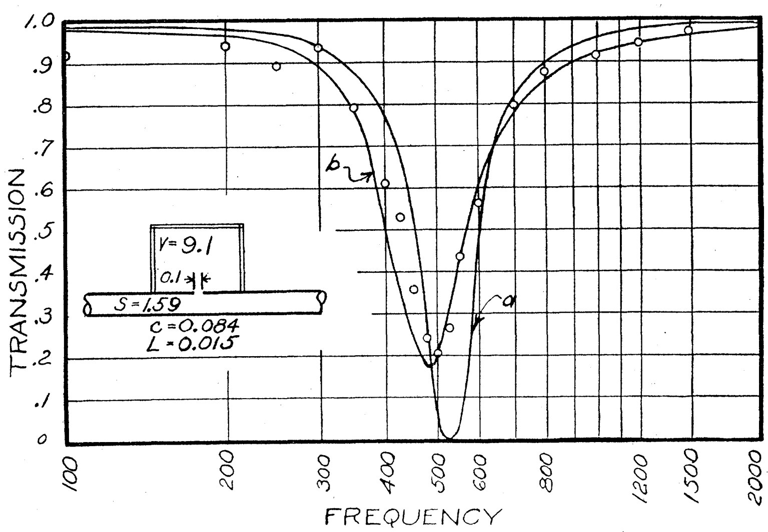

Fig. 2. Effect of viscosity in the orifice.

of Z_1 and Z_2 of Eq. (8) in Eq. (1) but first assuming $\mu = 0$. If the value, $\mu = .00018$, be used, no appreciable change in the curves results, even with the smallest orifice. Curve 4, Fig. 3, does not agree with the observed points and this is doubtless because, as the orifice becomes larger, the value of c is less and less approximately the value for the orifice opening into an infinite plane. If its value is increased arbitrarily to 0.74, the agreement with observations is satisfactory. Obviously there is here suggested a method of measuring c. It should be mentioned that this experimental value of 0.74 proves to give agreement of theory and experiment in the case of a Helmholtz resonator as a branch having the same orifice. In order to test the influence of radiation from the orifice into the open, the value of Z_1, Eq. (8), is placed equal to zero in Eq. (1).

The resulting changes in the lowest curve are shown by the dotted curve in Fig. 4. This is the case of the largest orifice. *The change is surprisingly small.* It should be stated that in the theory the radiation is assumed to occur from a hole in an infinite plane wall, an approximation not unsatisfactory because of the small amount of radiation. Fig. 4 also shows the data and computations at higher frequencies using the largest orifice.

The above comparisons of theory and experiment present several points of interest. First, orifices in a conduit diminish transmission with a magnitude increasing with decreasing frequency. Second, neither

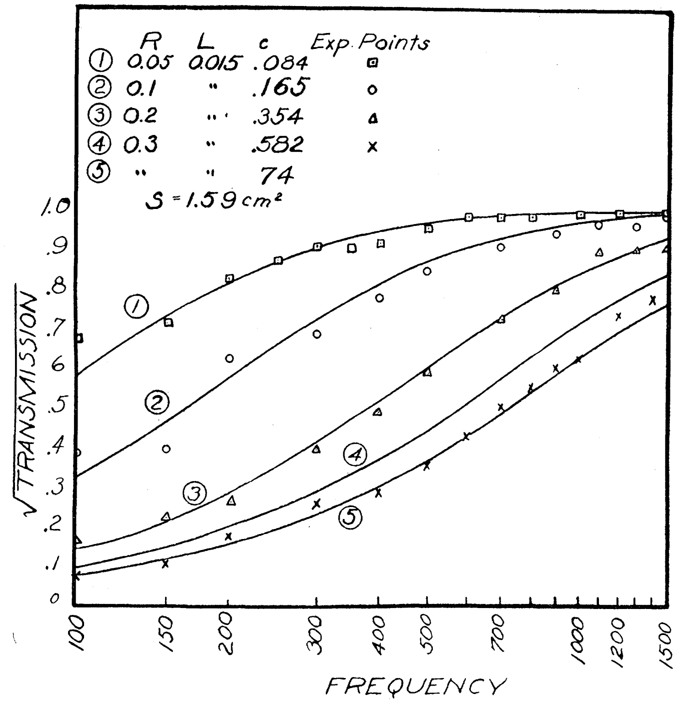

Fig. 3. Orifices as branches of a conduit.

viscosity nor radiation seriously influences the value of transmission in the cases cited. Third, the diminution in transmission is not caused by a loss of energy but by the inertance of the orifice which produces a reflected wave.

While the foregoing refers to a wave transmitted in one direction, the general effect produced in a resonant musical instrument can be understood. Consider the opening of a key on a clarinet. There is a reflected wave from the opening, as if it were an open end of a pipe and this condition, as is well known, enables the *resonant* tone for that pipe length to be established. The intensity of the incident wave becomes very large and consequently the sound escaping from the bell of the clarinet is made sufficiently large. Thus it is noticed that most of the sound escapes from the bell of the clarinet and not from the orifice. This wave reflected from the orifice would be a detriment to the establishment of any tone not in resonance. The radiation from the hole is small in comparison with the transmission along the tube and subsequent radiation from the bell. This phenomenon may be explained in the following terms: the

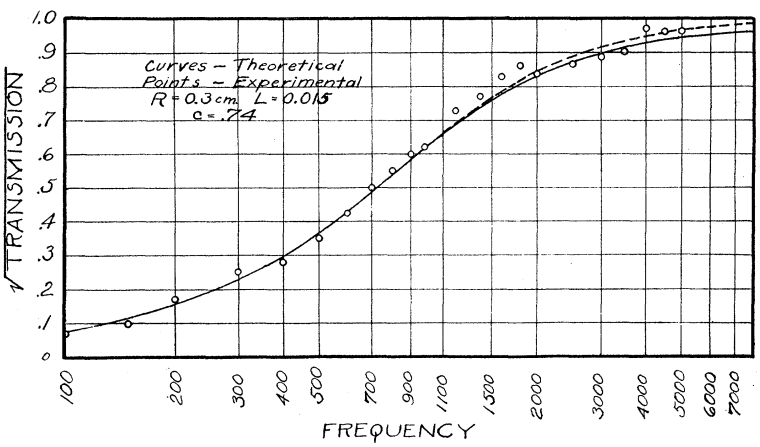

Fig. 4. Difference in transmission caused by radiation.

impedance of an orifice is approximately, but not entirely, imaginary, and hence practically wattless, whereas the impedance of an infinite pipe or conduit is real and has no wattless component. A number of years ago Boehm[7] noted that the transmission along the axis of the flute was much greater than through an orifice having an area as large as the cross-section of the conduit. The above discussion supplies the explanation.

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[7] Theobald Boehm, "On the Flute," translated and published by D. C. Miller. See p. 25.