THEORY OF CURRENT TRANSFERENCE AT THE CATHODE OF AN ARC

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Abstract

The relatively low temperature of the cathode in the case of such metals as copper and mercury, suggests that thermionic emission from the cathode is not essential and that some other factor may be more important in determining the current carried to the cathode in such cases. It appears that if the gas next to the cathode is sufficiently hot, thermal ionization in accordance with Saha's equation may account for much of the current to the cathode. Calculation shows that an upper limit for the required temperature is somewhat in excess of 4000°K for Ca, and 6000°K for Cu.

Introduction

Although it is generally accepted that a cathode hot enough for thermionic emission is a necessary condition for the existence of an arc, a number of observations have been made which cast serious doubt upon the universal necessity of this condition. By an arc, is here meant a self-maintaining discharge with a drop at the cathode very considerably less than the normal cathode drop for the particular cathode material and adjoining gas in question. Thus H. Stolt[1] found that by sufficiently rapid motion of an arc over its cathode, the cathode would give no evidence of having been subjected to a very high temperature, and the author[2] has shown that transition from a glow discharge to an arc may take place while the cathode is still cold.

The case for the necessity of a thermionically active cathode has been very ably presented in a notable paper by K. T. Compton[3] who concludes that the greater part of the current carried at the cathode is by electrons emitted thermionically. However, the most convincing examples which Compton gives are the carbon and tungsten arc, in which because of the refractory properties of the materials, the cathodes may reach such high temperatures that their thermionic emission is sufficient to account for most of the current. For more volatile materials, such as mercury or copper, the thermionic theory meets a very serious difficulty in that it calls for a cathode temperature far in excess of the boiling point of the cathode material, whereas Hagenbach and Langbein,[4] in the case of

[1] Stolt, Zeits. f. Physik **26,** 95 (1924).
[2] Slepian, Jour. of Franklin Inst. (Jan. 1926).
[3] K. T. Compton, Phys. Rev. **21,** 266 (1923).
[4] Hagenbach and Langbein, Arch. Gen. **461,** 329 (1918).

copper, iron, and nickel find by optical methods that the temperatures of the cathodes are actually very near their boiling points.

Thermal Ionization in an Arc, and Theory of Cathode

In the paper by K. T. Compton referred to above, the theory is very convincingly presented that in the positive column of an arc, the ionization is maintained principally by the high temperature existing there. Using Saha's equation[5] Compton calculates the degree of ionization to be expected at 4000°K, and finds it sufficient to account for the observed conductivity in the positive column of the arc.

It appears now that this theory of thermal ionization in the arc may be used also to explain the passage of current to the cathode, so that it is no longer necessary for the cathode to have a large electron emission or to be heated to a higher temperature than its boiling point. In fact, the thermal ionization in the gas, with reasonable assumptions as to temperature, is such that the arc current may be carried to the cathode by thermally generated positive ions.

Quantitative Test of Theory of Cathode

To be sound, this theory of the cathode must meet certain quantitative tests. First, the observed cathode drop must be sufficient to carry the necessary current density of positive ions to the cathode. Second, the product of cathode drop and current density must more than make up for the heat lost from the hot gas to the relatively cold cathode.

Langmuir[6] has shown that when a plane electrode in contact with an ionized gas is given a sufficiently negative potential to repel all electrons, the current density flowing to the electrode and carried entirely by positive ions will be independent of the potential and equal to the random positive ion current density in the gas. This random current density, I, is defined as the current corresponding to the number of positive ions per second which pass in one direction through a square centimeter of an imaginary plane in the body of the gas. According to Langmuir,

$$I=\tfrac{1}{4}\, n\, v\, e \qquad (1)$$

where n is the number of positive ions per cm^3, v is the average translational velocity of the thermal agitation of the ions, and e is the electronic charge.

The theory of the cathode here being developed, then, clearly requires that I given by (1) shall also be the current density at the cathode.

[5] Saha, Phil. Mag. **40**, 472 (1920).
[6] Langmuir, G. E. Rev. **27**, 449 (1924).

If n_1 is number of molecules per cm^3 in a gas at temperature T, then

$$n = n_1 \cdot x/(1+x) \tag{2}$$

where x is the fraction of molecules thermally ionized and is given by Saha's equation for atmospheric pressure,

$$\log \frac{x^2}{1-x^2} = -\frac{5050\ V_i}{T} + 2.5 \log T - 6.69 \tag{3}$$

V_i being the ionizing potential of the gas in volts.

For the velocity of agitation v of the ions, we have

$$\frac{1}{2}\frac{M}{N}v^2 = \frac{1}{2}kT\ , \tag{4}$$

M being the molecular weight of the ion, N the number of molecules in a gram molecule of substance, and k Boltzman's constant. Also

$$n_1 = n_0 \cdot 273/T \tag{5}$$

where n_0 is the number of molecules per cm^3 of a gas at 273°K. Substituting (2), (4) and (5) in (1), we obtain

$$I = \frac{273}{4} n_0 e \frac{x}{1+x}\sqrt{\frac{Nk}{MT}} \tag{6}$$

For calcium we have $M=40$, $V_1=6.01$, and for copper $M=63.5$ and $V_i=7.69$. Substituting these values in (3) and (6), and taking $n_0=2.70 \times 10^{19}$, $e=4.77\times 10^{-10}$, $N=6.06\times 10^{23}$, $k=1.37\times 10^{-16}$ the curves of Fig. 1 have been calculated.

Now the current density at the cathode of an arc is of the order of 100 amperes per cm^2 at atmospheric pressure. Examining Fig. 1, we see that for calcium and copper the temperatures of the vapors need to be from 4000° to 5000°, and 6000° to 7000° respectively for the positive ion density to be sufficient to carry the current to the cathode.

These temperatures are rather higher than those determined experimentally, but two considerations will serve to make these figures acceptable. First of all, this high temperature needs to exist only in a thin layer of gas next to the cathode and will therefore be very difficult to detect experimentally. Secondly, Saha's equation, which is the basis of this calculation, was derived on the supposition of thermodynamic equilibrium. But an ionized gas in an electric field is not in thermodynamic equilibrium, since the electrons take energy from the electric field and maintain themselves with a greater energy of agitation than that of the neutral molecules. Thus Langmuir[7] finds that in a mercury arc the electrons have a temperature of over 10,000°K in a gas of less than 400°K. For a gas in an intense electric field therefore, the temperature (of the neutral molecules) necessary for a given degree of ionization will be much less than that called for by Saha's equation.

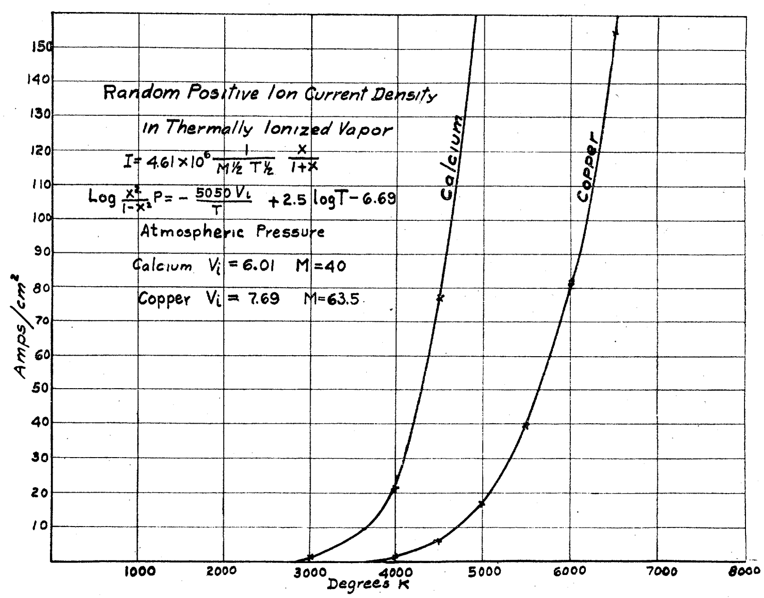

Fig. 1. Calculated variation of the temperature of the thermally ionized vapor with current density.

The electrode receiving the current of positive ions, as Langmuir has shown, is separated from the main body of ionized gas by a sheath containing only positive ions and neutral molecules, the electrons being repelled out of this sheath by the electric field. The thickness of this sheath varies with the applied potential, the applied potential being balanced by the space charge in the sheath. Langmuir gives the equation

$$I = \frac{2.336\times10^{-6}}{\sqrt{1834M}} \frac{1}{x^2} v^{3/2} \left(1 + .0247\sqrt{\frac{T}{V}}\right) \tag{7}$$

where V is the applied potential in volts, x the sheath thickness in cm, I the current density in amperes/cm², M the molecular weight of the ions, and T the absolute temperature.

[7] Langmuir, G. E. Rev. **27**, 544 (1924).

If we take $I=100$, $V=15$, approximately the cathode drop for copper, $M=63.5$, and $T=6000°$, we find for the sheath thickness,

$$x=7.7\times10^{-4}\text{ cm} . \tag{8}$$

Within this sheath it is not necessary for the gas to be thermally ionized, but at the edge of the sheath the random positive ion current must have the full value of 100 amperes per cm². The vapor then must have a temperature of 6000° within 7.7×10^{-4} cm from the cathode.

Since this separation from the cathode of 7.7×10^{-4} cm is comparable with the mean free path in the gas at 6000°, the rate of heat flow across the space into the cathode can not be calculated from the thermal conductivity of the vapor in bulk, but an upper limit to this heat flow may be determined from the kinetic theory. If we assume that all the molecules which enter the positive ion sheath reach the cathode and there give up all the energy which they have in excess of that corresponding to the cathode temperature, the heat thus calculated as given up to the cathode will be in error by being too large, inasmuch as collisions with molecules of gas in the positive ion sheath, and partially elastic collisions with the cathode are not allowed for. Letting n_1 be the number of molecules per cm³ at temperature T at the edge of the sheath remote from the cathode, the number of molecules which reach the cathode per second will be approximately

$$n_2=\tfrac{1}{4}n_1v_1 \tag{9}$$

v_1 being the mean translational velocity of the molecules, and since each molecule gives up the energy $3/2k(T-T_0)$, T_0 being the temperature of the cathode, we have as an upper limit for the heat lost to the cathode per second,

$$W=\tfrac{3}{8}n_1v_1k(T-T_0) \tag{10}$$

Letting $n_1=7.38\times10^{21}$, $v_1=1.14\times10^{3}T^{1/2}$ (copper), $k=1.37\times10^{-16}$, $T=6000°$, $T_0=2580°$ (boiling point of copper) we have in watts

$$W=1770\text{ watts} \tag{11}$$

If the cathode drop is 15 volts and the current density is 100 amperes per cm², the electrical input is 1500 watts. A large part, very likely the larger part, of this electrical input is communicated by bombarding positive ions directly to the cathode as heat, but not all of it, inasmuch as many of these positive ions will be reflected with a large part of their oncoming velocity, and many will collide with gas molecules in the space of 7.7×10^{-4} cm of Eq. (8). Now remembering that (11) is much too high for reasons given, we see that the electrical input is quite sufficient to make up for the heat lost to the cathode.

Conclusion and Summary

The theory of thermal ionization in the electric arc, as advanced by Compton to explain the conductivity in the positive column, can also account for the passage of current to the cathode without requiring thermionic emission from the cathode. Calculations made for the copper arc, using equations of Langmuir and Saha, show that with a reasonable assumption of temperature of the vapor, the ionization is sufficient for the positive ions to carry the whole current to the cathode. The heat lost to the cathode by the hot vapor is of such an order that it can be made up for by the electrical input at the cathode.

A hot cathode is therefore not necessary for an arc. High temperature appears to be essential, but it may be in the gas immediately adjacent to the cathode, and need not be in the cathode itself.

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