MARCH, 1926 *PHYSICAL REVIEW* *VOLUME 27*

THE ABSORPTION OF X-RAYS OF WAVE-LENGTH DOWN TO .08A

BY S. J. M. ALLEN

ABSTRACT

Mass absorption coffiecients of C, paraffine, S and 16 metal elements from Al to U for wave-lengths .56 to .08A have been measured in continuation of previous work[1] which covered most of these elements in the range .71 to .10A. The general radiation from a tungsten target (operated at voltages up to 200 kv) was dispersed by a crystal into an ionization chamber. By using Soller multiple slit collimators about ten times as great intensity was obtained as with a single slit of the same resolving power. The main results of the previous paper were confirmed and extended, especially the steady decrease in the "jump" of the absorption in going from the L to the K regions, as the atomic number increases. The scattering of paraffine wax is distinctly greater than that of C or Al, probably due to the richness of H in paraffine. The empirical formula $\mu/\rho = C\lambda^{2.92}N^4/A + \sigma/\rho$, applied to the experimental results gives values of σ/ρ which increase in general with atomic number, becoming about 1 for the heavier elements, W to Bi. The values of C obtained are .0132 for the K series and .00181 for the L series; these give calculated values which agree within 5 percent with the experimental. *Experiments with end-filtered high voltage rays.* Without collimators or crystal, the ionization chamber in the path of the direct beam of rays filtered through 1 cm tin and 2 cm aluminum, showed increased ionization when a carbon block was interposed, but when two Soller collimators were used with planes crossed so as to eliminate all oblique rays, normal absorption coefficients were obtained corresponding to .075A. This method may be useful for the shortest rays.

IN a previous paper[1] the author gave results of experiments on the absorption of homogeneous x-rays from $\lambda = .71$ to .10A following the standard method of reflecting the general radiation of a Coolidge tube from a crystal, and intercepting a narrow portion of the spectrum with an ionization chamber attached to the spectrometer.

It was shown that as the wave-length approached .10A, the K absorption coefficients of the high weight elements were decreasing steadily according to the laws of fluorescent absorption, but that in the case of the light elements fluorescent absorption being practically gone, further decrease was very slow. Also, that in the critical absorption regions of the elements the "jump" of the absorption coefficients from the L series to the K series became steadily less with increasing atomic number. This was especially marked in the case of the elements from W to Bi, with

---

[1] Allen, Phys. Rev. **24,** 1 (July 1924).

the result that the K absorption coefficients of these elements were nearly the same.

It was further shown that the Bragg-Owens' empirical absorption law, $\mu/\rho = C\lambda^3 N^4/A + \sigma/\mu$ was no longer able to predict values of μ/ρ for the high weight elements which agreed with the experimental values obtained. In all the discussion which follows in this paper μ/ρ will represent the experimentally determined mass absorption coefficient, τ/ρ the fluorescent absorption coefficient of the electronic orbits, (K, L, M, etc.), and σ/ρ the remainder of the absorbed energy, which may or may not be entirely due to pure scattering.

The present paper is a continuation of the above work, and carries the wave-length down to .08A. While these experiments were carried out in the same general manner as the former ones, the technique of operation was so different as to necessitate a brief description.

The source of the rays, up to a potential of 120 kv was a Coolidge Universal tube,and up to 200 kv a Coolidge high tension tube. The tube, generator, and all high tension connections were placed by themselves in a closed room, completely shut off from the adjacent room, where the spectrometer was placed, by a brick wall two feet thick and lined with lead. A hole was pierced through the wall and two lead slits 1 cm square placed at each end, defined the beam of rays from the tube.

In place of the usual single slit method, the multiple slit collimators, devised in my laboratory by Soller[1] were employed. The first collimator was placed directly in front of the crystal with its axis passing through the axis of rotation of the spectrometer and perpendicular to it. The second collimator was attached to the ionization chamber, and in the zero position its axis was coincident with that of the first. Each collimator was 8 cms long, 1 cm wide, 1 cm high and had slits about .2 mm wide.

The general radiation passing through the first collimator emerged as a number of thin beams parallel to each other in a vertical plane. This beam falling upon a calcite crystal 2 cms thick passed through it, the reflected radiation being obtained from the interior 100 planes. The zero position having been determined, the ionization chamber was swung around to some angle 2θ. The crystal was now rocked back and forth until maximum radiation was received, when it was assumed that radiation of wave-lengths between λ and $\lambda + d\lambda$ alone passed through the second collimator. The mean wave-length λ was calculated from the relation, $\lambda = 2d \sin \theta$.

---

[1] Soller, Phys. Rev. **24,** 158 (1924).

The resolving power was quite equal to that obtained with single slits of .1 to .2 mm.

It was found that this method gave about ten times the intensity that could be obtained with single slits at the same resolving power. Another great advantage of this method lies in the fact that the zero position once having been obtained does not change with penetration of the rays into the crystal, δ being obtained solely from 2θ the angle between the axes of the two collimators.

The rest of the experiment was carried out in the usual manner, the absorbing layers being placed on the side of the wall next to the tube, thus ensuring that no scattered radiation from these layers could enter the ionization chamber.

Although rays reflected from the crystal at values of θ less than that corresponding to $\lambda = .081$A were observed, difficulties of resolution were encountered which rendered them somewhat uncertain, and so results are here given only down to $\lambda = .081$A. It is hoped that a redesign of

TABLE IA

Mass absorption coefficients μ/ρ

λ in A	C	Paraffine	Al	S	Fe	Ni	Cu	Zn	Pd
.081	.143	.156	.145	.155	.235	.265	.270	.310	.73
.085	.145	.158	.153	.170	.242	—	.280	—	.80
.090	.148	.160	.160	.182	.250	.290	.295	.350	.86.
.095	.150	.165	.164	.186	.262	—	.305	—	—
.102	.150	.171	.169	.192	.280	.337	.335	.390	1.11
.113	—	—	—	—	.317	—	.405	—	—
.118	.151	.175	.172	.196	.360	—	.440	.520	—
.126	—	—	—	—	.410	.480	.510	—	—
.135	—	—	—	—	.460	—	.600	.690	—
.140	.152	.180	.195	.250	.500	—	.650	—	—
.145	—	—	—	—	.540	—	.710	.820	—
.148	—	—	—	—	—	—	—	—	—
.151	.153	—	—	—	.595	.690	.780	—	—
.160	—	.185	.212	.295	.660	—	.900	1.02	—
.164	—	—	—	—	—	—	—	—	—
.173	.155	.190	.235	.330	.780	1.00	1.09	1.24	—
.185	—	—	.249	.360	.90	1.18	1.30	1.47	—
.194	—	—	.265	—	1.00	—	—	—	—
.209	.166	.196	.295	.440	1.26	1.63	1.71	2.01	—
.220	—	—	.310	.500	1.40	—	2.00	2.32	—
.240	.170	.200	.356	.580	1.75	2.28	2.50	2.80	—
*.280	—	—	.475	.800	2.75	3.38	3.70	4.30	—
*.320	.200	.220	.630	1.08	3.95	4.87	5.25	6.20	—
*.340	—	—	.730	1.23	4.65	5.75	6.38	7.30	—
*.400	.245	.245	1.11	1.78	7.25	—	8.80	11.6	—
.430	—	—	1.33	2.10	—	—	12.5	14.5	—
.458	.288	—	1.54	—	—	—	14.7	17.2	—
.511	—	.34	2.06	—	—	—	20.5	24.0	—
.560	.40	.36	2.65	—	—	—	26.5	30.8	—
*.631	.55	—	3.78	6.9	—	—	37.8	43.0	—
*.710	.68	—	5.35	9.9	—	—	53.7	60.0	—

* Taken from previous paper.

the apparatus will in the future enable me to obtain results for shorter wave-lengths.

In Tables IA and IB are tabulated results of this work, and also a number of the results of the preceding work, which are indicated by asterisks. The vertical double arrows indicate the regions of critical absorption, the upper limit being the highest K values, and the lower the lowest L values. In the cases of Ba, Th, and U, the values given are the values which probably would have been obtained if these substances had been in the pure state. They were calculated assuming the additive law. Paraffine was in the form of paraffine wax.

TABLE IB

Mass absorption coefficients μ/ρ

λ in A	Ag	Sn	Ba	W	Pt	Au	Pb	Bi	Th	U
.081	.74	.80	—	2.40	2.50	2.44	2.53	2.50	—	—
.085	.83	.88	—	2.60	2.75	2.65	2.78	2.76	—	—
.090	.90	.95	—	2.80	2.95	2.85	3.00	3.00	—	—
.095	1.02	1.02	—	3.10	3.38	3.30	3.45	3.45	—	3.53
.102	1.17	1.20	—	3.50	3.80	3.76	3.90	3.90	4.00	↕
.113	—	1.52	—	4.30	4.50	4.48	4.70	4.70	↑	1.78
.118	1.57	1.77	1.95	4.75	5.00	4.90	5.13	4.90	↓	—
.126	1.68	2.05	—	5.35	5.70	5.50	5.23	↑	1.86	1.90
.135	—	2.40	—	6.25	6.55	6.40	↑		2.10	—
.140	2.40	2.62	2.80	6.75	7.08	6.90		↓	2.25	2.56
.145	—	2.90	—	7.30	7.55	7.40		2.25	2.48	—
.148	—	—	—	7.75	7.70	↑	↓	—	—	—
.151	2.65	3.15	3.40	8.00	↑		2.45	2.48	2.72	3.78
.160	—	3.60	4.00	8.90		↓	2.70	2.90	3.10	—
.164	—	—	—	↑	↓	2.70	—	—	—	3.24
.173	3.75	4.32	4.90		2.90	3.02	3.25	3.50	3.75	—
.185	4.40	5.05	5.80	↓	3.48	3.55	3.85	4.14	4.45	—
.194	5.00	—	7.0	3.20	3.90	—	4.40	—	5.05	—
.209	6.50	6.95	8.2	3.93	4.70	4.87	5.35	5.60	6.10	—
.220	7.40	8.30	9.2	4.20	5.25	5.50	5.90	6.48	6.90	—
.240	9.60	10.80	11.7	5.10	6.65	6.95	7.40	8.30	8.60	—
*.280	14.8	15.5	17.2	7.20	9.60	—	11.5	12.5	—	—
*.320	21.1	22.0	24.2	10.1	13.5	—	16.2	17.7	—	—
*.340	24.5	—	6.0	12.0	15.8	—	19.7	21.0	—	—
*.400	38.2	—	9.0	19.8	24.5	—	31.8	—	—	—
		↕								
.430	46.5	8.0	11.5	24.7	30.0	—	39.0	—	—	—
.458	60.0	9.0	13.7	29.7	35.5	—	46.0	—	—	—
	↕									
.511	10.0	12.8	19.0	—	—	—	—	—	—	—
.560	15.0	16.5	—	—	—	—	—	—	—	—
*.631	20.5	—	—	75	87	—	101—	—	—	—
*.710	28.5	—	—	—	119	—	140	—	—	—

* Taken from previous paper.

Table II gives the values of the critical absorption wave-lengths as obtained from the data of Tables IA and IB. These values agree in general to within one percent with those published by other observers.

A detailed study of the absorption coefficients of Tables IA and IB shows that they are of the same character as those of the preceding work. The "jumps" in the values of τ/ρ from L series to K series steadily decrease with increasing atomic number, that for U being about 4. The K absorption coefficients from W to U are nearly all the same, for the same wave-lengths, and it would require an accuracy of one percent or better to decide whether there is any real difference. The decrease in μ/ρ with λ is about the same for all. The intermediate elements, such as Ag, Sn and Ba, show values of σ/ρ which decrease much more slowly with λ, indicating that σ/ρ which decreases slowly is becoming of increasing importance in comparison with τ/ρ. The values of μ/ρ for the light elements as $\lambda=.08$A is approached show such small rates of change, that for these elements τ/ρ is fast disappearing as an important factor, and σ/ρ is now the predominating factor. In the case of C the change in μ/ρ from $\lambda=.160$ to $\lambda=.081$A is only a few percent, and even this small rate of decrease is becoming smaller as we approach .08A. The same is true of paraffine, Al and S. μ/ρ for paraffine wax at $\lambda=.081$A is distinctly greater than that of C or Al, whilst at $\lambda=.40$A it is equal to that of C and much less than that of Al. This can be probably explained by assuming that σ/ρ for H is greater than that for C, for since paraffine wax is rich in H, it will show correspondingly larger values of σ/ρ and smaller values of τ/ρ than C.

TABLE II

Critical absorption wave-lengths λ_c

Element:	U	Th	Bi	Pb	Au	Pt	W	Ba	Ag
λ_c in A:	.105	.113	.132	.139	.154	.159	.178	.330	.484

The same phenomenon has been found for substances rich in H by other observers at longer wave-lengths.

There is no evidence that σ/ρ is experiencing any sudden or rapid decrease with λ.

If we are justified in concluding from the above results that the true mass scattering coefficient for light elements will continue to decrease at such a slow rate it is difficult to see how it can arrive for $\lambda=.02$A at such a low value for C as .056 for the filtered γ rays of radium C, which the recent work of Ahmad,[2] Ahmad and Stoner,[3] and that of Owen, Fleming and Fage[4] would seem to indicate. It is possible that there estimations of λ are much too high.

---

[2] Ahmad, Proc. Roy. Soc. **A105,** 507 (1924).

[3] Ahmad and Stoner, Proc. Roy. Soc. **A106,** 8 (1924).

[4] Owen, Fleming and Fage, Phys. Soc. Proc. **36,** 355 (August, 1924).

In Table III are expressed some estimated values of, σ/ρ, calculated from the data of Tables IA and IB by aid of the formula, $\sigma/\rho=\mu/\rho-k\lambda^{2.92}$. If this formula holds for these short wave-lengths for both the K series (excluding elements W to U) and the L series, the results indicate that σ/ρ increases with N, reaching in the L series of the heavy elements values possibly as high as 1.0. Similar results are obtained if we use λ^3, only they are larger.

TABLE III

Mass scattering coefficients, $\sigma/\rho=\mu/\rho-k\lambda^{2.92}$

Element	λ.08A	λ.120A	λ.210A	λ.320A	λ.440A
		K series			
Carbon	.142	.148	.151	.148	.145
Paraffine	.155	.155	.184	.180	.169
Aluminum	.136	.144	.150	.130	.140
Sulfur	.138	.144	.170	.150	—
Iron	.165	.160	.200	.21	.25
Copper	.175	.160	.24	.03	—
Zinc	.195	—	—	—	—
Silver	.370	.43	.50	.3	—
Tin	.380	.45	—	—	—
Barium	—	.51	.70	—	—
		L series			
Tungsten	.9	.75	1.0	—	—
Platinum					
Gold	to	1.0	1.2		
Lead		1.1	1.1		
Bismuth	1.0	.9	1.0		

Now, if σ/ρ, which here merely means the rest of the energy lost after fluorescent absorption has been allowed for, is completely due to pure scattering, then it should decrease in general with increase of N, being proportional to N/A in any of the theoretical formulas of, Thomson, Compton, or Jauncey.

It is evident that either one cannot use these empirical formulas to predict the values of τ/ρ at these short wave-lengths, or, if one can, then the values of σ/ρ so obtained are much larger in many cases than the theoretical formulas of pure scattering would indicate.

It is also possible that the experimental values of σ/ρ are not wholly due to pure scattering, but may involve some form of absorption, either true absorption, or scattering, or both, not associated with the K, L, etc. orbits, such as perhaps nuclear absorption.

It is important to call attention to the fact that Barkla and his associates in several papers on relative scattering state that the relative scattering of heavy elements to light elements increases rapidly with N, and also with λ.

Also Debye[5] from theoretical considerations predicts, that in the case of scattering from atoms where the electrons are not free but concentrated near the center, the atomic scattering coefficient would be proportional to a higher power of N than the first, in the limiting case (electrons at the center) being proportional to N^2. For mass scattering this would give σ/ρ proportional to $N^2/2$ instead of N/A.

Until one has obtained by a direct method true values of σ/ρ, it may not seem very fruitful to try to distinguish, in the region of short wave-lengths, between the various empirical absorption formulas. The generally accepted one is

$$\left.\begin{array}{lr} \mu/\rho = C\lambda^3 N^4/A + \sigma/\rho \ldots\ldots\ldots\ldots\ldots & \text{(mass)} \\ \mu_a = C_a\lambda^3 N^4 + \sigma_a \ldots\ldots\ldots\ldots\ldots\ldots & \text{(atomic)} \end{array}\right\}(1).$$

The author has proposed the following—

$$\left.\begin{array}{lr} \mu/\rho = C\lambda^{2.92} N^4/A + \sigma/\rho \ldots\ldots\ldots\ldots\ldots & \text{(mass)} \\ \mu_a = C_a\lambda^{2.92} N^4 + \sigma_a \ldots\ldots\ldots\ldots\ldots\ldots & \text{(atomic)} \end{array}\right\}(2)$$

Within the limits in which these formulas may be used to predict results to about five percent, I have, from the data of these two papers, and from those given by Bragg and Pierce,[6] Hewlett,[7] Richtmyer,[8] Windgarth,[9] and Stoner and Martin,[10] calculated the best values of C and C_a. These are expressed in Table IV.

TABLE IV

Values of C and C_a

	K series		L series	
	Eqs. (1)	Eqs. (2)	Eqs. (1)	Eqs. (2)
C	1.36×10^{-2}	1.32×10^{-2}	1.86×10^{-3}	1.81×10^{-3}
C_a	2.24×10^{-26}	2.19×10^{-26}	3.06×10^{-27}	3.00×10^{-27}

The experimental data of Tables 1A and IB discussed above are also illustrated by the graphs of Figs. 1, 2, 3, and 4. Figs. 1 and 2 show the relations between μ/ρ and $\lambda^{2.92}$. Figs. 3 and 4 show the relations between μ/ρ and N^4/A. Values of J from the graphs of 1 and 2 are tabulated. Also values of C_{K} and C_{L} are tabulated from Figs. 3 and 4.

In order to ascertain what reliance could be placed upon "end" filtered rays at high voltage, the following experiments were made at 200 kv.

---

[5] Debye, Ann. d. Physik **46**, 809 (1915).

[6] Bragg and Pierce, Phil. Mag. **8**, 608 (1914).

[7] Hewlett, Phys. Rev. **17**, 284 (1921).

[8] Richtmyer, Phys. Rev. **18**, 13 (1921).

[9] Windgarth, Car Bloms Boktrycheri, Lund (1923).

[10] Stoner and Martin, Proc. Roy. Soc. **A107**, 1925.

The two collimators being removed temporarily, and the ionization chamber turned to the zero position, a filter of 1 cm tin and 2 cms aluminum was placed next to the tube. A reading of the electroscope was then taken in arbitrary units, say 100. A block of carbon was then

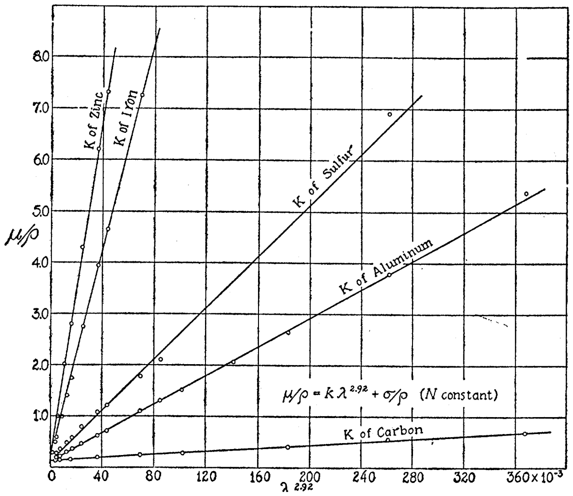


Fig. 1. Variation of μ/ρ with $\lambda^{2.92}$.

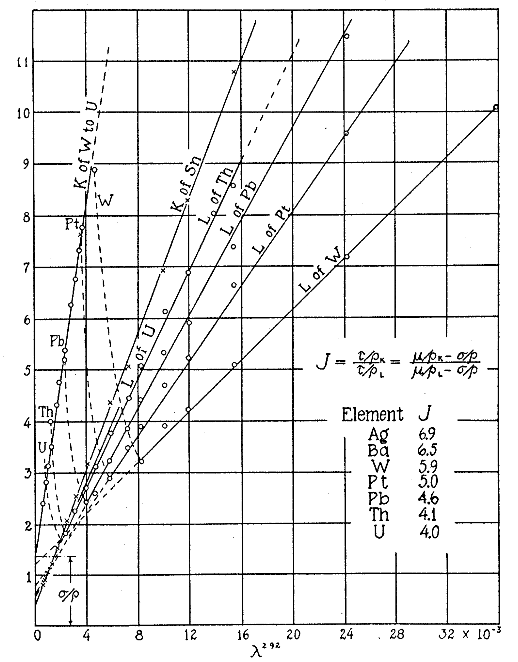


Fig. 2. Variation of μ/ρ with $\lambda^{2.92}$.

placed directly behind the filter to intercept the rays before they passed through the wall. The somewhat surprising result was now obtained that the ionization was increased to 300 instead of being decreased.

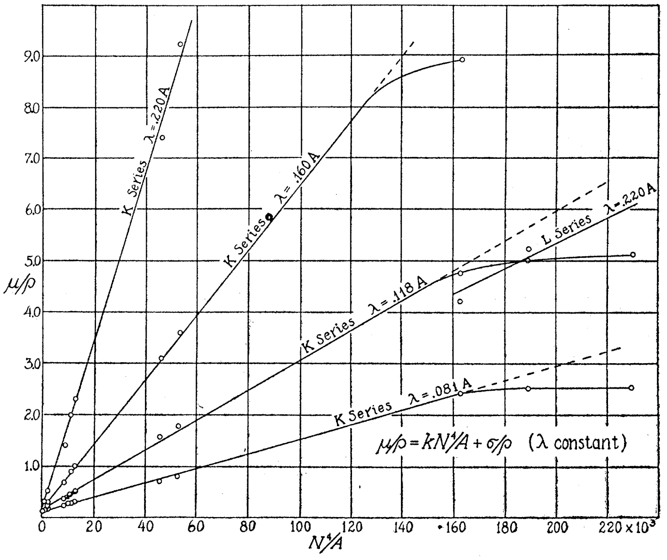
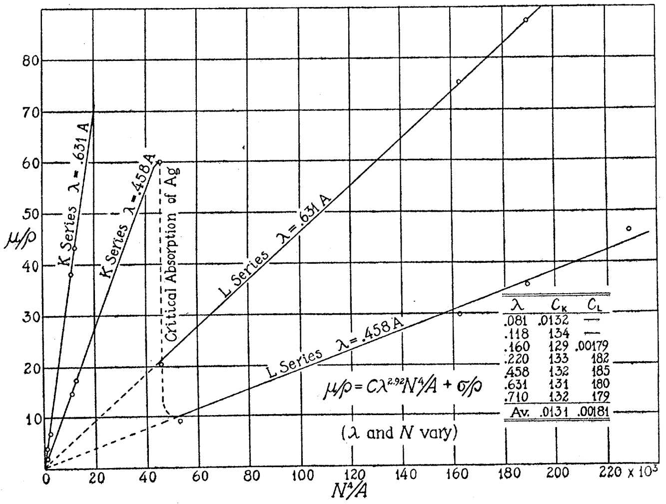


Fig. 3. Variation of μ/ρ with N^4/A.



λ	C_K	C_L
.081	.0132	—
.118	134	—
.160	129	.00179
.220	133	182
.458	132	185
.631	131	180
.710	132	179
Av.	.0131	.00181

Fig. 4. Variation of μ/ρ with N^4/A.

The same result was obtained with other elements, but in a lesser degree for the heavier elements. It is thus seen that more rays were scattered in the direction of the direct beam, than were scattered away. Under these conditions absorption coefficients would have no true significance.

Next the collimators were placed in position, with axes coincident but with their planes crossed at right angles. Readings taken now, show normal results, and it makes no difference where the absorbing layer is placed. Only those rays which pass straight forward can enter the ionization chambers, all oblique rays in both horizontal and vertical directions being eliminated.

In Table V are given the results of this last experiment. The effective wave-length viz. $\lambda = .075$A was calculated from μ/ρ for Ag and Sn. The values in this table are in good conformity with those obtained by the crystal method at $\lambda = .08$A.

TABLE V

X-rays filtered through Sn, voltage 200,000, $\lambda_{eff.} = .075$ A

Element:	C	Par.	Al	S	Fe	Ni	Cu	Zn	Ag	Sn	Pt	Au	Pb	Bi
μ/ρ	.140	.145	.147	.155	.22	.24	.25	.28	.69	.72	2.02	2.12	2.33	2.42

Since at these high voltages the energy of the spectrum is largely concentrated in the short wave-lengths, "end" filtering with crossed collimators will give results of some importance, when one is no longer able to obtain homogeneous rays from crystal reflection, at least in those elements which are far removed from their critical absorption regions.

Absorption coefficients obtained in the past from filtered "end" rays which have seemed to indicate very small values of μ/ρ at high voltages are of doubtful value, unless precautions similar in effect to those discussed above were taken.

In conclusion it may be of advantage briefly to review the chief points brought out by the experimental results of these two papers.

A careful analysis of the absorption coefficients of the elements from C to U for values of λ between .71 and .08A, shows quite clearly that no general formula of absorption of the Bragg-Owens type can predict values which agree with experimental ones in both the L and K series by simply changing the constant. In the L series it will predict values increasingly too low, and in the K series increasingly too high, as we proceed to the high atomic weight elements. The experimental fact, that the decreasing jumps in the absorption coefficients from the L series to the K series, which results in the coefficients of the elements from W to U being nearly the same for the same wave-lengths, will explain the failure in the case of the K series.

In the case of the L series, the difference σ/ρ between the experimental value of μ/ρ and the calculated value of τ/ρ is too great to be entirely accounted for by pure scattering according to the theories of Thomson, Compton, and Jauncey. It therefore seems necessary to raise the question whether in addition to fluorescent absorption and pure scattering there may exist nuclear absorption of some kind, which in the case of high weight atoms is great enough to be experimentally distinguished.

In the case of light atoms where fluorescent absorption has practically disappeared, the experimental results show that the absorption is decreasing with λ at a very slow rate.

Until one is able to determine in a region of wave-lengths where τ/ρ is very large in comparison with σ/ρ, the exact variation of τ/ρ with λ, the method of this paper in estimating σ/ρ may be open to some question. The author is at present engaged in measuring the absorption coefficients of wave-lengths greater than 1A and it is hoped that the results of this investigation in the near future may throw more light on the problem.

UNIVERSITY OF CINCINNATI,
November 3, 1925.