

MEASUREMENT OF THE REFRACTION OF X-RAYS  
IN A PRISM BY MEANS OF THE DOUBLE  
X-RAY SPECTROMETER

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## ABSTRACT

In the double spectrometer used, x-radiation was reflected from a crystal surface  $A$  through a slit  $S$  on to a second crystal surface  $B$ , and thence to the ionization chamber. Surfaces  $A$  and  $B$  were obtained by splitting a single crystal of calcite. The curve obtained for Mo  $K\alpha$  by rocking  $B$  is a triangle with base about 8 seconds wide. The effect of interposing a *prism of aluminum* (angle  $166^\circ$ ) between  $S$  and  $B$  was to shift the curve  $5.62''$  on the average, giving  $1-\mu=1.68\times 10^{-6}$ . The value calculated from the Lorentz dispersion formula is  $1.77\times 10^{-6}$ . For Cu  $K\alpha$  the shift was  $5.53''$  for a  $116^\circ$  prism, giving  $1-\mu=8.4\times 10^{-6}$  as compared with the calculated value  $8.36\times 10^{-6}$ . The accuracy of this method is thus within 5 percent.

**E**VIDENCE of the direct refraction of x-rays on passing through a prism was long sought for without success. The effect is so minute that its direct detection is difficult.

Recently, however, the direct bending in a prism has been measured by the photographic spectrometer method in the laboratory of Professor Siegbahn.<sup>1</sup> Shortly afterward the direct refraction in a prism was measured by the authors,<sup>2</sup> using the single ionization spectrometer. A serious defect of the single ionization spectrometer for this purpose arises from the fact that the maxima of the energy curves are shifted not only by refraction but by the unsymmetrical absorption of the beam in the prism.

Refraction at the surface of a crystal in the case of crystal reflection had already been quite accurately measured by Bergen Davis and R. von Nardroff<sup>3</sup> and by C. C. Hatley,<sup>4</sup> using the ionization spectrometer.

The present paper describes the application of the *double x-ray spectrometer* to the measurement of refraction in a prism. This method is capable of considerable accuracy. A bending of the beam of x-rays as small as one-tenth second of arc can be observed when conditions are favorable.

<sup>1</sup> Larsen, Siegbahn and Waller, Proc. Am. Phys. Soc., Phys. Rev. **25**, 235 (Feb. 1925)

<sup>2</sup> Bergen Davis and C. M. Slack, Proc. Am. Phys. Soc., Phys. Rev. **25**, 881 (June 1925).

<sup>3</sup> Bergen Davis and R. von Nardroff, Proc. Nat. Acad. Sc., **10**, (Feb. 1924) also (Sept. 1924).

<sup>4</sup> C. C. Hatley, Phys. Rev. **24**, 486 (Nov. 1924).

The method will be at once clear by reference to Fig. 1. *A* and *B* are two crystals of split Iceland-spar (calcite). The two reflecting surfaces are the divided halves of the same crystal. The surface of *B* on which the rays fall is the surface that coincided with the reflecting surface of *A* before cleavage. Care was taken to mount the crystals without abrading the reflecting surfaces in any way. The percent reflection and still more important, the narrowness of the "rocking" curves, depends on the virgin condition of these surfaces.<sup>5</sup>

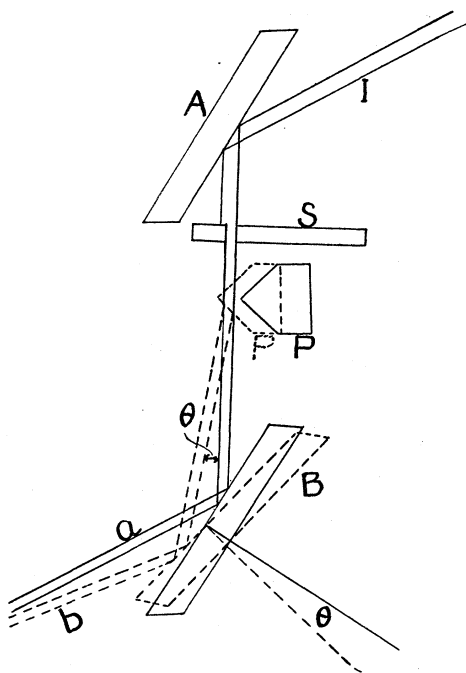


Fig. 1. The double x-ray spectrometer.

The narrow beam *I* of incident x-rays is reflected from crystal *A* through a limiting slit *S* to crystal *B*, and from *B* to the ionization chamber. Crystal *B* is mounted in such a way that it can be turned through a small measured angle as desired. This is accomplished by means of a long arm attached to the crystal table which is operated by a tangent screw. The hand wheel that turns the tangent screw is so divided that one scale division on the edge of wheel represents one-half second of arc of rotation of crystal. The details of this arrangement have been previously described (see Fig. 2 of Davis and Stempel<sup>5</sup>).

<sup>5</sup> Davis and Stempel, Phys. Rev. 17, 608 (May 1921).

The beam of homogeneous x-rays after passing through slit  $S$  falls on crystal  $B$  and is reflected into the ionization chamber. This beam is marked  $a$  in Fig. 1. Maximum reflection is obtained when the reflecting planes of  $B$  are parallel to those of  $A$ . If now crystal  $B$  be turned (rocked) a small measured angle each side of this maximum position, energy distribution curves are obtained (curve  $a$ , Fig. 2). By careful adjustment of

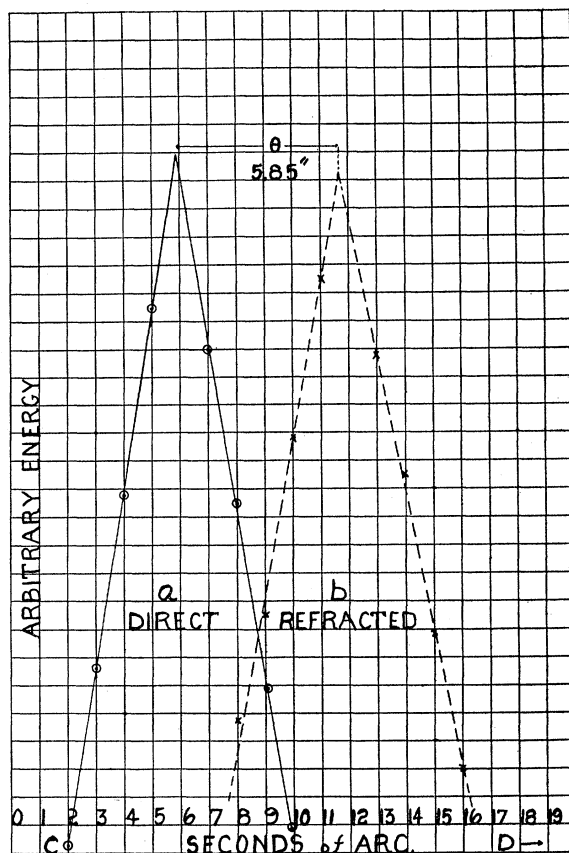


Fig. 2. Specimen curves showing the refraction in an aluminum prism.

the crystals these rocking curves could be made quite narrow. The width at half-maximum was less than  $6''$  of arc in the case of  $\text{Mo K}\alpha$  radiation. The position of the maximum of the curves could be determined to about one-tenth second of arc.

The prism  $P$  in which refraction is to be measured is mounted on a carriage with a fine screw so that it may be run in and out of the path of the x-ray beam as desired. When the prism is in the path of the beam,

the ray is refracted through an angle  $\theta$  as shown. Crystal *B* is now rotated to reflect this refracted beam. A new rocking curve is obtained. The angular distance of the peak of this curve is equal to  $\theta$ , the angle of refraction. The great advantage of the double x-ray spectrometer over the single is that the unequal absorption of the rays in the prism does not affect the position of the peak of the curves at all. This was directly tested by experiment. A piece of lead with a flat edge (not a prism) was mounted in place of the prism. When this piece of lead was pushed partially across the beam, the rocking curve was diminished in intensity, but the position of the peak was not altered.

Specimen curves for refraction in an aluminum prism are given in Fig. 2. Curve *a* was obtained with the direct beam *a* (Fig. 1) and curve *b* with the refracted beam *b*. Measurements were made for two frequencies, Mo  $K\alpha$  ( $\lambda$ .7078) and Cu  $K\alpha$  ( $\lambda$ 1.537). The refracting angle *R* of the prism was  $166^\circ$  in case of  $\lambda$ .7078 and was  $116^\circ$  in case of  $\lambda$ 1.537.

When the prism is symmetrically placed (position of minimum deviation) the index of refraction  $\mu$  may be expressed by

$$\mu = 1 - \delta; \delta = -\theta/2 \tan \frac{1}{2}R$$

The sign of  $\theta$  is negative for bending in the direction indicated in the figure.

The results obtained are given in the table.

$\lambda$	<i>R</i>	$\theta$ (obs)	Mean $\theta$	$\delta \times 10^{-6}$ (obs)	$\delta \times 10^{-6}$ (calc)
.7078	$166^\circ$	5.85"	5.62"	1.68	1.77
		5.46			
		5.75			
		5.50			
		5.55			
1.537	$116^\circ$	5.40	5.53"	8.4	8.36
		5.60			
		5.60			

The last column of the table gives the values of  $\delta$  computed from the Lorentz dispersion formula. In the case of the radiation here used, the frequency is so far removed from the natural resonance frequencies of the aluminum atom that the simplified form

$$\delta = \frac{e^2}{2\pi m} \frac{1}{V^2},$$

was used for the computations.

The accuracy obtained in these measurements is well within  $\pm 5$  percent. This accuracy might be increased by repeated measurements and in particular by maintaining constant temperature.

It was found that the position of the peaks of the curves often shifted during the course of a run. It was observed that the temperature also changed at the same time. On those occasions when the temperature was practically constant the shift was quite small. This shift apparently arises from an unequal expansion or contraction of the various parts of the apparatus, particularly the parts of the mechanism operating crystal *B*. Another source of error was the measurement of the angle *R* of the prism and the placing of it in a symmetrical position.

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