ON ELECTRIC CHARGES CARRIED BY INDIVIDUAL MICROSCOPIC PARTICLES*

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Abstract

Criticism of conclusions as to the existence of an elementary electronic charge, from experiments of Derieux with mercury droplets.—In these experiments observations of the same droplet were made at different pressures. The size of one droplet which apparently was constant in mass and not disturbed by air currents, is computed and a result obtained different from that obtained by Millikan's method of computation. The charges computed by using the size so determined are not multiples of a unit charge. This confirms Ehrenhaft's conclusion that other electric charges are stable on microscopic and submicroscopic metallic particles than those stable on particles of dielectrics.

(A reply by J. B. Derieux follows this article.—Ed.)

F. EHRENHAFT[1] and his pupils working with particles of precious metals (silver, platinum, gold and mercury) suspended in inert gases (N, Ar etc.), found on them electric charges down to 10^{-12} electrostatic units. This led him to the conclusion that the existence of an elementary charge is not yet proved by experiment.

R. A. Millikan[2] and his pupils on the other hand are of the opinion that they have succeeded in proving that all static charges both on insulators, such as oil (Millikan) and shellac (Lee[3]), and on conductors, e. g. mercury (Derieux[4]) are built up of elementary charges.

Two very strong objections, however, may be urged against the method used by R. A. Millikan and his pupils. The first is that all measurements are made on big droplets, their radii being always greater than 3×10^{-5} cm. The second is that these authors a priori presuppose an equality of charges on all test particles, i.e. the existence of an elementary charge, in determining the size of the charged particles, which is

* Contrary to the general editorial policy of the Physical Review, this criticism is published in spite of the fact that it is believed to be erroneous, because of the importance of the subject, and because it was submitted by Professor F. Ehrenhaft. A reply by J. B. Derieux and a discussion by R. A. Millikan follow this criticism.—G. S. F.

[1] F. Ehrenhaft, Anz. Wiener Akademie, March 4, 1909; April 21, 1910; May 12, 1910; Wien. Ber. **119,** 882 (1910); Phys. Zeits. **11,** 940 (1910); Ann. der Phys. **44,** 657 (1914); **56,** 1, (1918); **63,** 773 (1920).

[2] R. A. Millikan, Phys. Zeits. **11,** 1097 (1910); Phys. Rev. **32,** 349 (1911); Phys. Zeits. **14,** 736 (1913); Phil. Mag. **34,** 1 (1917).

[3] I. Y. Lee, Phys. Rev. **4,** 420 (1914).

[4] J. B. Derieux, Phys. Rev. **11,** 203 (1918).

the essential element of the droplet method. In the following a method will be put forward by which the radius of a particle is determined without any assumption of the existence of an elementary charge.

The Stokes-Cunningham law of motion

$$(4/3)a^3\pi(\sigma-\rho)g=6\pi\mu av_f(1+Al/a)^{-1}$$

can be written in the form $\alpha+\beta l=v_f$ if $(2/9)(\sigma-\rho)ga^2/\mu\equiv\alpha$ and $(2/9)(\sigma-\rho)gaA/\mu\equiv\beta$. It postulates a linear relationship between v_f and l. Since it is possible to measure the speed of fall of the same particle at various gas-pressures[5] the individual straight line is fixed by means of $\alpha=(v_1l_2-v_2l_1)/(l_2-l_1)$ and $\beta=(v_2-v_1)/(l_2-l_1)$, which quantities can be determined experimentally. If all assumptions hold[6], we can compute the radius of the particle from the equation

$$a=\sqrt{(9/2)\mu\alpha/(\sigma-\rho)g}.$$

This method of determining the size of the carrier of the electric charge is independent of every assumption about its charge.[7]

Now a pupil of R. A. Millikan, J. B. Derieux, has succeeded in measuring the same mercury droplet at varying gas pressures. But he did not use the above-mentioned method of calculating the radius of the droplet without recourse to assumptions about an elementary charge. Therefore by calculating the radii also according to this method, it is possible to examine R. A. Millikan's method which was used by J. B. Derieux for the determination of the radii of his particles.

On considering the data given by Derieux we find at first sight the astonishing fact that among eleven mercury-droplets (record numbers 51-61) which could be measured at varying gas-pressures, there are three for which the times of fall are longer at reduced pressures than at atmospheric pressure. Such a behavior seems at first to be unintelligible. For a more detailed discussion we plotted the reciprocals of the times of fall (speeds of fall, since distance of fall is 1 cm) as ordinates against the mean free paths l as abscissas. Through every pair of points belonging to one particle straight lines were drawn. The diagrams thus found are analogous to those already given by R. Baer.[8] Subsequently the radius and the density of these particles were computed according to the method used by R. Baer. For this computation the value of the constant

[5] First stated by E. Meyer and W. Gerlach, Ann. der Phys. **47,** 224 (1915) and I. Parankiewicz, Phys. Zeits. **19,** 280 (1918).

[6] (a) Sphericity of the test particles; (b) validity of the Stokes-Cunningham law with a constant A; (c) density of the particles equal to that of the material in bulk.

[7] Compare Zeits. f. Phys. **16,** 34 (1923).

[8] R. Baer, Ann. der Phys. **59,** 394-403 (1919).

$A=0.708$, given by R. A. Millikan for mercury-air, was assumed. The following values were found

Drop No. :	53	55	56	57	58	59	60	61
$a(10^{-5}$ cm) :	72.73	15.59	24.37	14.44	5.149	5.805	4.213	5.051
σ(g cm^{-3}) :	1.233	1.718	0.961	2.623	11.19	9.719	20.54	19.53

In analogy to R. Baer's criticism[8] of Ehrenhaft's results, it would seem possible to conclude that the mercury-droplets observed by Derieux and used as evidence for the existence of an elementary charge by R. A. Millikan are either non-spherical or are of spongy structure. Such a conclusion is, however, absurd since the particles were produced by condensing the vapor of boiling mercury. We therefore must try to find a more plausible explanation for this curious behavior. Now the air in the condenser was pumped off through six holes in the center of the upper plate only, so it could happen that equilibrium between the air in the condenser and the air in the pressure-cylinder[9] was not reached when the time-measurements began. Therefore a rising current of air could have remained in the condenser. The result of this would be that the observed values of v_f would be found too low. Since an approximately constant current in the condenser would not influence the values of v_1+v_s,[10] these values were plotted as ordinates against the mean free paths l as abscissas. It appeared that all the straight lines which were drawn through related points showed a uniform upward direction, with a few curves crossing the others. The reason for this could not be found, because the particles are not recorded in detail (detailed records are given only for droplets No. 59 and 60).[11]

The criterion for proving the absence of a current or another disturbance of the measurements is to be found in the constancy of the balancing-potential[12] which can be computed separately for the different pressures (from the equation $f^*=([v_f/(v_f+v_s)]f)$. The detailed recorded measurements of the droplets No. 59 and 60 were examined with regard to this condition. We find

Drop No.:	59			60			
Pressure in mm Hg:	750.9	348.1	196.3	751.0	430.7	299.5	180.0
f^*:[13]	3.346	3.128	3.016	4.702	4.731	4.721	4.672

[9] Compare J. B. Derieux, l.c.[4] p. 205.

[10] Computed from the recorded radii of the particles by means of the equation $(4/3)a^3\pi(\sigma-\rho)g=ef^*$.

[11] Perhaps the evaporation of the droplets which according to J. B. Derieux is greater at reduced pressures accounts for the deviation.

[12] R. Baer, Ann. der Phys. **67**, 182, 1922. By balancing-potential that potential is understood at which the weight of the drop is balanced by the electric field. The above-mentioned formula follows from the equations $ef^*=mg$, $mg=v_f/B$ and $ef-mg=v_s/B$ (f=field-strength in the condenser.)

[13] Computed for the charge which Derieux considers as built up of four elementary charges.

For droplet No. 59 there results a maximum fluctuation of ten percent against only one percent for droplet No. 60. If we compute the mean errors of the balancing-potentials in the usual way we find an error of one percent at most. Consequently, the fluctuations for droplet No. 59 fall beyond the limits of error, which means the presence of a disturbance (current in the condenser, inconstancy of the mass of the droplet caused by gradual evaporation etc.). On the other hand, the fluctuations of the balancing-potential for droplet No. 60 lie within the limits of error and therefore the measurements can be taken as free from disturbances (no current in the condenser, constancy of the mass of the particle). Having thus gained an opinion about the trustworthiness of the measurements we can determine the size of these droplets.

By plotting their times of fall as ordinates against the mean free paths as abscissas it is shown that the linear relationship between v_f and l postulated by the Stokes-Cunningham law exists indeed. Therefore α and β belonging to each of the straight lines were computed by means of the method of least squares. It is seen from the equation $ef-mg=6\pi\mu av_s(1+Al/a)^{-1}$ or $\alpha'+\beta'l=v_s$, that a linear relation must also exist between v_s and l. It should be $\beta'/\alpha'=\beta/\alpha=A/a=c$. α' and β' were computed in the same way as α and β. But the batteries ran down and a little current might after all have been in the condenser. To eliminate these small sources of error α'' and β'' belonging to the values of $(v_f+v_s)/f$, not touched by these sources of error, were computed by means of the method of least squares. It was found[14]

		v_f	v_s	$(v_f+v_s)/f$	v_f	v_s	$(v_f+v_s)/f$
Drop No.	:	59			60		
$(\beta/\alpha)10^{-3}$	:	147.3	239.6	220.4	165.1	167.9	171.2
$a\times10^5$	:	4.794	4.481	4.671	5.196	5.184	5.169
$e\times10^{10}$	:	4.588	3.962	4.243	4.156	4.124	4.091
A	:	0.706	1.074	1.029	0.858	0.871	0.885
a(Derieux)	:			4.815	5.445		
A(Derieux)	:			0.832	0.803		

The non-agreement of the values of β/α gained for droplet No. 59 shows once more the presence of a source of error, as already inferred from the inconsistency of the balancing-potential.[15] The fluctuations of the values of β/α for droplet No. 60, however, lie within the limits of error,

[14] It should be mentioned that the electric charges measured on these big droplets are as follows: particle No. 59: 8.471; 12.717; 16.970.$\times10^{-10}$; particle No. 60: 12.310; 16.624; 20.795; 25.110$\times10^{-10}$ electrostatic units. The exact multiples of the electronic charge would be: 9.548; 14.322; 19.096; 23.870; 28.644$\times10^{-10}$ electrostatic units.

[15] Nevertheless, the radius computed from the values of $(v_f+v_s)/f$ cannot be in error of more than one percent under the assumption that it is essentially a current that produces the disturbance of the measurements.

as it can be shown, and the radius computed from the values of v_f cannot be in error by more than one half percent.

Thus we find that the radii of the droplets computed by us by means of a method free from assumptions about an elementary charge, and the radii computed by Derieux according to the method of R. A. Millikan from the equation $mg = ef^*$ differ by more than four percent. Therefore the electric charges carried by these mercury-droplets show deviations of more than 14 percent[16] from the electronic charge and its multiples.

These deviations are in agreement with those observed by Ishida[17] on oil-drops and Silvey[18] on mercury-droplets.

To sum up it could be shown that all the measurements taken by Derieux at different gas-pressures were disturbed by causes that, however, could not be found, because the particles are not recorded in detail. Only the measurements taken on particle No. 60 could be shown to be free from these disturbances (no current in the condenser, no evaporation of the droplet). Therefore a value for the constant A could be computed from the measurements on this particle. The value of the constant A was found to be 0.858 against the value 0.708 given by Millikan. The sphericity of the particles being beyond doubt, the only assumption for this computation is that the density of the particle is equal to that of the material in bulk. On the other hand the assumption of a constant $A = 0.708$ would mean a density of this mercury droplet $\sigma = 21.2$.

As it is well-known, R. A. Millikan always brought forward the paper of J. B. Derieux as a proof[19] for the theory that all electric charges carried by mercury-droplets are of the same value (4.774×10^{-10} e.s.u.) as those carried by insulators. The present investigation shows the fallacy of this argument.

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[16] At least ten percent of this lies beyond the limits of error.
[17] Y. Ishida, Phys. Rev. **21**, 561 (1923).
[18] O. W. Silvey, Phys. Rev. **7**, 102 (1916).
[19] R. A. Millikan, Phys. Rev. **8**, 620 (1916); "The Electron," p. 176, 1917.