MEASUREMENT OF THE FOUR MAGNETIC TRANSVERSE EFFECTS

By Edwin H. Hall

Abstract

This paper presents in detail the experimental arrangements and technique used by the author in measuring the four magnetic transverse effects (the Hall, Ettingshausen, Nernst and Leduc effects) on the same strip of metal, in succession. The strip 5 cm long by 2 cm wide by .01 to .06 cm thick depending on the metal, is soldered to two brass end blocks which may be independently cooled or heated with water or steam. The pole pieces, 4.5 cm in diameter and 0.7 cm apart, are thermally insulated from the strip by flannel and mica. The temperature of these pole pieces is adjusted to approximately that of the middle of the strip, by water cooling or heating. Ten thermo-junctions are soldered at various points along one edge and along the median line of the strip and one at the middle of the other edge. These enable longitudinal and transverse potential and temperature gradients to be measured. Various corrections are discussed, including (a) effect of size of thermo-junctions, (b) the effect of the limited length of the strip with reference to the width, (c) masking influence of one transverse effect on another, (d) effects of non-uniformity of temperature gradients due to heat leakage to or from the strip. Also certain precautions in the measurement of the separate effects are given. Results for Au, Pd, Ni, Co, previously published in summary are here given with some details, including corrected values for the *Ettingshausen effect for palladium.*

I PROPOSE to exhibit in this paper the methods and devices which I have used in recent experimental work[1] upon the four "transverse" effects of the magnetic field, the definitions and sign conventions of which are illustrated by Figs. 1-4.

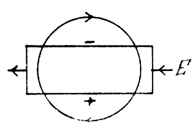

Fig. 1
Hall effect,
R or $_eT_e$
positive.

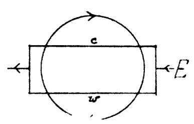

Fig. 2
Ettingshausen effect,
P or $_eT_h$
positive.

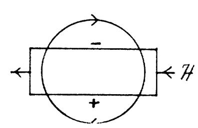

Fig. 3
Nernst effect
Q or $_hT_e$,
positive.

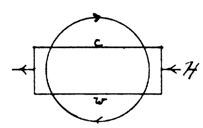

Fig. 4
Leduc effect,
S or $_hT_h$,
positive.

Apparatus

The pole pieces of my magnet are about 4.5 cm wide, and I usually have them 0.7 cm apart. The strip or plate of metal to be studied is commonly about 5 cm long and 2 cm wide, and it is mounted in such a

[1] E. H. Hall, Proc. Nat. Acad. Sci. 11, p. 416 (July 1925).

way that, barring accidents, it need not be removed from its position between the poles, or even be disturbed there, during the measurement of all four effects at various temperatures. Thus, in Fig. 5, the scale of which is half full size, *WW* is a board (a small drawing-board, 9 inches wide, is suitable) in which a hole is bored just large enough to admit the magnet pole *P*; it is backed by a wooden boss of such thickness as to bring the outer surface of the board a little beyond the face of the pole; it carries two brass blocks *BB* bolted to it, and to the thinned opposed ends of these blocks the ends of the experimental metal strip *m* are soldered. The distance of *m* from the pole *P* is about 0.25 cm and from the pole *P′* somewhat greater, this greater distance allowing room fcr

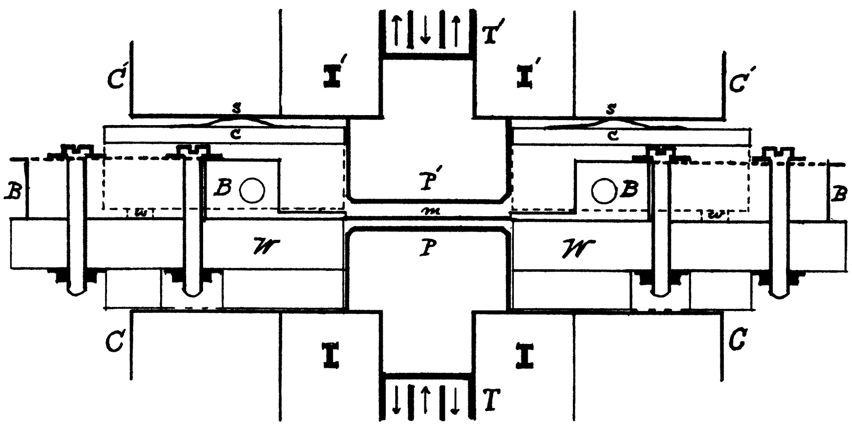

Fig. 5. The experimental plate *m*, and the method of mounting between the poles of the electromagnet, all as seen from above.

the somewhat elaborate wire connections indicated in Fig. 6. A wooden cover *cc*, channeled in such a way as to bridge over the blocks *BB*, is fastened by screws, not shown, to the board *WW*, *ww* being washers of such thickness as to prevent pressure on the tops of the bolts passing through *BB*. Springs, indicated or suggested by *ss*, pressing against the face of the coil *C′C′*, hold the rear surface of the wooden boss, behind *WW*, against the face of the coil *CC*, thus fixing the position of *m* between the poles. Everything shown between the face of *CC* and the face of *C′C′* can be handled as one coherent *ensemble*. Before measurements begin, crevices leading into this *ensemble* or existing between it and the faces of the coils are stuffed with cotton, to reduce currents of air, which otherwise are likely to make trouble when a considerable difference of temperature exists between the interior and the exterior.

Fig. 6, which is also half full size, shows one corner of the board WW and the things it carries. The right-hand block B is channeled, and its channel leads into a short horizontal brass tube connecting with a larger vertical one, to which is attached, by means of the coupling K, a stuffing-box S carrying a thermometer t. Water, entering through a tube at T, circulates through the channel of B and escapes at T', just above the bulb of the thermometer. When steam instead of water is used, it enters at T' and comes out at T. G is a brass guide piece intended to maintain

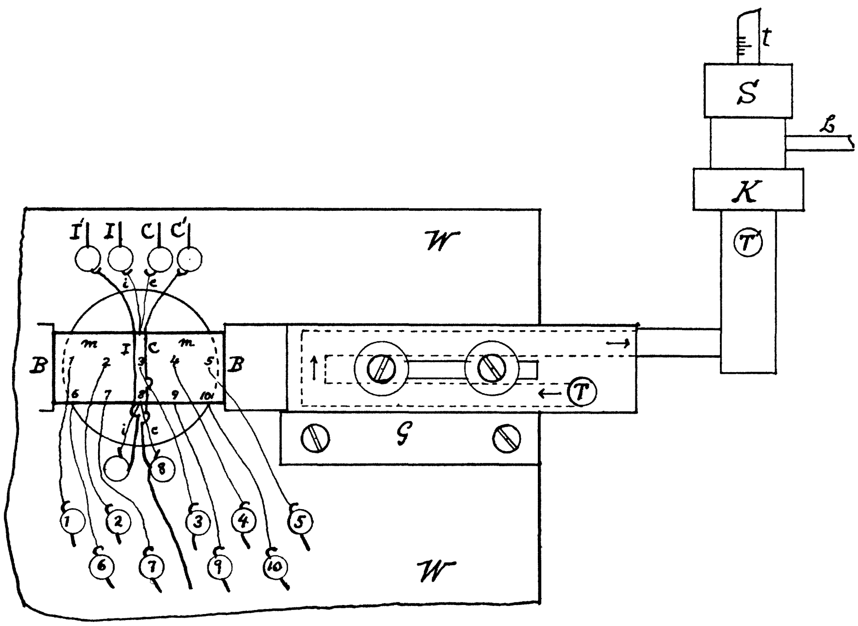

Fig. 6. *WW* is one corner of the board carrying the experimental plate *mm* and its attachments. The circle behind *mm* represents one pole of the electromagnet.

the proper alignment of B when the bolts fastening the latter to the board are loosened. The other block B, the end only of which is shown in Fig. 6, is, with its connections, a left-hand counterpart of the right-hand block.

When one of the electromagnetic effects, the Hall or the Ettingshausen, is to be measured, streams of the same temperature are sent through both channels so as to give both of the blocks B, and thus both ends of the metal strip *mm* connecting them, the same temperature. Then an electric current is led in or led out by the wire L and a corresponding wire at the other end of the apparatus. Of course, the mid-length part of *mm* will not necessarily have the same temperature as the ends. The whole matter of the temperature control of the middle part will be discussed at length farther on.

When one of the thermo-magnetic effects, the Nernst or the Righi-Leduc, is to be measured, electric connection is broken at each L and streams of different temperature, one being sometimes a flow of steam, are sent through the two channels, thus producing a temperature gradient along the plate *mm*.

At point 8, mid-point of the lower edge of the plate *mm*, a thermo-electric junction, of No. 40 copper wire and No. 40 "ideal" wire twisted together, is soldered. A similar couple is attached in the same way at the mid-point of the upper edge of *mm*. Much care should be taken with these junctions. They should be small, compact, firm, and firmly attached to the plate, with no superfluity of solder. A method of making them is indicated in Fig. 7. A is a fixed bar carrying two screws 16 cm apart, beneath the heads of which the fine wires, one of copper, the other of "ideal," are fastened. B is a movable bar about 8 cm from A. As shown

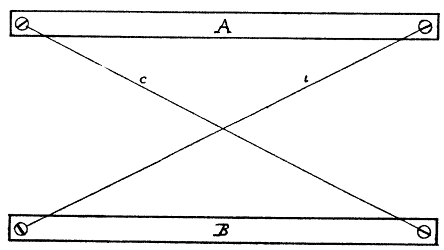

Fig. 7

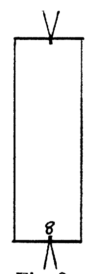

Fig. 8

Fig. 7. Illustrating the method of making the thermo-electric junctions.

in the figure the two wires are merely crossing each other in contact. Bar B is now turned, in a vertical plane, through a complete revolution and thus reaches its original position, a gentle tension on the wires being all the time maintained. Each wire is thus twisted once completely around the other in a close spiral about 0.5 mm long. This junction is now soldered, the wires having been cleaned before twisting. Then with a sharp knife the wires on one side, say the side toward B, are cut off close to the junction, which is thus ready to be soldered to the plate. (See "8" in Fig. 8, corresponding to the point marked in the same way in Fig. 6.) The two opposite junctions, each extending about 0. 5 mm on the plate 2 cm wide, reduce its effective width to an extent not easy to determine with precision, but the maximum must be less than 5 percent. I suggest that a correction, to be called *correction* (a), be added to each of the observed transverse differences of potential or of temperature, this correction being such a percentage of the observed value as the length of *one* junction is to the width of the plate.

If a junction does not have very good and close attachment to the plate, we can not be sure that its changes of temperature follow satisfactorily those of the plate at the point of attachment. In fact, under the best practicable conditions the temperature of a junction cannot be controlled solely by that of the plate, as the packing material, and the fine wires leading away from the junction, must have some influence. This condition tends to make the observed transverse temperature differences, from which the Ettingshausen and the Righi-Leduc coefficients are estimated, somewhat less than the real differences existing in the plate.

The wires indicated in Fig. 8 should be so arranged as to make the circuits into which they enter as nearly non-inductive as may be, for changes of intensity of the magnetic field. Otherwise, a slight change of strength of the magnetizing current, say 1 percent, occurring suddenly by reason of an imperfect connection, may produce a very disturbing deflection of the galvanometer needle.

The copper wires only of these two junctions are used when transverse potential differences, the Hall and the Nernst effects, are to be measured. More will be said farther on concerning the thermo-electric use of the junctions.

The other wires shown in Fig. 6 as attached to the plate *mm* are for the purpose of measuring the longitudinal temperature differences along two lines of the plate, the median line and the lower boundary line. These wires should not be larger than No. 40, about .08 mm in diam., and must be of such material as to give effective thermo-electric couples with the material of the plate. Thus, if the latter is of nickel, palladium, cobalt, etc., the wires should be of copper. If the plate is of copper, or of gold or silver, these wires may be of constantan ("ideal"). The points of attachment of the wires are intended to be 1 cm apart, but since accurate placing is difficult, measurements of distances are made after the soldering, and these actual distances are used in calculating temperature gradients. At no great distance from the plate *mm* each of the fine wires attached to it is soldered to a thicker wire of the same material to lessen electrical resistance and the danger of breaking.

Calibration of thermo-electric couples. On the advice of Mr. W. P. White of the Carnegie Geophysical Laboratory, I use the soft, "ideal," constantan wire that is produced by the Electrical Alloy Company of Morristown, N. J. My own calibration tests of couples made of No. 40 "ideal" and No. 40 copper wire show, all the way from 0°C to 85°C, sensitivity values about 6 percent greater than those given in the tables of Mr. L. H. Adams for copper-constantan couples. Couples made of copper and the coarser "ideal" wire mentioned above were about 3 percent less sensitive than those having the fine wire. I did not attempt to allow for this difference in using, as described above, circuits in which coarse wires were joined to fine wires, but took the values given by the fine wire couples as holding in my measurements of the transverse effects.

I made a special test for any effect of the magnetic field, about 9000 gauss, on the sensitiveness of a copper-ideal couple, both junctions of which, besides the whole length of the constantan wire, were between the poles. The effect of the field, if there was any effect, was only a small fraction of one percent of the total power of the couple.

For couples in which constantan was not one of the metals, I made no calibration tests but calculated the thermo-electric powers from data given by Bridgman in his paper[2] on "Thermo-electric Qualities Under Pressure."

The question of strip dimensions; correction (*b*). The best length and width of the metal strip to be used will naturally depend on the width of the magnetic field that can be regarded as fairly uniform. With pole pieces 4.5 cm in diameter I use, as has been said above, a strip about 2 cm wide and 5 cm long, between lines of attachment to the blocks B and B. Considerable width is desirable, in order to reduce the importance of the encroachment of the junctions upon the plate, as in Fig. 8; but, on the other hand, great width tends to increase the short-circuiting influence of the ends which lie outside the intense magnetic field, and of the blocks BB, to which these ends are fastened. This short-circuiting, which doubtless occurs with all four of the transverse effects and is probably about equally great, relatively, for all of them, was observed in 1886 by Ettingshausen and Nernst[3] working with a plate of bismuth and by myself[3] in 1888 working with a plate of soft steel, in experiments showing that the Hall effect is, other things being equal, notably smaller in a short strip of metal than in a long strip. I have recently made more careful and extended observations for the purpose of determining, approximately, what percentage of correction should be made for the error due to the fact that the strip I use is of limited length. I studied the Hall effect in a thin gold strip of variable effective length.

Fig. 9 illustrates the apparatus used. WW is, as before, a flat board, with a hole to receive the pole piece P. A thin glass plate gg is sunk into a depression in this board and on this plate rests a gold strip m, 2 cm wide, 5.1 cm long and about 0.013 cm thick. Semi-cylindrical blocks of

[2] Bridgman, Proc. Amer. Acad. **53,** 269-386.

[3] See Campbell, "Galvanomagnetic and Thermomagnetic Effects" (Longmans), pp. 31 and 32.

brass, *cc*, somewhat longer than the width of the gold, rest on *m* and are pressed firmly against it by the stiff springs *ss*. The distance between these blocks, beginning with 0.5 cm as shown in Fig. 9, could be increased by convenient stages till it equalled almost the whole length of the strip. It is assumed that the Hall effect in the thick movable blocks is negligible

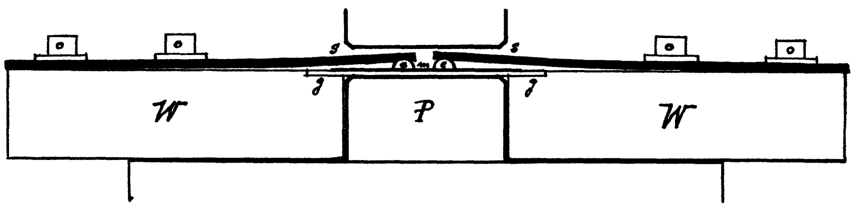

Fig. 9. This shows, as seen from above, the device used for varying the effective length of the experimental strop.

and that the gold immediately under and in contact with a block can be treated as a part of the block. The values of the Hall coefficient *R* found are given in Table I.

TABLE I

Variation of Hall coefficient with the effective length x of the strip

x	R(obs.)	R (calc.)	Contribution to R of	
0.5 cm	−0.000500	−0.000531	1st 0.5 cm	−0.000531
1.0	−0.000594	−0.000593	2nd "	−0.000062
1.5	−0.000622	−0.000627	3d "	−0.000034
2.0	−0.000646	−0.000649	4th "	−0.000022
2.5	−0.000669	−0.000665	5th "	−0.000016
3.0	−0.000683	−0.000678	6th "	−0.000013
3.5	−0.000688*	−0.000688	7th "	−0.000010
4.0	−0.000695	−0.000695	8th "	−0.000007
4.8	−0.000705	−0.000705	9th (0.8 cm)	−0.000010

* This is the mean of two values 691 and 685 obtained several days apart.

Representing by x the variable block-distance in centimeters, I found by an empirical process the following formula, which corresponds fairly well with the observations.

$$R = -0.00077(1 - 0.23^{x^{1/3}}). \tag{1}$$

It must be noted that the distance x was not very accurately measured and that any given error in the value of x will have its greatest effect when this distance is small. The values calculated with this formula are also given in Table I.

Of course formula (1) indicates that with an infinite value of x, in a magnetic field of uniform strength, the maximum value of R would be −0.000770. As the value of x is about 5 cm in my ordinary experiments, Table I, which gives $R = -0.000705$ for $x = 4.8$ cm, indicates that I should add about 8 percent to my ordinarily observed values of R in order to make due correction for the limited length of the metal strip used. This is for a strip 2 cm wide. The correction to be applied increases with the width of the strip.

As this correction is due to the short-circuiting effect of transverse conduction around the ends, and as such conduction should have about the same proportional effect in the case of transverse thermal difference as in the case of transverse potential difference, I assume that to every transverse gradient *actually existing* at mid-length of a strip 5 cm long and 2 cm wide, an addition of 8 percent should be made, to take proper account of this short-circuiting influence. This correction, then, *which hereinafter will be called correction* (b), applies to all four of the transverse effects. In some cases, where other corrections to be applied are large, it is a matter for consideration whether this 8 percent shall be added before or after other corrections. The words "actually existing," which I have italicized, are decisive in this connection.

There is, however, another way of looking at this matter, a way which I find useful in another connection, as will presently appear. This is to think of the middle half-centimeter of the strip as contributing a certain amount to the transverse gradient, whether electrical or thermal, which is found there, and of each successive element of the length of the strip, toward either end, as contributing also its share, these contributions, for a given increment of length, decreasing as distance from the middle increases. The last column of Table I was made in accordance with this view from the values given under R (calc.).

Thermo-electric complication in measurement of transverse potential difference; correction (c). If in the measurement of a transverse potential (Hall effect or Nernst effect), the side-connection wires used are not of the same metal as the strip m, these wires, together with the part of m which connects them, constitute a thermo-electric couple which, because of the difference of temperature of its two junctions (Ettingshausen effect or Righi-Leduc effect), tends to make the Hall effect or the Nernst effect appear either greater or less than it should be. Correction of the observed transverse potential difference must be made accordingly, and so the Ettingshausen effect must be measured before the final value of the Hall effect can be known, and the Righi-Leduc effect before the final value of the Nernst effect can be known.

This correction, *which will be called correction* (c), may be very large. Thus in one case, with copper wires leading from a cobalt strip, the final value of the Nernst coefficient Q was about 70 percent of the value found without this correction.

Longitudinal temperature gradient; corrections (d) and (e). The temperature conditions of the strip m are difficult to control satisfactorily. Through a packing of cotton flannel and mica disks there is a notable flow of heat between the strip and the neighboring pole-faces, unless the latter are in some way brought nearly to the same temperature as the strip itself. Unwin[4] interposed between m and the poles a thin metal channel through which water of any desired temperature flowed, and I have tried a similar device, but on the whole I do not like it. Unless one makes the distance between the pole pieces greater than the distance I have used, 0.7 cm, he must make the water channel so thin that it is likely to become obstructed by bubbles when hot water is used.

In my more recent experiments I have endeavored to keep the temperature of the pole pieces as nearly as possible the same as that of the middle of the strip m. For this purpose I have sent through the double brass tubes T and T' (Fig. 5), which fit into the bore of the magnet cores, water of suitable temperature. But this device, used alone, operates rather slowly, and I am planning to use magnet coils the windings of which will consist of copper tubing carrying a stream of water, cold or hot according to the pole temperature desired. A thermo-electric couple, one junction of which presses against the side of a pole piece, enables me to keep watch of its temperature changes.

It is evident from what has been said that a uniform temperature gradient cannot be maintained from one end to the other of the strip m. If the pole pieces have the same temperature as the mid-length of the strip, the warmer end of the strip will lose heat to the poles and the cooler end will gain heat from the poles, the result being that the longitudinal temperature gradient of the strip will be flattened in the middle but steeper at each end, the general shape being that shown by the curve in Fig. 10. This is a line of double curvature, the point of change from concave to convex being near the middle of the strip, in fact just at the middle, if this has exactly the same temperature as the pole faces. If the poles are cooler, the point in question is displaced toward the cooler end, if they are the warmer, toward the warmer end, of the strip.

For each temperature stage in the measurement of Q or S a careful study should be made of such a temperature-gradient curve. Data for plotting it are obtained by measuring the temperature differences of points (1) and (2), (2) and (3), (3) and (4), (4) and (5), (6) and (7), (7) and (8), (8) and (9), (9) and (10), of Fig. 6, in the order given and then in the reverse order, first with the gradient running from right to left,

[4] Unwin, Proc. Roy. Soc. of Edin. **34**, p. 210 (1913-14).

then with it running in the reverse direction. The gradient along the (1) to (5) line is not exactly the same as that along the (6) to (10) line, the edge of the strip being rather more affected by the inflow and outflow of heat than the median lengthwise line. Assuming that the gradient along the upper edge is the same as that along the lower edge, and so letting the lower-edge observations represent both edges, I have taken the mean of the (1) to (5) and the (6) to (10) observations, thus making the median line of equal importance with the two edges in estimating the general temperature-gradient. In palladium the temperature-gradient near the mid-length of the plate was usually about 10 percent less along the lower edge than along the median line, and, accordingly, taking the mean of these values was equivalent to reducing the first estimate of this gradient (made from the observed temperature differences between points (2) and (4) on the median line) about 5 percent. *This correction will be called correction* (*d*).

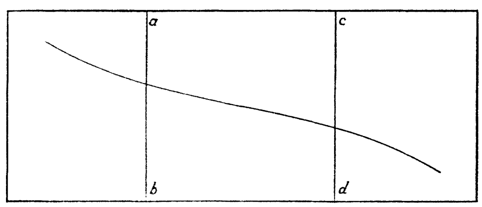

Fig. 10. The curve illustrates the variation of longitudinal temperature gradient along the experimental strip, in observations of the Ettingshausen and the Righi-Leduc effects.

Moreover, I have averaged the right-to-left gradient observations with the left-to-right gradient observations, making due allowance for inaccuracies of placing of the contact points. Thus I have obtained data for plotting a representative hot-to-cold curve corresponding to each mid-point temperature at which I undertook to measure Q and S. For example, I have plotted three such curves in the case of palladium, for the three mid-point temperatures 25°, 44.5° and 65°, approximately. In fact, I plotted two such curves with a mid-point temperature near 45°, one with a mid-section temperature-gradient of about 10.5° per cm, the other with a gradient about half as great. For the first of these I used in one of the blocks B (Fig. 5) a stream of water at about 11°C. and in the other a stream of steam. For the second I used two water streams, one near 30° and the other near 75°.

All these curves were carefully plotted and carefully studied. The temperature corresponding to the point of inflexion of any curve I took to be the temperature of the pole-faces. It was usually a little below that of the mid-point of the plate. Taking any given section of the curve, I could from the difference of gradient at the ends of this section estimate the amount of heat that leaked out from or into the corresponding section of the plate m. Then, knowing the mean distance from the plate to the pole-faces and, approximately, the mean difference of temperature between these faces and the section of the plate, I could make a rough determination of k, the thermal conductivity of the packing between the poles and the plate. Of course, slight inaccuracies in drawing the temperature-gradient curve might affect seriously the value found for k at any one section, but the errors of different sections tended to neutralize each other. Values found for k in the case of palladium, with a packing consisting mainly of cotton flannel, with one thin layer of mica, are given below:

TABLE II

Thermal conductivity of packing around the palladium strip

Middle temp.	Middle gradient	k below mid-point	k above mid-point	Means
25°C	4 .3°/cm	0 .00021		
		0 .00016		
		0 .00019		
				0 .00019
44 .5	10 .7	0 .00028	0 .00014	
		0 .00018	0 .00026	
		0 .00014	0 .00020	
				0 .00020
45 .5	5 .5	0 .00035		
		0 .00021		
		0 .00012		
				0 .00023
65 .1	5 .5	0 .00019	0 .00028	
		0 .00015	0 .00017	
		0 .00008	0 .00010	
				0 .00016
			Final mean	0 .00020

Kaye and Laby give 0.00023 as the thermal conductivity of flannel. It appears probable, therefore, that the leakage of heat was maintained, for the most part, by simple conduction between plate and poles, and not by convection due to air-currents moving upward or downward past the plate. That is, one can make a helpful estimate, in advance, of what the leakage will be by assuming that it will be caused by conduction only through the packing.

It is to be observed that, in my experiments on the Nernst and on the Righi-Leduc effect, I measure the difference of temperature of the two points (2) and (4) of Fig. 6, which are 2 cm apart, and assume, provisionally, that a uniform temperature gradient extends from one to the other, thus finding a value for the gradient at the mid-point. Careful study of the temperature gradient curves shows, however, that the mid-point gradient arrived at in this way—that is, by taking the mean gradient between points (2) and (4)—is somewhat too large, perhaps 5 or 10 percent greater than the true value. Should we, then, make correction to the full extent of this divergence? This raises the question whether the effective longitudinal temperature-gradient, the gradient value which appears as Δ in the formulas $Q=\delta_e/H\Delta$ and $S=\delta_h/H\Delta$, for the Nernst and the Righi-Leduc coefficients, respectively, should be the precise value which holds at the mid-line of the plate between the junction point (8) in Fig. 6 and the opposite point on the upper edge of the plate. The discussion already given of "short-circuiting" due to parts of the plate or its connections in which the potential or temperature gradient is not the same as at the middle, requires us to answer this question in the negative. The proper value of Δ to be used in the formulas above given is to be arrived at by a synthetic process in which the middle-half centimeter length of the plate will have a predominating but not an exclusive influence.

I shall assume that if we start at the middle cross-line of the plate and extend the length under consideration by 0.5 cm stages, the boundaries moving to right and left equally, each successive half cm length will affect the transverse gradient (of potential in the Nernst effect, of temperature in the Righi-Leduc effect) to an extent proportional to the product of the mean temperature-gradient (longitudinal) of this half cm by a *weight-factor* indicated by the significant figures set opposite this half cm length in the last column of Table I. I shall call the value of Δ found in this way the *synthetic* value, or Δ_s, and this is the value to be used in the formulas for Q and S.

The relations of Δ_s to various gradients taken from the four temperature curves for palladium, $\Delta_{0.5}$ for the middle 0.5 cm, Δ_1 for the middle 1 cm, etc., are shown in Table III.

TABLE III

Longitudinal temperature gradients at different distances from the median line

$\Delta_{0.5}$	Δ_1	$\Delta_{1.5}$	Δ_2	Δ_s
4.04	4.14	4.17	4.31	4.26
9.24	9.74	10.16	10.76	10.07
4.88	5.00	5.21	5.52	5.25
4.76	4.94	5.18	5.48	5.17

It appears from this table that, on the average, Δ_s was about 8 percent greater than $\Delta_{0.5}$, 4 percent greater than Δ_1, 0.5 percent greater than $\Delta_{1.5}$, and 4.5 percent less than Δ_2. Accordingly, the values first found for Q and S, in the case of palladium, were to be increased about 4.5 percent to make allowance for the too great vaue of Δ provisionally used. *This will be called correction (e).*

Transverse tmperature-gradient; correction (f). The method of measuring the difference of temperature actually existing at any time between point (8), of Fig. 6, and the opposite point on the upper edge of the plate, will be explained later. My view of the matter is that the Ettingshausen effect or the Righi-Leduc effect is all the time tending to increase the transverse temperature difference and that the heat conductivity of the plate m, assisted by the conductivity of the packing lying between the plate and the pole-faces, tends to reduce it. What we measure is the compromise result of these tendencies. We are now concerned with the question how much greater this difference would be if, other things being equal, the packing were thoroughly non-conductive for heat.

Study of the longitudinal temperature curve helps us here. For simplicity let us assume, for the present, that the temperature of the mid-point of the plate m is the same as that of the pole-faces, this bringing the point of symmetry of the temperature-curve to lie on the median transverse line of the plate, as in Fig. 10. Then the slope of the curve, at the line ab of Fig. 10, 1 cm to the left of the middle line, will be the same as the slope at the line cd, 1 cm to the right of the middle. This implies, if we neglect the slight difference of thermal conductivity due to difference of temperature, that the amount of heat carried by the plate across line ab is the same as the amount carried by it across the line cd. Moreover, this amount of heat is equal to that which would be carried from ab to cd *by the plate alone*, if the *temperature-gradient* existing at ab and at cd existed all the way from one line to the other, and it is considerably greater than the amount that would flow with a uniform gradient descending from the *temperature* of ab to the *temperature* of cd. In fact, if ΔT represents the temperature of ab minus the temperature of cd, and if γ represents the *temperature-gradient* at ab and cd, the ratio of the heat actually conveyed from ab to cd, by plate, packing, etc., to that which would be conveyed by the plate alone, with the existing temperatures at ab and cd, is $2\gamma/\Delta T$. In some instances I have found this ratio to be as great as 1.19.

Turning now to the *transverse* conveyance of heat, let us suppose the line ac to be warmer than the line bd, and that we have measured their difference of temperature and found it to be δT. Assuming the plate to be of standard width, 2 cm, we find line ac analogous to ab and bd to cd, and so we conclude that, if the packing were non-conductive, the transverse temperature difference, in order to carry as much heat transversely as is now carried by plate and packing together, would have to be approximately, $\delta T \times (2\gamma/\Delta T)$. Accordingly, we amend our observed transverse temperature-difference by means of the correcting factor $(2\gamma/\Delta T)$. *This correction will be called correction (f).*

I have provisionally made the assumption, in constructing Fig. (10), that the mid-point of the plate has the same temperature as the pole-faces, so that the line *ac* will be as much above the temperature of the poles as the line *bd* will be below that temperature. If this condition is not exactly maintained, and usually it will not be, a slight divergence from it will not seriously affect the accuracy of the correction just described. For example, if the pole-faces are a degree or two colder than the transverse mid-line of the plate, we shall have superposed on the equal out-and-in flows of heat which I have assumed, a flow outward from all parts of this transverse line. Such a flow will affect the transverse *gradient* in the plate but little, though it is better to keep it as small as practicable—that is, the poles should be kept as nearly as may be at the temperature of the middle of the plate.

Thickness and method of support of plate. It is evident that, other things being equal, the disturbing influence of heat leakage through the packing will be less the greater the thickness and the heat conductivity of the plate. The nickel plate I have used is about 0.058 cm thick, the cobalt plate about 0.053 cm, the gold plate about 0.0124 cm, the palladium plate about 0.0105 cm. Very thin plates are objectionable, even for the Hall effect, as it is difficult to measure their effective thickness with accuracy, and they are liable to serious mechanical injury unless very securely supported. Of course, where temperature gradients play no part, there is no objection to fastening the metal with cement to a glass plate; but such a backing is not desirable, in general, where temperature differences are concerned. Accordingly, my practice, when all four effects are to be measured, is to support the metal plate merely by its attachments to the blocks *BB*, Figs. (5) and (6). In the case of cobalt, however, since this metal, so far as my acquaintance with it goes, is very brittle, I felt obliged to use a backing of glass, which made the heat leakage greater than it would have been with a packing of flannel only.

In order to prevent buckling of the cobalt plate on heating, one of the leads *B* (see Fig. 6) was left slightly loose under its retaining screws, alignment being maintained by means of the guide *G*.

Measurement of magnetic field strength. Fundamentally, my method of measuring the field strength is to withdraw suddenly from between the pole-pieces a test-coil of known integral area, in circuit with a standard induction and a ballistic d'Arsonval galvanometer. But use of the test-coil is hardly practicable when a plate with all its small-wire attachments is in place between the poles. Accordingly I use the test-coil in advance of the final installation of the plate, making observations of the galvanometer throw with various strengths of magnetizing current, and so getting data for plotting a curve showing the relation between the strength of this current and the intensity of the field. Two or three such curves should be made, each for a different temperature of the magnet pole-pieces, according to the range of temperature to be used in the main experiments.

When a magnetic metal plate is to be studied, it or an equally thick piece of like material should be in place between the poles when these observations with the test-coil are made.

Of course care should be taken to see that the distance between the pole-pieces is the same during the main experiments as during the preliminary test-coil work. I gauge this distance by means of a steel cylinder turned to the required thickness. Moreover, as the electro-magnet which I use suffers a perceptible decrease of pole-distance, unless care is taken to prevent it, when a strong magnetizing current is applied, I use a stop between the faces of the two magnetizing coils, near their upper edge.

Measurement of the Hall Effect

This operation is simple and easy, after the plate is properly mounted and connected. I have used a non-astatic Dubois-Rubens "armored" galvanometer of about 11 ohms series resistance, which rests on a platform suspended by three stout wires from a bracket bolted to a brick partition wall of the laboratory. Suspended beneath the platform is a vertical cylinder about 8 cm in diameter, which is partly submerged in an oil bath, to dampen vibrations. On the upper surface of the platform is a sheet of hair-felt about 3.5 cm thick, on which is a board about 2 cm thick carrying four rubber "sponges" compressed to a thickness of about 1 cm by the load they carry, which is a sheet of slate about 1.3 cm thick on which rest three glass cups receiving the feet of the galvanometer. I mention all these details because this Julius suspension arrangement, for which I am indebted to my former colleague Dr. E. R. Schaeffer, has proved eminently satisfactory, in a building subject to much disturbance, and has saved me a vast amount of work and worry.

The galvanometer, as I have used it, is not very sensitive. With a scale distance of 1.8 meters, 1 cm deflection is given by about 1.8×10^{-8} ampere. My ordinary method of determining its sensitivity is to find by trial how great a fraction of one volt is required to produce 1 cm deflection under the conditions of the actual Hall effect observations—that is, with a greater or less amount of "introduced" resistance in the galvanometer circuit.

The galvanometer needle, provided with an air vane, has a vibration period of about 20 seconds, and is nearly "dead-heat." Readings with a reversal of the magnetic field at the plate m, are taken 45 seconds apart, a considerable interval being needed for all conditions to reach a final state after the reversal. As it is usually impracticable to arrange the side connecting wires at the plate in such a way as to make that part of the galvanometer circuit non-inductive, this circuit is opened just before the magnet reversal and closed a second or two after this reversal, a procedure which saves the needle from a too disturbing impulse.

It sometimes happens, especially at forced temperatures, that an appreciable longitudinal temperature-gradient exists at the middle of the plate during the Hall effect observations, thus superposing a Nernst effect. To eliminate error from this cause, two sets of observations should always be made, one with the longitudinal current running from left to right and one with its direction reversed. Another reason for this reversal is that it tends to exclude error that might otherwise come through a slight leakage of electric current from the magnet circuit into the galvanometer circuit. If reasonable precautions are taken, such leakage is not likely to be troublesome when the laboratory is artificially heated, but during the damp summer months it may become apparent and bothersome.

The temperature of the mid-part of the plate m during Hall effect observations can be found by means of a thermo-electric couple consisting of one of the junctions, let us say the top one, soldered to the plate (see Fig. 6) and another junction kept at a known temperature outside the magnetic field. The condition of this second junction is indicated in Fig. 11, which shows, one third full size, a double-walled glass tube, through the outer compartment of which runs water of the desired temperature while the inner contains oil of a sort that will not act injuriously on the wires that are immersed in it. An "ideal" wire I, leading from the junction on the plate, enters a narrow glass tube held in a cork stopper and extending nearly to the bottom of the oil. Near the bottom of this tube the ideal wire is soldered to a copper wire C, which, together with the copper wire coming directly from the junction on the plate, leads off to the galvanometer. A thermometer T, with the middle of its bulb on a level with the junction in the oil, gives the temperature of this junction.

The value of the Hall effect coefficient R as found directly from the observations made, is subject to the three corrections, (a), (b), and (c), already described. Corrections (a) and (c) should be applied first and then (b), the latter being taken as a certain percentage of the value obtained by applying the (a) and (c) corrections. As correction (c) depends on the size of the Ettingshausen effect, the value of P, or rather the transverse temperature-difference actually existing because of the Ettingshausen effect, must be found before that of R can be finally determined.

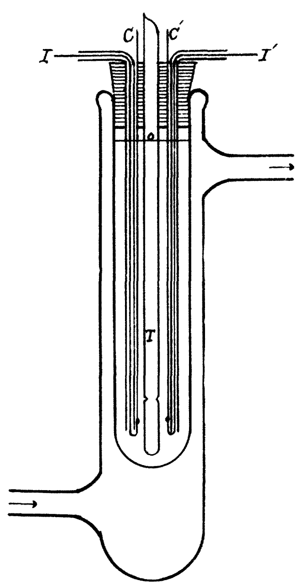

Fig. 11. Arrangement for controlling the temperature of thermo-electric couples.

The strength of the longitudinal current sent through the strip m should be adapted to the case in hand, the weakest current that will give satisfactorily large transverse effects being the best. Through a gold strip about 2 cm wide and 0.0125 cm thick, I used at times a current of about 30 amp.; but a current of equal strength sent through a palladium strip of like dimensions heats the middle many degrees, and is therefore objectionable unless a high temperature is desired.

Within the limits of accuracy of my observations, the transverse effect, whether of potential or of temperature, is, other things being equal, proportional to the strength of the longitudinal current. I do not, however, take this proportionality for granted in dealing with any new metal, and it is my custom to test it by experiment.

Measurement of the Ettingshausen Effect

This effect consists of a small difference of temperature, sometimes not more than 0.01 degree in my experiments, between the same two points at which the attachments are made for measurement of the Hall effect. In fact, unless unusual precautions are taken to prevent the combination, these two effects always coexist. To measure the Ettingshausen effect alone, Unwin,[5] who has done good work on all four of the transverse effects, did not establish metallic contact of thermo-electric junctions with the two points in question, but used an interposed thin sheet of mica, making correction for the failure of this device to establish

[5] Unwin, Proc. Roy. Soc. Edin. **34**, p. 208 (1913-14), and **41**, p. 44 (1920-21).

complete equality of temperature between the junctions and the points on the plate at which they were applied. Thus he had only a single thermo-electric circuit and could connect this with an ordinary sensitive galvanometer in the ordinary way.

My method, much less simple in some aspects, has been to employ two independent thermo-electric couples, one having a junction soldered to one of the two points on the plate (see Fig. 8), the other having a junction soldered to the other of these points, and to connect these two couples to the coils of a galvanometer used differentially. The second junction of each couple is maintained at a known temperature, by being placed beside the other in an oil bath like that shown in Fig. 11.

My galvanometer, the same that I have used for measuring the Hall effect, was not intended for differential use, having only a single needle-group. To use it differentially for purposes of careful measurement was a somewhat novel and venturesome experiment, involving a rather complicated and laborious operation for determining the sensitiveness of the instrument as thus used. This operation ran as follows: With the two thermo-electric circuits sending their currents in opposite directions, one through coil A, the other through coil B of the galvanometer, the deflection produced by the introduction of a known e.m.f. into the circuit of coil A was measured, and next, with both the opposed currents reversed in the galvanometer coils, the effect of the same e.m.f. was measured again. Then corresponding measurements were made by introducing a known e.m.f. into the circuit of coil B. The variation of sensitiveness indicated by comparison of any two of these four measurements was often many percent, but the steadiness of the mean value of the four measurements through months of observation gave me great confidence in the substantial correctness of this mean. Nevertheless, I hope to use a properly wound differential galvanometer for measurement of the Ettingshausen and the Righi-Leduc effects hereafter.

Assuming it possible to keep the two oil-bath junctions (in Fig.11) at the same temperature, one might suppose it to be a matter of indifference what this temperature was to be, but in practice it is found best to keep this temperature as nearly as may be the same as the mean temperature of the junctions on the plate m. If the difference between these two temperatures becomes large, there is so great an e.m.f. in each of the two opposing circuits that a slight change of resistance in one of them, such as may be due to local temperature influence, may upset their equilibrium enough to produce very troublesome galvanometer deflections. It is true that a compensating e.m.f. from a storage cell may be introduced into each circuit in such a way as to make the net e.m.f. in each nearly zero, and of this device I make habitual use; but even so it is best to have the temperature difference in question as small as practicable; for storage cells are not always sufficiently constant in their e.m.f. to be depended on for much "compensating" action.

The two junction-points near the bottom of the oil-bath should be brought nearer together than they are shown to be in Fig. 11, due care being taken to prevent conductive contact of the two circuits, and these junctions should be symmetrically placed, so that, if possible, neither will be affected before the other by any change of temperature of the water stream surrounding the oil bath. Moreover, all thin wires in either circuit, and especially all junctions of thin wires, should be covered in such a way as to protect them from sudden temperature changes, such as a varying current of air or changing illumination might produce. Attention to such details may save a great deal of time and worry in the galvanometer measurements.

After the transverse temperature difference existing between the two side junctions on the plate *m* has been found by experiment, correction (a) should be applied, and to the value thus found corrections (b) and (f) should be added.

The Nernst Effect

This effect, a transverse potential-difference accompanying a longitudinal temperature-gradient across the magnetic field, is not usually difficult to discover or even to measure. The longitudinal temperature-gradient, however, is naturally less convenient to deal with than a longitudinal electric current. I have said about all that I need say in regard to this matter, in my discussion of corrections (d) and (e). I do not, as a rule, have much trouble with inconstancy of this temperature-gradient, though it should be measured frequently.

Within the limits of accuracy of my observations, the Nernst transverse potential-differences, other things being equal, are proportional to the steepness of this gradient.

To reach the final value of the coefficient Q, corrections (a), (d), (e), (c) and (b) should be applied. The application of correction (c) requires knowledge of the transverse temperature difference, due to the Righi-Leduc effect, which accompanies the Nernst effect.

The Righi-Leduc Effect

This effect, a transverse temperature-difference accompanying a longitudinal temperature-gradient, across the magnetic field, is perhaps even more troublesome to measure than the Ettingshausen effect, but I have already discussed most of the difficulties likely to be encountered here.

Within the limits of accuracy of my observations, the transverse temperature difference is, other things being equal, proportional to the longitudinal temperature gradient. The corrections to be applied are (a), (d), (e), (f), and (b).

Quality of Metals; Numerical Results

The four metals that I have studied especially during the past two years are gold, palladium, nickel and cobalt.

The gold is the purest and softest obtainable from the S. S. White Dental Manufacturing Company of Boston.

The palladium is the purest that could be furnished me in February 1922, by Baker and Company of Newark. It was "of special high purity but not guaranteed chemically pure."

The nickel plate was given me by Leeds and Northrup and probably is of a high degree of purity, though its chemical analysis has not been made.

The cobalt was given me in 1913 by Dr. H. T. Kalmus of the Research Laboratory of the School of Mining, Kingston, Ont. He wrote "our cobalt runs very erratically from substantially pure metal to metal with nearly 1 percent of impurities." The plate probably contains some iron and nickel; as I have already indicated, it is decidedly brittle.

A summary table of the results obtained with these metals was given in the Proceedings of the National Academy of Science for July 1925, but as the values there given for P in palladium are all about 7 percent too large, the whole table, with the necessary correction, is here repeated.

Table IV

Values of R, P, Q and S in certain metals

Metal	Temp.	$R\times10^6$	$P\times10^9$	Temp.	$Q\times10^6$	$S\times10^9$
Gold	17°	− 705	− 1.2	25°	− 181	−300
	72	− 696	− 0.4	57	− 181	−263
Palladium	26	− 845	+ 16.2	25	+ 327	− 48.7
	45	− 855	+ 17.8	45	+ 326	− 41.4
	64	− 844	+ 20.6	67	+ 335	− 37.8
Nickel	22	−4520	+ 60.6	38	+2590	−528
	57	−5910	+105.7	57	+3040	−494
	86	−7280	+154.2	77	+3660	−447
Cobalt	19	+3550	+ 81.0	25	+1900	+377
	48	+4440	+109.0	47	+2190	+429

A more detailed statement for one of the metals, palladium, may be of interest here, as it will give some idea of the precision of the measurements, their agreement or disagreement among themselves, and of the range of experimental conditions. In the following table, T indicates the temperature at mid-point of the plate under examination, I is the current density in amperes per square centimeter of cross-section of the plate, G is the longitudinal temperature-gradient. The intensity of the magnetic field varied from about 8700 to about 9200.

TABLE V

Sample results for palladium

These were obtained on various dates in Nov. and Dec. 1924 and in Jan. 1915 as indicated in parentheses; (N4) = Nov. 4 etc.

T	I	$R\times10^6$	$P\times10^9$	T	G	$Q\times10^6$	$S\times10^9$
28.0°C	414	−852(N4)		21.5°C	4.2	+327(D9)	
24.0	571	−836(D1)		24.7	4.4	+328(D23)	
28.0	410		+16.1(N4)	26.2	3.8	+326(J31)	
25.0	567		+16.5(D4)	24.5	4.2		−46.5(D10)
43.6	399	−862(N6)		26.1	4.4		−50.8(D22)
44.0	976	−846(D1)		45.8	10.9	+329(D30)	
47.0	633		+17.6(N28)	45.1	5.5	+322(J9)	
45.0	962		+18.2(N29)	45.4	10.6		−412(J2)
64.0	952	−844(N26)		45.2	5.6		−414(J7)
63.0	957		+20.3(N24)	67.0	5.5	+336(J13)	
64.0	943		+21.1(N26)	68.0	5.3	+332(J28)	
				66.0	5.4		−38.3(J15)
				66.8	5.0		−37.5(J30)

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