THE SCHOTTKY EFFECT IN LOW FREQUENCY CIRCUITS

By J. B. Johnson

Abstract

(1) *In the absence of space charge.* This effect, discovered by Schottky, which depends on the probability fluctuations of electron emission from a filament, has been measured over a considerable range of conditions in resonant circuits of which the natural frequency was varied from 8 to nearly 6000 p.p.s. The effect is much larger in the lower range of frequencies than the theory predicts. *With a tungsten filament,* the ratio of observed to theoretical effect e'/e is about .7 for frequencies above 200, but increases rapidly to 50 at 10 cycles per sec. *With an oxide coated filament,* the ratio increases from 1 at 5000 cycles to 100 at 100 cycles. This is interpreted to mean that the emission of electrons is not strictly chaotic but is influenced by irregular temporal changes in the cathode emissivity. In a high frequency circuit these changes become imperceptible and the emission is effectively random. (2) *When current is limited by space charge* the Schottky effect decreases because of the interaction of the electrons, and other disturbances may act upon the space charge so as to completely mask the remanent Schottky effect. The magnitude of the disturbances in amplifying vacuum tubes can therefore not be predicted from measurements on the true Schottky effect.

THE thermionic current in a vacuum tube is not a smooth flow of electricity, but is subject to rapid and irregular fluctuations in magnitude. These fluctuations, discovered by W. Schottky and called by him the "Schrot-Effekt" (small shot-effect) are caused by the *random* emission of electrons from the cathode, and are made manifest by voltage or current fluctuations in any circuit into which the tube is connected. When these irregularities are sufficiently amplified by other vacuum tubes to be perceivable by means of a telephone receiver, they give rise to a continual sound with no definite pitch, like that usually associated with the conch shell, or like the faint sound that is heard when the hand is cupped to the ear.

The mathematical theory of the Schottky effect[1] is based upon the assumption that *the electrons pass from the cathode to the anode of the tube independently of one another.* Interpreting this condition by the simple laws of probability, it has been shown that the magnitude of the fluctuations of voltage or current in the circuit associated with the tube should depend only upon the charge on the electron, the average space current,

[1] W. Schottky, Ann. der Phys. **57,** 541 (1918); idem. **68,** 157 (1922).
J. B. Johnson, Ann. der Phys. **67,** 154 (1922).
T. C. Fry, J. Franklin Inst. **199,** 203 (Feb. 1925).

and the circuit constants. This prediction has hitherto either failed of verification[2] or has been verified only in part.[3]

It is the purpose of this paper to present the results, first, of a considerable series of measurements on the Schottky effect as it appears in circuits of comparatively low natural frequency, and secondly, of some preliminary studies upon the possible relation between the Schottky effect and the "noise" in vacuum tube amplifiers.

The measurements made under the conditions in which the Schottky effect should appear, i.e., in two electrode tubes with no appreciable space charge, show some striking deviations from the theory. While in the higher range of frequencies that were studied or with very low space currents the effect approached the predicted value, at the lower frequencies it is much too large. Particularly interesting is the behavior of oxide coated cathodes which give an effect up to hundreds of times too large at the lowest frequency, remarkable when we consider that this state of affairs can only be caused by a sequence of electron emissions different from the postulated purely random one. Except for the fact that this larger effect varies with the frequency and the space current, the circuit responds to it much the same as to the purer Schottky effect at the higher frequencies.

In amplifier tubes on the other hand, space charge is always present and makes the electrons pass in a sequence more orderly than the purely chaotic one. In fact, in these tubes the Schottky effect is so profoundly affected by the presence of space charge that even in the same tube there is little correlation between the noise with and without space charge. Furthermore, when space charge limits the current in a tube the opportunity also exists for agencies to come into play that can alter the current by acting upon the space charge. Tungsten filament tubes are usually subject to disturbances of this kind, so large and frequent that they entirely mask the reduced Schottky effect. Wehnelt cathodes are much less productive of this kind of disturbance so that consistent measurements can be obtained with tubes having oxide coating filaments.

The measurements here reported were done with apparatus in which the final indicating instrument was a thermocouple and micro-ammeter. Since the measured effect did not in general agree with the theory, the

[2] C. A. Hartmann, Ann. der Phys. **65,** 51 (1921); Phys. Zeits. **23,** 436 (1922). Hartmann's method of measurement could not give correct results, as pointed out by R. Fürth (Phys. Zeits. **23,** 354, 1922).

[3] A. W. Hull and N. H. Williams, Science, Aug. 1, 1924, p. 100; Phys. Rev. **25,** 147 (Feb. 1925). These authors obtained a check with the theory using a circuit of high natural frequency.

measurements were carried out for a number of tubes and covered a considerable range of frequencies, current values and circuit constants. The evaluation of the data was done according to the exact relation derived by Fry rather than by the approximate formula of Schottky.

Method of Measurement and Computation

For the purpose of measuring the Schottky effect the space current of the tube was passed through the inductance and resistance of a damped resonant circuit. The voltage fluctuations set up across this circuit were amplified with comparatively small distortion, by an amplifier[4] which terminated in a vacuum thermocouple and micro-ammeter. The measuring system therefore indicated the effective square of the voltage surges over the resonant circuit.

This measuring circuit, comprising the amplifier and thermocouple, was calibrated by sinusoidal alternating voltage over the whole range of frequencies of the variable resonant circuit, extending from about 8 p.p.s. to nearly 6000 p.p.s. The calibrating voltage was obtained from a vacuum

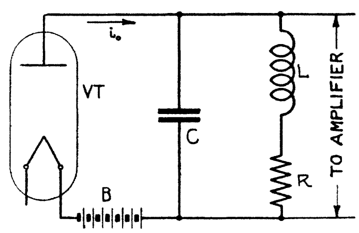

Fig. 1. Experimental circuit.

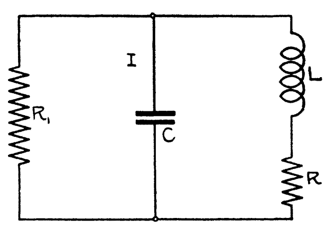

Fig. 2. Equivalent experimental circuit.

tube oscillator and was attenuated to the necessary small fraction of a measured initial value by a two-step resistance potentiometer. The resistances in this network were arranged and shielded with special care with a view to avoiding induction effects between the branches.

The circuit immediately connected to the experimental tube is shown diagrammatically in Fig. 1. The tube itself is represented by VT, B is a dry cell battery, usually of 360 volts, while L, C and R are the inductance coil, condenser and resistance in the resonant circuit. The condenser C was a mica condenser of negligible effective resistance. It was variable in steps from $1\mu f$ to $.001\mu f$. For the lowest frequencies this was supplemented by paper condensers of up to $7.5\mu f$ capacity of known effective resistance. The resistance R was a non-inductive decade box. For the

[4] The amplifier consisted of four stages with Western Electric 102-D tubes and one stage using a 101-D tube, all resistance-capacity coupled. The total voltage amplification was about 3×10^5 and the power amplification was 10^{14}.

inductance L four separate coils were used, each calibrated over the pertinent range of frequency and direct current value for effective inductance and series resistance. One of these coils, A, whose inductance was .100 h, was wound on a toroidal wood core. The other three were wound on cores of silicon steel with air gaps, and their inductances were about .1 h, .025 h, and 38 h, respectively and will be referred to as coils B,C and D.

The experimental circuit includes also the input elements of the amplifier, but electrically it can be represented by Fig. 2. In this diagram R_1 is the resultant of three resistances in parallel, namely, the resistance to alternating current of the experimental tube at any particular condition, the grid leak resistance of the first amplifier stage and the effective input shunt resistance of the tube in this stage. In the value of C must be included also the input capacity of the first amplifier stage, usually negligible. These various values were obtained by suitable measurements.

The measurements that were made upon the Schottky effect in this circuit will, in what follows, be expressed in terms of the mean square voltage $\overline{V^2}$(obs.) across the resonant circuit at the input of the amplifier. This quantity is given simply by the product of the deflection of the thermocouple meter (less the zero reading caused by inherent amplifier noise) and the calibration constant of the measuring system, which was calibrated in terms of voltage squared.

This measured quantity is to be compared with the corresponding quantity $\overline{V^2}$(calc.) which is obtained from the known conditions and circuit constants as shown by Fry.[1] Fry has developed a general formula for the Schottky effect which is applicable to any circuit. However, the specific example given by Fry covers exactly the arrangement of apparatus shown in Fig. 2, so that his Formula (13) can be used directly in this case. With certain changes in the notation,[5] made for convenience in the presentation, this formula reads,

$$\overline{V^2}\,(\text{calc.}) = \frac{ei_0}{2C^2}\,\frac{L}{R(1+L/RR_1C)}\left[1+\frac{C}{L}\,\frac{R^2}{(1+R/R_1)}\right].$$

in which

e = charge on the electron;

i_0 = average space current;

L = effective inductance;

C = total capacity;

R = effective resistance in the inductance branch;

[5] Fry's formula is expressed in terms of S, the mean power in the thermocouple, instead of the mean square voltage over the resonant circuit.

R_1 = total shunt resistance, including that of the experimental tube, grid leak and amplifier tube.

Great accuracy was not striven for in this work because of the large differences found to exist between measurements and theory. Taking into account all the possible sources of error which could enter the observations, calibrations and measurements of circuit constants, the probable error of a single comparison between an observed and a calculated value of V^2 is about five percent.

Procedure and Results

For certain preliminary work the thermocouple in the measuring circuit was replaced by a pair of telephone receivers which could also be connected to a variable source of current giving a sound similar in character to that produced by the amplifier. By the use of this modified circuit a considerable number of tubes were examined under identical conditions in order to determine qualitatively the influence of structural factors upon the amplitude of "noise" produced when the space current is limited by the cathode temperature so that space charge is practically absent. These tubes differed from one another, in the first place, as to the arrangement of the electrodes, some being standard audions while others were special tubes having the cathode and anode in various dispositions. Some of the tubes had the electrodes insulated with extreme care, and one tube had the electrodes enclosed in a grounded metal cylinder so that no part of the glass was exposed to the light or the electron current from the filament. The anode material was nickel in most of the tubes, but in some platinum or tungsten was used. The tubes had filaments of different materials such as platinum coated with rare earth oxides by various methods, commercial tungsten, pure tungsten and single crystal tungsten.

In the amplitude of sound which they produced these tubes differed from one another considerably, by factors perhaps as large as ten. Among all the variables, however, only that of the filament material showed a definite correlation to the sound. The sound from the tubes with oxide coated filaments was on the average several times larger than that from the tungsten filament tubes, but of the same quality so far as could be judged. In one tube the gas pressure was varied while the sound was observed, but there was no appreciable increase in the loudness until the pressure was so high that a glow began to be visible in the tube. In another case the tube was exposed to various amounts of illumination without any effect on the sound.

From the tubes thus tested, three were selected as representative of their groups, for the quantitative measurements. Two of these tubes were standard Western Electric 102-D audions with plane-parallel plates and grids and a V shaped ribbon of oxide coated platinum as cathode. These will be referred to as tubes No. 2 and No. 3. Tube No. 1 was similar in structure but had a filament of pure tungsten. When they were used as two-electrode tubes the grid and plate were connected together so as to form but one anode.

The results of the observations on these tubes will be presented so as to show the relation between the observed and the calculated value of $\overline{V}^2$, first as the circuit constants C, L and R, the natural frequency of the resonant circuit, and the space current i_0, are varied in the absence of space charge; and secondly, as space charge limits the current in the two-

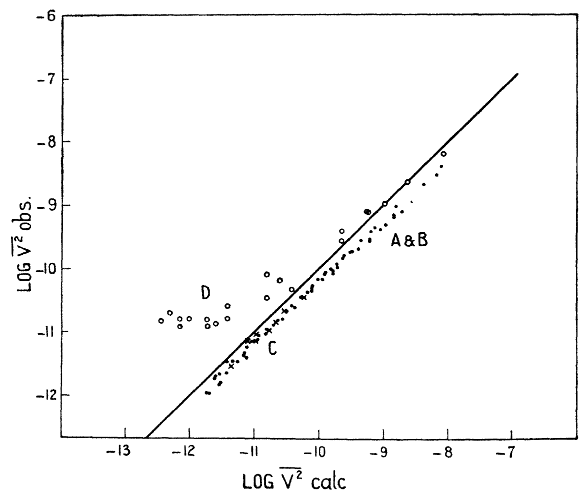

Fig. 3. Capacity and inductance variation for tube No. 1, tungsten filament; current 5 m-a.; various inductances A, B, C and D; frequency 8 to 6000 p.p.s.

electrode or three-electrode tube. The solid line in the diagrams is in each case the locus of the theoretical value of $\overline{V}^2$. The letters A, B, C and D on the diagrams refer to the inductance coils that were used in the resonant circuit for the groups of points which the letters designate.

A. The Schottky Effect in the Absence of Space Charge

In Figs. 3 and 4 is shown the relation of the observed to the calculated values of $\overline{V}^2$ for tubes No. 1 and No. 2 respectively, the capacity of the resonant circuit being varied between the limits 7.5μf and .01μf. The observations for the tungsten filament tube with the smaller inductance lie nearly parallel to the theoretical line but somewhat below it, as if the charge of the electron were about two-thirds of what it is known to be. With the 38 henry coil the observed values of $\overline{V}^2$ become increasingly too high with increasing capacity, being about 50 times larger than the

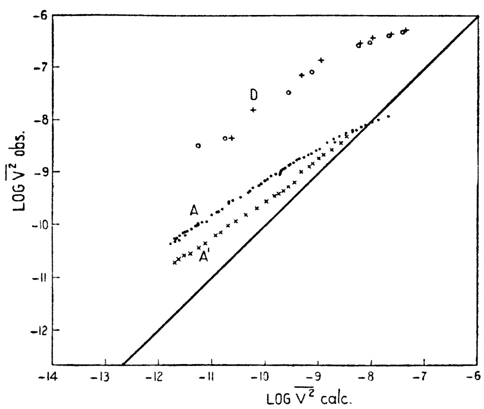

Fig. 4. Capacity and inductance variation for tube No. 2, coated filament; current 5 m-a.; two inductances A and D. Curve A′ was taken at a different time.

theoretical value at the largest capacity. The oxide coated filament tube yielded values that are too high over nearly the entire range and approach the theoretical values only at the lower capacities. Oxide filaments also

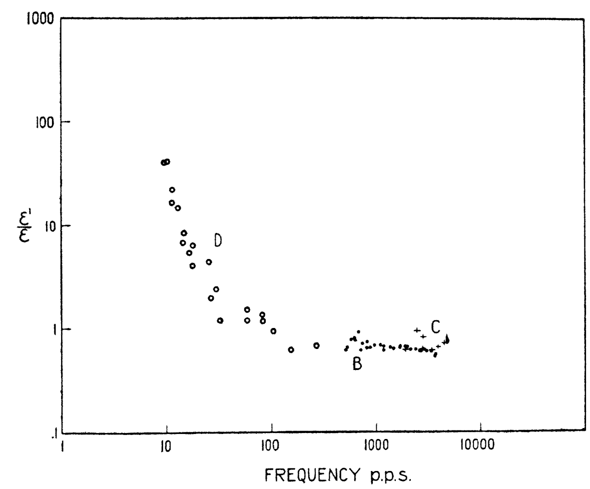

Fig. 5. Frequency variation for tube No. 1, tungsten filament; same data as in Fig. 3 plotted to frequency scale.

show a greater change with time than has been observed with tungsten filaments, as illustrated by the curves A and A′ of Fig. 4, obtained with the same tube under identical conditions but at different times.

The variable which causes the divergence from linearity of the curves of Fig. 3 and Fig. 4 and which does not appear in the theory is the frequency of the resonant circuit. This is definitely shown upon plotting to a frequency scale, as has been done for the previous data in Figs. 5 and 6, and in Fig. 7 for other data. The ordinates are the ratios of the apparent to the actual charge of the electron, which is the same as the ratio of the observed to the calculated values of $\overline{V^2}$. The logarithmic scales of Figs. 5 and 6 cover a larger range, while the linear scale of Fig. 7 brings

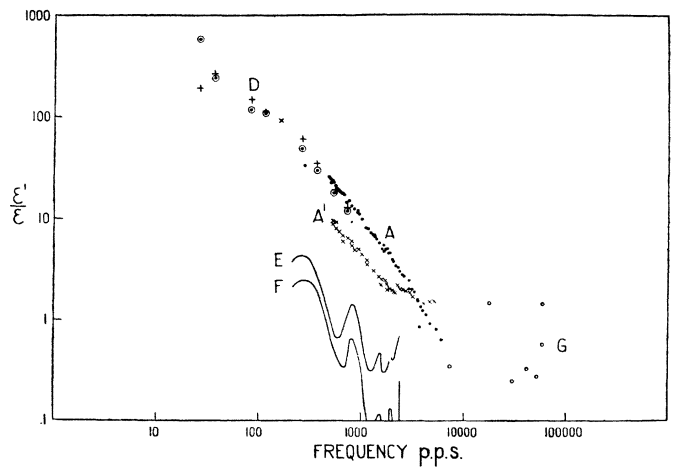

Fig. 6. Frequency variation for tube No. 2, coated filament; same data as in Fig. 4 plotted to a frequency scale; curves E and F give Hartmann's results for 2 m-a. and 20 m-a.; points G were obtained with less steady measuring circuit.

out more strongly the course of events in the higher frequency range. The fact that the curves obtained with the various inductances join up where they overlap in frequency shows clearly that the *frequency* of the resonant circuit is the determining factor in the large discrepancy between observation and theory.

The points marked G in Fig. 6 were obtained with a less reliable circuit and are included merely to show that the order of magnitude, at least, of the observed ratio does not lie far from unity up to the natural frequency of 100,000 p.p.s. The solid lines E and F in the same figure are plotted, for a comparison, from Hartmann's data for a tantalum filament at the space currents of 2 milli-amperes and 20 milli-amperes.[6]

Even when the ratio of the observed to the calculated value of $\overline{V^2}$ is considerably larger than unity, it is still independent of the effective damping resistance in the resonant circuit. Fig. 8 presents a series of

[6] C. A. Hartmann, Phys. Zeits. **23**, p. 436 (1922).

observations in which the resistance R was varied from 29 ohms at the upper end of the curve to 1029 at the lower end. This last value is about half the critical damping resistance of the circuit, and yet the points lie on a straight line within the errors of measurement.

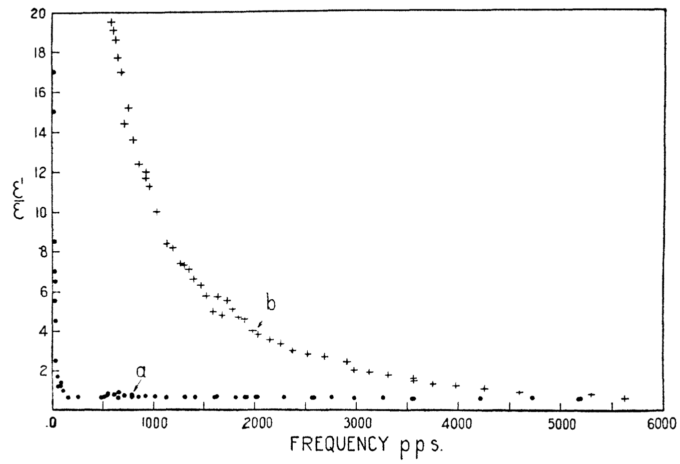

Fig. 7. Frequency variation plotted to a linear frequency scale.
a. Tube No. 1, tungsten filament; inductance A; current 5 m-a.
b. Tube No. 2, coated filament; inductances B and D; current 5 m-a.

The curves of Figs. 3 to 8 have to do with the dependence of the observed effect upon the circuit constants and show that except in so far

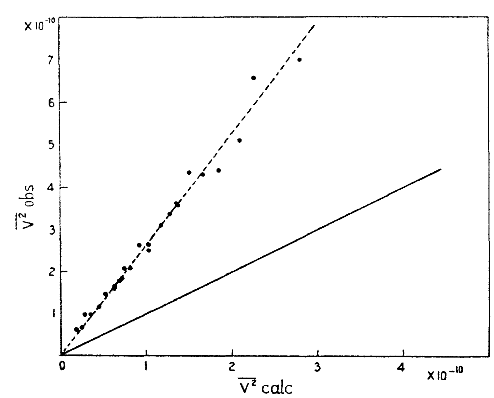

Fig. 8. Resistance variation for tube No. 3, coated filament; inductance A; capacity .07μf; frequency 1900 p.p.s.; current 5 m-a.; resistance 29 to 1029 ohms.

as the circuit constants determine the *frequency* of the circuit their influence is that predicted by the theory. This fact, as well as that different tubes give different results, shows that the deviations of the observed ratio from unity is not caused by failure of the theory to use the circuit constants correctly but by the inadequateness of the original assumptions at the basis of the theory.

Curves relating the observed and calculated voltage fluctuations as the space current is varied throw interesting light on the subject of these deviations, especially again in the case of the oxide coated filaments. Fig. 9 gives two such curves for a tungsten filament, and Fig. 10 gives a series of curves obtained with an oxide coated filament, the space current being controlled by the filament temperature. In the measurements on the tungsten filament tube the current i_0 ranged from 10 milli-amperes to .01 milli-ampere, and in this thousand-fold range there is no marked deviation of the curve from linearity although the observed values are

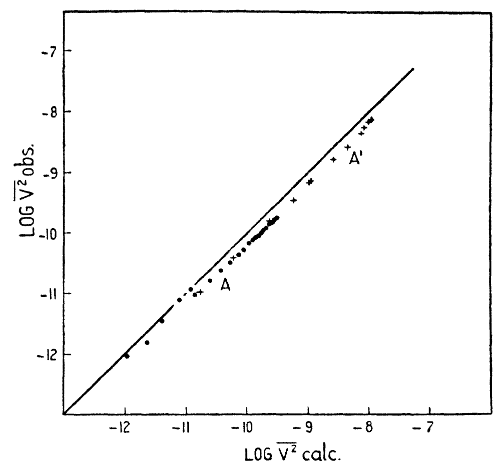

Fig. 9. Current variation for tube No. 1, tungsten filament; inductance A; capacity .10μf for A, .01μf for A_1; frequency 1590 and 5300 p.p.s.; current 10 to .01 m-a.

low by about the same amount as they are in Fig. 3. Experimental obstacles prevented getting a curve at the low frequencies where, as in Fig. 3, the observed $\overline{V^2}$ is much too high.

In the corresponding curves for the oxide coated filaments, Fig. 10, the space current range was from 5 milli-amperes down to .1 milli-ampere in most cases. The remarkable feature about these curves is that they all approach the theoretical line as the space current is decreased. It is as if with decreasing current, or perhaps decreasing filament temperature, the ideal conditions were approached which are assumed in the theory and which for the same current values more nearly exist in the tungsten cathode tubes.

B. Schottky Effect and Space Charge

The modification of the Schottky effect caused by the appearance of space charge in a two-electrode tube is shown typically in Fig. 11. For this

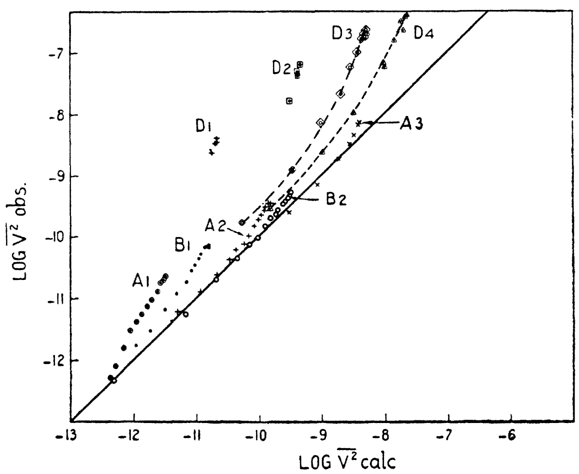

Fig. 10. Current variation for tube No. 2, coated filament; inductances as shown by letters; capacities A_1, .80μf; A_2, .10μf; A_3, .010μf; B_1, .20μf; B_2, .02μf; D_1, 1.00μf; D_2, .10μf; D_3, .0101μf; D_4, .0022μf; current range 5 to .1 m-a.

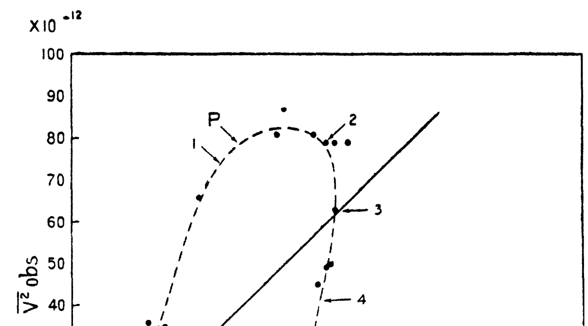

Fig. 11. Effect of space charge for tube No. 2; inductance A; capacity .10μf; frequency 1590 p.p.s.; current in m-a. given by figures.

experiment the anode potential was reduced to 9 volts and the space current was controlled by the cathode temperature. The space current values, in milli-amperes, are indicated by the figures attached to the arrows. The curve relates to tube No. 2, with oxide coated filament. As was the case with the higher anode voltage the observed values of $\overline{V^2}$ increase faster than the calculated. At the current of one milli-ampere, however, space charge begins to be appreciable and the observed values of $\overline{V^2}$ decrease until at 10 milli-amperes it is a small fraction of the calculated value.[7] Above ten milli-amperes[8] the observed values again increased and became very erratic, and the sound when heard in the telephone was characteristically changed.

Tungsten filament tubes show this effect of increasing noise in the space charge region to a much larger degree and at a much sharper limit. At the current value where the effect of space charge is just discernible on the space current-temperature diagram, corresponding to the point P in Fig. 11, current pulses begin to be superimposed on the more regular Schottky effect. These become more frequent as the filament temperature is increased until thermocouple measurements are difficult or impossible. An attempt to estimate the magnitude of these pulses was made by comparing them with the effect of discharging a small condenser through some part of the circuit. The estimates indicated that the pulses were equivalent to the passage, or the failure to pass, of a charge of 10^5 to 10^8 electrons.

If these current pulses were produced by the sudden emission, or failure of emission of a group of electrons from the filament they would, like the Schottky effect, be most distinct in the absence of space charge and would be suppressed in the space charge region. Since the opposite is the case they must be caused by something which acts upon the space charge and thereby varies the current. The evidence points to the conclusion that the "clicks" are caused either by ions which pass through the space charge or by ions which are held in the space charge region near the filament for some time. In the case of a current pulse of 10^8 electrons, a calculation shows that this could be produced by the passage from the filament to the plate of 2.3×10^6 hydrogen ions, of 1.7×10^5 singly charged tungsten atoms or of one singly charged tungsten particle 2×10^{-3} mm in diameter. The large size of the particle seems to rule out the last hypothesis, but there is as yet no other discriminatory evidence.

The phenomena which space charge introduces in the two-electrode tube are also present in the three-electrode tube where the current is

[7] The calculated value of $\overline{V^2}$ fails to rise linearly with increasing space current because of the lowered internal resistance of the tube, which in turn greatly lowers R_1.

[8] The saturation current for the oxide coated filament is not well defined, and the higher currents were reached by heating the filament considerably above the normal temperature.

controlled largely by the artificial space charge of a grid. Fig. 12 contains the data obtained with the tungsten filament tube No. 1, plotted similarly to those of Fig. 11 for the two-electrode tube. The space current, denoted as before by the figures attached to the arrows, was controlled by the filament temperature. As in the case of the two-electrode tube the noise decreases with the appearance of space charge, but here only to about one-third of the theoretical value as against nearly zero for the two-electrode tube. The fluctuation which remains, however, gives rise to a sound in a telephone of the same character as does the pure Schottky effect and is no doubt a Schottky effect which is modified by the space

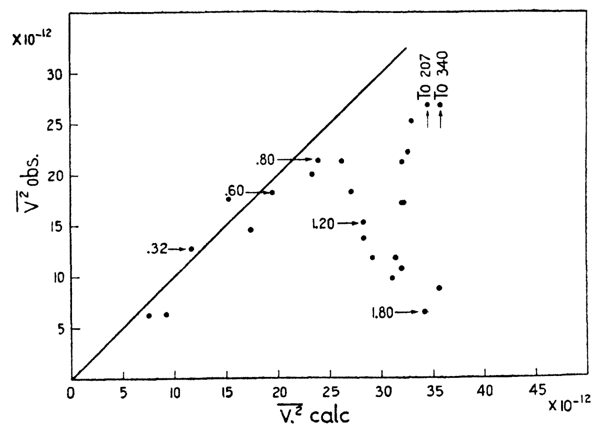

Fig. 12. Space charge in three-electrode tube No. 1; inductance A; capacity .10μf; current in m-a. given by figures.

charge. This conclusion seems supported by the fact that there is a relation between the noise a tube produces and the amplification as measured in the same condition, such that the ratio of noise to amplification is never less than a certain value. This is made clear by Fig. 13 in which each circle indicates in arbitrary values the measurement of noise and of amplification of one tube. A wide variety of tubes were used for this set of measurements. The circles of the diagram lie to the right of a straight line through the origin with only one or two exceptions. It seems probable that this line represents the residual of the Schottky effect in tubes when other disturbances are absent.

The inherent "noise" in vacuum tube amplifiers cannot, therefore, be estimated by the theory of the Schottky effect as it now stands. This is so partly because the theory has not been extended to take into account the presence of space charge, and still more because the remanent Schottky effect is usually masked by other disturbances which become effective when space charge appears.

Discussion

The experimental results cited here give force to the contention of Fry[1] that the Schottky effect is not a proper means for measuring the charge on the electron. It is rather a phenomenon which enables us, with the aid of *the already known value of e*, to determine the truth or falsity of the basic assumptions underlying the theory. The assumption which has the least foundation on previously known facts, instinctive though it may be, is that which concerns the random emission of electrons, and it is to this assumption that the test of the experiments should be, and here has been, applied. The discrepancies existing between theory and experiments, with the striking introduction of the frequency as an independent vari-

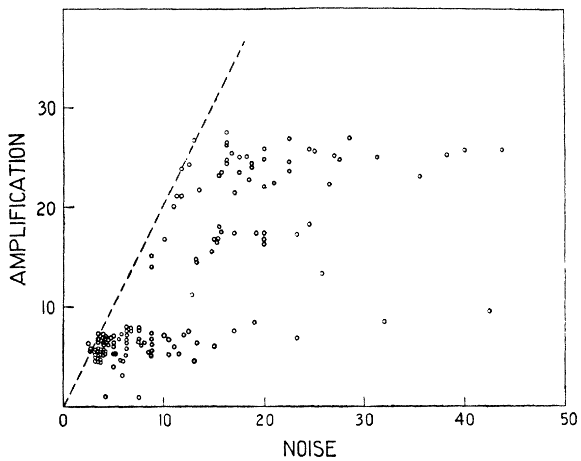

Fig. 13. Amplification as a function of noise in three-electrode tubes; noise in arbitrary units; each point represents a tube.

able, are clearly caused by the fact that the electron emission does not strictly follow the chaotic sequence assumed in the theory. However, since this deviation takes a much simpler course than Hartmann's results indicated, it yields to an explanation which seems more plausible than that offered by Hartmann and Schottky.[9]

The Hartmann-Schottky hypothesis is that the cooling produced on a small surface of the cathode by the emission of one electron makes the emission of another electron from the same area less probable for a time comparable with the natural period of the resonant circuit. The present data, however, do not show the maxima and minima in the $\overline{V^2}$-frequency curve that are so prominent in Hartmann's curves (Fig. 6). Further-

[9] C. A. Hartmann, Ann. der Phys. **65**, 51 (1921);
W. Schottky, Ann. der Phys. **68**, 157 (1922).

more, although such low frequencies were attained (down to 8 p.p.s.) that the time required for the small surface cooled by the emission of one electron to reach equilibrium must be very small compared with the period of the circuit, yet even at this low frequency the ratio of the observed to the calculated quantity was rising rapidly. The above hypothesis seems, therefore, to be no longer either necessary or tenable.

An assumption of a somewhat different nature leads to conclusions which agree better with the facts. We may suppose that the chance of an electron emission is influenced not so much by previous emissions as by other factors which change irregularly with the time, chiefly the activity of the filament. The electron emission at any time depends upon the condition of the cathode surface, and the surface is probably in a continual process of change due to such causes as evaporation, diffusion, chemical action, structural rearrangements and gas ion bombardment. These changes would go on at different rates in different parts of the surface and would cover areas very large compared with that involved in the emission of one electron. The changes involving the greater area or larger amplitude might be expected to require a longer time, while changes so small as to affect only a few electrons would lose their significance in the general statistical emission. The general effect of these changes would be a variation in the total space current superimposed upon the fluctuation of the Schottky effect, having little influence at the high frequencies and an increasing effect as the natural frequency of the measuring circuit becomes lower. Again, the effect might well be greater for the composite surface of an oxide coated cathode than for one of a relatively pure metal, and the effect might be more dependent upon the temperature with a cathode of the former type.

The sum of these considerations, therefore, seems favorable to the view that the large discrepancies between theory and experiment at the lower frequencies are caused by secular changes in the surface conditions of the cathode. The values of $\overline{V^2}$ which are too low, on the other hand, sometimes as low as one-fifth of the theoretical value, require a different explanation, but no adequate explanation has been found.

BELL TELEPHONE LABORATORIES, INCORPORATED,
NEW YORK, N. Y.
January 8, 1925.*

* Received April 11, 1925—Ed.