

A REINVESTIGATION OF THE WAVE-LENGTHS AND  
RELATIVE INTENSITIES IN THE MOLYBDENUM  
K SERIES X-RAY SPECTRUM

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ABSTRACT

The x-ray beam from a water-cooled target of a tube carrying 25 m-amp. from a 50 kv storage battery, was limited to 2 minutes and reflected from a calcite crystal of a spectrometer with a Genevoise scale reading to 2 seconds, into an ionisation chamber. The 3rd to 5th orders were used. The angle of reflection for each wave-length was determined by readings on both sides of the direct beam. Corrections were applied for the temperature of the crystal above 18°, for the eccentricity of the scale and for refraction. *The wave-lengths* are found to be (taking  $d=3.028\text{\AA}$ ):  $\alpha_2=.711835$ ,  $\alpha_1=.707525$ ,  $\beta_3=.631354$ ,  $\beta_1=.630791$ ,  $\gamma=.619526$ . From 3 to 5 determinations were made for each line, the average deviation from the mean being .000025. The mean deviation from the last results of A. Leide is .00003 except for  $\beta_1$  for which his value seems too high. The separation  $\beta_3-\beta_1$  is .000563 in good agreement with the separation  $L\beta_3-L\beta_4$  determined by Coster. The wave-lengths taking  $d=3.02904\text{\AA}$  and the  $\nu/R$  and  $\sqrt{\nu/R}$  values for the K lines and for the corresponding energy levels in the Mo spectrum are also given. The K absorption limit was found at .61830A. *Relative intensities.* Assuming each component of the  $\beta$  doublet to be a single line of the same width, the relative intensities come out  $K\beta_1/\beta_3=2.0$ , in agreement with the theoretical predictions of Sommerfeld and of Coster and Goudsmit. In the case of the  $\beta$  and  $\gamma$  doublets the total intensity of each doublet was taken proportional to the area under it, correcting for the K absorption in the case of the  $\gamma$  peak. The relative intensities then come out  $K\beta/\gamma=7.7$ , which is greater than the corresponding ratio for tungsten, indicating a decrease with increasing atomic number.

APPARATUS AND METHOD OF MEASURING WAVE-LENGTHS

THIS paper is a report of experiments carried out in the laboratory of Professor William Duane at Harvard University. We have re-measured the wave-lengths of the lines in the molybdenum K series spectrum, and have redetermined the relative intensities of some of the lines which are not widely separated in wave-length.

The apparatus available for this investigation had several advantages over that with which the previous work on the molybdenum spectrum in this laboratory was performed. (1) We had at our disposal an ionisation spectrometer with a Genevoise scale, on which settings could be made to 2 seconds of arc. (2) Tubes with water-cooled targets were employed which permitted the use of much more power than that used

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in the previous work. (3) A new 50,000 volt high capacity storage battery made it possible to operate at higher voltages and currents for longer intervals than before.

The new storage battery and general arrangement of the apparatus have been described.<sup>2</sup> The spectrometer measurements were made by the ionisation method, as used previously in this laboratory. The position of the crystal was read on the accurate silver scale with a traveling microscope, and the ionisation chamber was set approximately by reading its position on a brass scale concentric with the silver scale. Approximate settings of the ionisation chamber were sufficient to follow the reflected beam since the slit in front of the chamber was quite wide (6-8 mm). All angular readings used in the calculation of wave-lengths were settings of the crystal table. The x-ray beam incident upon the crystal was limited by two slits 37 cm apart. The slits were of approximately the same width, and limited the angular breadth of the beam to about 2 minutes of arc. The vertical height of the beam was limited by a stop 1 cm high placed between the x-ray tube and the crystal 5 cm from the crystal.

In the experiments on the wave-lengths of lines, the x-ray tube was run at 25 milliamperes and a constant battery voltage of about 50 kilovolts. The ionisation currents were read on an electrometer and scale system whose sensitivity was about 3 meters per volt.

The wave-length measurements reported here were all obtained in higher orders of reflection from calcite. In the crystal used there was no good evidence of broadening of the lines in higher orders. In a determination of wave-length, the crystal was rotated in steps of 15 seconds each through the angular range in which the center of a line had been found to lie by previous trials. Two settings could thus be located at which the ionisation current was higher than for any other two in the vicinity, and from the relative intensities at these two settings the position of the line could be estimated within 2 or 3 seconds of arc.<sup>3</sup> The crystal table and ionisation chamber were then at once shifted so that the same wave-length was reflected from the crystal on the opposite side of the direct beam, and again the setting of the crystal for the center of the line was determined. The temperature was taken during each observation. The two determinations of angle were made as nearly simultaneously as possible because it was found that from day to day, from unknown causes, the position of the crystal at which maximum reflection in a line was obtained varied through an angular range which

<sup>2</sup> Armstrong and Stifler, *J. Opt. Soc. Amer. and Rev. Sci. Inst.* **11**, 509 (1925).

<sup>3</sup> Cf J. C. Hudson, *J. Opt. Soc. Amer. and Rev. Sci. Inst.* **9**, 259 (1924).

in some cases was as large as a minute of arc. We have made determinations of angles of reflection in which the setting of the crystal was varied in steps of less than 15 seconds, but we believe that no additional accuracy is thereby gained.

In the measurement of the molybdenum K critical absorption wavelength, we used general radiation from a tube with a water-cooled copper target containing a button of tungsten. The absorbing substance was a screen of ammonium molybdate. The width of the beam in the absorption measurements was slightly over 4 minutes of arc. The angular measurements were made in the same way as those for line wavelengths, except that the angular reading corresponding to an absorption wavelength is that of a point on the discontinuity half way between the base-lines of the radiation of shorter and longer wavelengths.

#### CORRECTIONS AND SOURCES OF ERROR

It is well known that in this method of measurement, in which the wavelengths are calculated from angular settings of the crystal, the penetration of the rays into the crystal before reflection introduces no error. Since, also, the face of the crystal may be considered plane, no appreciable error is introduced if the point of effective reflection of the beam does not lie exactly in the axis of rotation of the spectrometer. In our experiments the method of making the point of effective reflection coincide approximately with the axis of rotation of the spectrometer was to limit by a narrow slit the beam entering the ionisation chamber, and then to vary the position of the crystal and other slits until the angle through which the ionisation chamber was turned to obtain maximum intensity for the beam reflected from the crystal was just twice the angle traversed by the crystal over a large range.

If the sleeve which carries the microscopes and crystal does not rotate about the same vertical axis as that which carries the scale, the angle as read on the scale by one microscope will not be the true angle between the two settings of the crystal. The true angle may be ascertained by reading two microscopes, *A* and *B*, at each setting. *A* and *B* on our instrument are very nearly 180° apart. The true angle through which the crystal is turned is the average of the apparent angles as read on *A* and *B*. Before measuring the wavelengths of the molybdenum lines, we determined the correction to be applied to the scale in the region in which angular settings would be taken. We found that a correction of 6 seconds should be added to the apparent angle (35° to 70°) as read on microscope *A*. The eccentricity, of course, varied somewhat with the settings, but the variations were well within the limit of error of locating the

peaks, and we have corrected all readings on microscope *A* in this range by 6 seconds. In the measurement of the wave-lengths of lines, all settings were made with vernier *A* alone. This technique is probably not the best, as it is known that the eccentricity varies somewhat with temperature and other factors; therefore it would be advisable to read both microscopes for every wave-length determination. After the wave-lengths had been measured, we redetermined the eccentricity in the same part of the scale as before and found that it was not the same, but that the variations were well below the error in locating the positions of the lines themselves.

In the measurement of the critical absorption wave-length, the apparatus was slightly modified so as to permit the reading of both microscopes at once. The readings were taken in a different part of the scale, in which the eccentricity correction to be applied to *A* was only two seconds.

The observed wave-lengths were corrected for the temperature of the crystal from the table in Siegbahn's "Spektroskopie der Röntgenstrahlen," p. 87. All corrections were made to a temperature of 18°C.

It is now known that a correction must be applied to the wave-length calculated by the simple Bragg law on account of the fact that the index of refraction of the crystal for the x-rays becomes appreciable when precision measurements are made. The true wave-length in air may be obtained by the formula<sup>4</sup>

$$n\lambda = 2d \sin \theta (1 - \delta / \sin^2 \theta) \quad (1)$$

where  $\lambda$  is the true wave-length in air,  $d$  is the grating space,  $\theta$  is the observed glancing angle, and  $\delta = 1 - \mu$ , where  $\mu$  is the index of refraction of the crystal for the x-rays. Hatley<sup>5</sup> has adopted a suggestion of Bergen Davis and by an ingenious method obtained a measurement of the index of refraction of calcite for Mo  $K\alpha_1$  x-rays. The value given by Hatley for  $\delta$  is  $2.03 \times 10^{-6}$ , which agrees well with that calculated by the Lorentz dispersion formula. In the higher orders of reflection, where  $\theta$  is large, it may be seen from Eq. (1) that the deviation from the Bragg law is very slight. In the first order it is about  $-.0001A$  for Mo  $K\alpha_1$  from calcite. We have neglected the change of index of refraction with wave-length throughout the molybdenum spectrum. We have instead based our calculations on the value given by Hatley, and have used for  $\theta$  an angle lying between the critical angles for Mo  $K\beta$  and Mo  $K\alpha$ . The extent of the correction applied is shown in the tables.

<sup>4</sup> A. H. Compton, *Phil. Mag.* **45**, 1121 (1923)

<sup>5</sup> C. C. Hatley, *Phys. Rev.* **24**, 486 (1924).

A possible source of error of unknown magnitude lies in the ruling of the scale of the instrument itself. It is quite possible that certain regular variations may recur in the rulings of these scales; through intervals of a degree or so five minute marks may be one or two seconds displaced. It seems unlikely that in the large angles measured such a periodic variation would introduce appreciable error. No calibration of the scale of our instrument has been attempted; accordingly our measurements are subject to such errors as may occur in the scale itself.

#### METHOD OF CALCULATION OF WAVE-LENGTH

Different investigators are not yet agreed on the proper value of  $d$  for calcite. Recently A. H. Compton, O. K. DeFoe, and H. N. Beets<sup>6</sup> have redetermined the density and rhombohedral angle of calcite, and have recommended for  $d$  the value used by Siegbahn in the calculation of his results, namely  $3.02904 \times 10^{-8}$  cm at  $18^\circ\text{C}$ . The wave-lengths previously measured in this laboratory and some determined by Bergen Davis and his collaborators have been calculated on the basis  $d = 3.028 \times 10^{-8}$  cm at  $18^\circ\text{C}$ . There seems to be some ground for the opinion that higher values of density should be given more weight than lower values in the averaging of a series of determinations. If this is true, then lower values of  $d$  are more likely to be correct. The wave-lengths reported here are given both on the basis  $d = 3.028\text{A}$  and on  $d = 3.02904\text{A}$  at  $18^\circ\text{C}$ .

Compton and his collaborators report that in their measurements of the densities of representative samples of calcite differences in density greater than the experimental error occur. This at once raises the question of the generality of wave-length determinations based on crystalline diffraction. It seems highly probable that the precision measurements of wave-length now possible should be found to depend on the particular calcite crystal used, on account of the varying composition of calcite. The varying amounts of impurity in calcite may be a source of the discrepancies found in the wave-length determinations by different investigators. We know nothing of the previous history of the sample of calcite which we used. It is a small, clear, museum specimen which was found by trial to give sharp lines in higher orders of reflection.

The method employed in the calculation of the wave-length in air at  $18^\circ\text{C}$  from the observed angle was as follows. From the observed glancing angle a preliminary wave-length was calculated on the basis  $d = 3.028\text{A}$ . This preliminary wave-length was then corrected to  $18^\circ\text{C}$ ; this correction was in general a small positive one, since the temperatures were usually above  $18^\circ\text{C}$ . After a number of these corrected preliminary

<sup>6</sup> Compton, DeFoe and Beets, *Phys. Rev.* **25**, 618-629 (1925).

wave-lengths had been obtained in the same order of reflection, they were weighted as to reliability and averaged. The true wave-length in air was obtained by correcting the order average for refraction. The temperature corrections were very small, as were also, on account of high order, those for refraction.

TABLE I

*Wave-lengths of Mo K series lines.*

Wave-lengths in this table are in angstroms and have been calculated on the basis  $d(18^\circ) = 3.028\text{\AA}$ .

	Order	Date	$\lambda$ at 18°C	Wt.	av. $\lambda$ for order	Corr. for refraction	$\lambda$ in air at 18°C	Wt.	Final $\lambda$
$K\alpha_2$	Third	Mar. 23	.711789	(1)	.711789	-.000012	.711777	(1)	
	Fourth	Mar. 26	.711834	(1)					
		Mar. 26	.711670	(0)					
		Mar. 27	.711897	(1)					
		Mar. 27	.711834	(1)					
		Mar. 27	.711862	(1)	.711857	-.000007	.711850	(4)	.711835
$K\alpha_1$	Third	Feb. 26	.707558	(1)					
		Mar. 21	.707590	(1)					
		Mar. 23	.707577	(1)	.707575	-.000012	.707563	(3)	
	Fourth	Mar. 21	.707535	(1)					
		Mar. 23	.707534	(1)					
		Mar. 24	.707484	(1)	.707518	-.000007	.707511	(3)	
	Fifth	Mar. 24	.707453	(1)					
		Mar. 24	.707532	(1)					
		Mar. 24	.707532	(1)	.707506	-.000004	.707502	(3)	.707525
$K\beta_1$	Fourth	Mar. 5	.630778	(1)					
		Mar. 7	.630801	(1)	.630790	-.000007	.630783	(2)	
	Fifth	Mar. 12	.630852	(1)					
		Mar. 17	.630772	(1)					
		Mar. 17	.630776	(1)	.630800	-.000004	.630796	(3)	.630791
$K\gamma$	Fourth	Mar. 13	.619543	(1)					
		Mar. 14	.619513	(1)					
		Mar. 17	.619543	(1)	.619533	-.000007	.619526		.619526
$K_{\text{abs}}$	Third	May 2	.61848	(1)					
		June 8	.61802	( $\frac{1}{2}$ )					
		June 8	.61832	(1)					
		June 8	.61828	(1)	.61831	-.00001	.61830		.61830

The later steps of the calculation are shown in Table I. Each of these determinations is an absolute measurement in the sense that  $2\theta$  was measured directly. All the wave-length determinations which were made are included in the table. One measurement of  $K\alpha_2$  gave a value so far from the mean of the others that we have not included it in the average and have weighted it zero.

In the fourth and, especially, the fifth orders, we obtained a good separation of the Mo  $K\beta$  doublet. This was announced in a preliminary

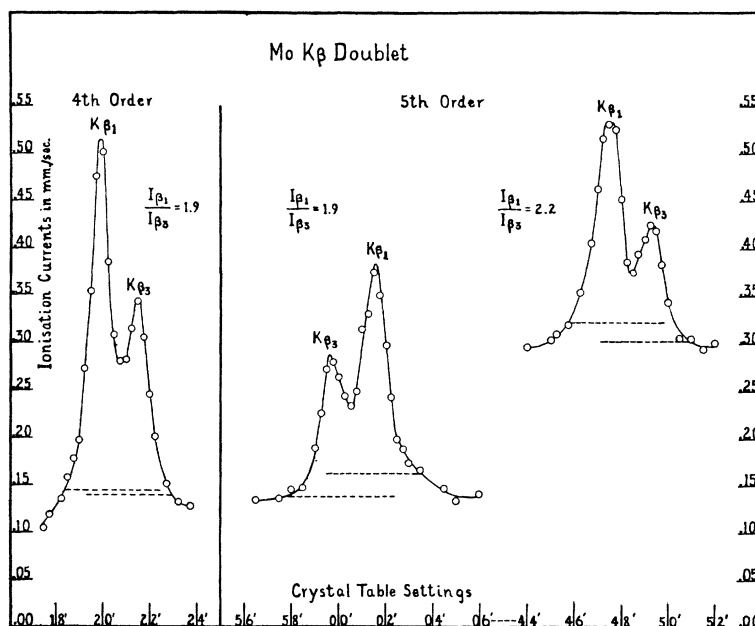


Fig. 1. Separation of Mo  $K\beta$  doublet.

report.<sup>7</sup> Some of the curves obtained are shown in Fig. 1 and the results in Table II. The doublet nature of this line has been experimentally

TABLE II

*Separation of Mo  $K\beta$  doublet and wave-length of Mo  $K\beta_3$ .*  
Wave-lengths in this table are in angstroms and have been calculated on the basis  $d(18^\circ) = 3.028 \text{ \AA}$ .

Order	Date	Separation	Wt.	$\Delta\lambda$	Average	Final $\Delta\lambda$	Mo $K\beta_3$
Fourth	Mar. 10	1'22'' (L)	(1)	.000545			
	Mar. 7	1'31'' (R)	(1)	.000606	.000576		
Fifth	Mar. 10	1'56'' (L)	(1)	.000580			
	Mar. 12	1'44'' (R)	(1)	.000520	.000550	.000563	.631354

observed in Mo and other elements by various authors.<sup>8</sup> According to the accepted energy level diagram the same frequency difference should

<sup>7</sup> Allison and Armstrong, Phys. Rev. **25**, 882 (1925).

<sup>8</sup> M. de Broglie, Compt. Rend. **170**, 1053 and 1245 (1920).

Duane and Patterson, Bull. Nat. Res. Council. Vol. 1, Part 6, p. 393 (1920).

C. B. Croft, Phys. Rev. **24**, 9 (1924).

A. Leide, Compt. Rend. **180**, 1203 (1925).

appear between the lines  $L\beta_3$  and  $L\beta_4$ . Coster<sup>9</sup> has measured the wave-lengths of Mo  $L\beta_3$  and Mo  $L\beta_4$  and has found for the wave-lengths 5.0002 and 5.0358A respectively. From these values it may be calculated that the wave-length difference in the  $K\beta$  doublet should be .000564A. Our observed value of .000563A agrees closely with this. The separation of .00039A reported by Leide<sup>8</sup> agrees less well with the measurements of Coster in the L series.

TABLE III

Wave-length,  $\nu/R$ , and  $\sqrt{\nu/R}$  values for Mo K spectrum.

Line	$\lambda$		$\nu/R$		$\sqrt{\nu/R}$	
	$d=3.028A$	$d=3.02904$	$d=3.028$	$d=3.02904$	$d=3.028$	$d=3.02904$
$K\alpha_2$	.711835A	.712078	1280.17	1279.73	35.7795	35.7734
$K\alpha_1$	.707525	.707768	1287.97	1287.53	35.8882	35.8821
$K\beta_3$	.631354	.631571	1443.36	1442.86	37.9915	35.9850
$K\beta_1$	.630791	.631009	1444.65	1444.15	38.0085	38.0020
$K\gamma$	.619526	.619737	1470.91	1470.41	38.3525	38.3460
$K_{\text{abs}}$	.61830	.61851	1473.83	1473.33	38.391	38.384

Table III contains a summary of the wave-length values and the  $\nu/R$  ( $R=109737$ ) and  $\sqrt{\nu/R}$  values calculated from them. Table IV contains some of the energy levels which can be computed from the observed values in the K series by means of the combination principle.

TABLE IV.

Energy levels in the Mo spectrum.

Level	$\nu/R$		$\sqrt{\nu/R}$	
	$d=3.028A$	$d=3.02904$	$d=3.028$	$d=3.02904$
$K_{11}=K_{\text{abs}}$	1473.83	1473.33	38.395	38.384
$L_{21}=K_{11}-K\alpha_2$	193.66	193.60	13.916	13.914
$L_{22}=K_{11}-K\alpha_1$	185.86	185.80	13.633	13.631
$M_{21}=K_{11}-K\beta_3$	30.47	30.47	5.520	5.520
$M_{22}=K_{11}-K\beta_1$	29.19	29.18	5.403	5.402
$N_{21}N_{22}=K_{11}-K\gamma$	2.92	2.92	1.71	1.71

In Table V we have given the results of other investigators who have measured the wave-lengths of the Mo K series. The values given by C. C. Hatley and by R. von Nardroff are corrected for index of refraction but apparently not for temperature of the crystal. As far as we know, these corrections have not been applied to the other previous measurements which we have listed. The later measurements of Leide agree well with the ones reported here, except for the line  $K\beta_1$ . In addi-

<sup>9</sup> D. Coster, Phil. Mag. **43**, 1070 (1922).



tion to the lines listed in Table V, Leide has reported a new line  $\beta_4$  which he attributes to the transitions  $\overline{O}_{21}\overline{O}_{22} \rightarrow K_{11}$ . One of us<sup>10</sup> has commented on this reported line, which should lie between  $K\gamma$  and the absorption limit. According to the prevalent electron distribution schemes, such a line would probably be semi-optical. The corresponding transition

TABLE V  
Comparison of results of different investigations of the wave-lengths  
of the Mo K series lines.

		K $\alpha_2$	K $\alpha_1$	K $\beta_3$	K $\beta_1$	K $\gamma$	K $\alpha_{\text{abs}}$
Duane and Patterson <sup>8</sup> (1920)	( $d=3.028$ )	.71212A	.70783	.63110	.6197	.61842	
Overn <sup>11</sup> (1921)	( $d_{NaCl}=2.814$ )	.7131	.7087	.6324	.6268	.6214	
Leide(1) <sup>12</sup>	( $d=3.02904$ )	.71187	.70759	.63075	.61927	....	
Hatley <sup>5</sup> (1924)	( $d=3.028$ )	....	.707717	....	....	....	
von Nardroff <sup>13</sup> (1924)	( $d=3.028$ )	....	....	.63102	....	....	
Leide(2) <sup>14</sup> (1925)	( $d=3.02904$ )	.71208	.70780	.63163	.63124	.61969	....
Present results	( $d=3.028$ )	.711835	.707525	.631354	.630791	.619526	.61830
	( $d=3.02904$ )	.712078	.707768	.631571	.631009	.619737	.61851

in the L series ( $L\gamma_4$ ) has not been observed in molybdenum.<sup>9</sup> In a photograph it might well be difficult to distinguish this line from the Mo K critical absorption which occurs on account of the absorption of the target for its own radiation. We have not observed this line, though we have taken readings in the fourth order through the region in which it should occur.\*

#### RELATIVE INTENSITIES OF THE LINES

It is now well known from the researches of Duane and Stenström, Duane and Patterson, and Siegbahn and Žáček,<sup>15</sup> that the relative intensity of the components of the K $\alpha$  doublet is 2/1 for a wide range of

<sup>10</sup> S. K. Allison, Nature **115**, 978 (1925).

<sup>11</sup> O. B. Overn, Phys. Rev. **18**, 350 (1921).

<sup>12</sup> A. Leide (1), Siegbahn, "Spektroskopie der Röntgenstrahlen," p. 102.

<sup>13</sup> R. von Nardroff, Phys. Rev. **24**, 143 (1924).

<sup>14</sup> A. Leide (2), loc. cit.<sup>8</sup>

\* *Note added to proof.* In a later publication (Thesis, Lund 1925) Leide no longer assigns this line (.61825A) to the transition  $\overline{O}_{21}\overline{O}_{22} \rightarrow K_{11}$ , stating that it probably results from a forbidden jump from the levels  $N_{IV}N_V$ . From photometer curves of the photographs taken by him it appears improbable that the presence of this line can be ascribed to a misinterpretation of the K critical absorption from the target.

<sup>15</sup> These results are summarized in Siegbahn, Spektroskopie der Röntgenstrahlen, p. 97.

elements. We have not attempted any experiments expressly designed to confirm this fact for molybdenum.

In our investigation of the Mo  $K\beta$  doublet we have been able to measure the relative intensities of  $K\beta_1$  and  $K\beta_3$ , and we find, within the limit of experimental error, the ratio 2/1 for  $\beta_1/\beta_3$ . M. de Broglie noted in his first measurement of the doublet that the component of longer wave-length was the one of lesser intensity, and our work confirms this observation. The very small difference in wave-length between the components makes corrections for differential absorption by the walls of the tube, etc., unnecessary. Since, however, the doublet is not completely separated even in the fifth order, it is essential to make some assumptions concerning the base-line under each peak. In the first place, we have assumed that each component of the doublet is itself a single line, and that both components have the same intrinsic wave-length breadth. In this case, then, it is permissible to take the ordinate from the tip of the peak to the radiation under the tip (from other sources) as proportional to the intensity of the component. In addition we have assumed that the peaks representing the components are symmetrical about the ordinates determined by their tips. Then, if the separation of the doublet is  $\Delta\lambda$ , we may take the radiation at a point corresponding to  $\lambda\beta_1 - \Delta\lambda$  as the base-line for  $K\beta_3$ . This will include the general radiation under  $K\beta_3$  and, by the symmetry assumption, also the amount of  $K\beta_1$  which, because of incomplete resolution, is under the tip of  $K\beta_3$ . In the same way the radiation at  $\lambda\beta_3 + \Delta\lambda$  may be used as the base-line for  $K\beta_1$ . The base-lines given by this method are shown as dotted lines in Fig. 1. The method can be applied only when the resolution is great enough so that there is no appreciable radiation from  $\beta_1$  at  $\lambda\beta_3 + \Delta\lambda$  and vice versa. The results of three of the most trustworthy measurements are given in Fig. 1 and Table VI.

TABLE VI  
*Relative intensity of the lines Mo  $K\beta_1$  and  $K\beta_3$ .*

Date	Order	Ratio $\beta_1/\beta_3$	Wt.	Average
Mar. 7	4	1.9	(1)	
Mar. 10	5	1.9	(1)	
Mar. 12	5	2.2	(1)	2.0

Duane and Patterson<sup>16</sup> have measured the relative intensity of Mo  $K\beta$  to Mo  $K\gamma$  in the first and second orders and have found 6.3 and 5.5 respectively. They obtained this ratio from the highest points on the two peaks. Although this method is sufficient for a first approximation,

<sup>16</sup> Duane and Patterson, *Phys. Rev.* **19**, 542 (1922); also, *Proc. Nat. Acad. Sci.* **8**, 85 (1922).

a better procedure is probably to consider the *area* under each peak, since  $\beta$  and  $\gamma$  are both undoubtedly doublets of unequal wave-length separation. We have therefore taken the area under each peak above the general radiation as proportional to its intensity. The area was measured by means of an integrator. A typical curve showing  $\beta$  and  $\gamma$  is given in Fig. 2. The slits limiting the beam incident upon the crystal were somewhat wider than those used for wave-length measurements. In these experiments on the  $\beta$  and  $\gamma$  relative intensities, the x-ray tube had a

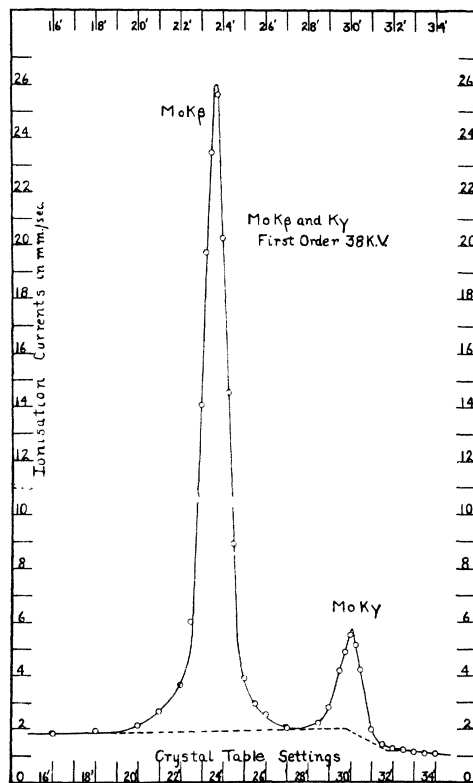


Fig. 2. Relative intensity of Mo K $\beta$  and K $\gamma$ .

thin blown-glass window to reduce absorption, and the x-rays entered the ionisation chamber through a mica window .002 cm thick. The presence of absorption by the target itself is shown by the lower baseline on the short wave-length side of  $\gamma$ . On the assumptions that the center of the critical absorption rise lies 45 seconds to the short wave length side of  $\gamma$ , and that the breadth of the beam is equal to the breadth of the peak, we have drawn in the discontinuity as it must exist in the general radiation and have thus corrected the area of K $\gamma$ . The critical

absorption and base-line are shown in dotted lines in Fig. 2. The results of several measurements of the relative intensity of  $K\beta$  to  $K\gamma$  are given in Table VII. In these determinations the voltage on the tube and the current through the tube were varied through rather wide limits, but no effect on the relative intensities of the lines could be detected. The relative intensities given are uncorrected for ordinary absorption effects.

TABLE VII

*Relative intensity of the lines Mo  $K\beta$  and  $K\gamma$ .*

Date	Order	Ratio $\beta/\gamma$	Wt	Average
Mar. 30	1	7.3	(1)	
Mar. 30	1	8.5	(1)	
Mar. 31	1	8.3	(1)	
Apr. 2	1	8.4	(1)	
Apr. 4	1	6.6	(1)	
Apr. 4	1	7.3	(1)	7.7

The relative intensity of  $K\alpha$  to  $K\beta$  has been measured for some elements in this region by Unnewehr<sup>17</sup> and by Siegbahn and Žáček.<sup>18</sup> We have not succeeded in obtaining a satisfactory value of this ratio for Mo because of the large correction for absorption which must enter.

#### DISCUSSION OF THE INTENSITY RESULTS

Recently Sommerfeld<sup>19</sup> and Coster and Goudsmit<sup>20</sup> have made theoretical predictions of the relative intensities of some doublets and "compound doublets" in x-ray spectra. They have based their predictions on the results of Burger and Dorgelo,<sup>21</sup> who have measured relative intensities in the optical region.

These investigators have found that the relative intensity of the components of the doublets in the principal series ( $1s - mp$ ) of the alkalis is 2/1. The well-known analogy of the x-ray levels to the optical levels of the alkalis would thus lead us to expect that the doublets of the K series would have the same relative intensity, 2/1. By this analogy the  $K\beta$  doublet should correspond to the second member of the principal series ( $1s - 3p_2, 3p_1$ ). The experimental intensity ratio reported here confirms the theoretical predictions of Sommerfeld and Coster and Goudsmit. It is also interesting to see that this simple ratio holds even though in falling from the M group to the K group the electrons pre-

<sup>17</sup> E. C. Unnewehr, *Phys. Rev.* **22**, 529 (1923).

<sup>18</sup> Siegbahn and Žáček, *Ann. der Physik* **71**, 187 (1923).

<sup>19</sup> A. Sommerfeld, *Ann. der Physik* **76**, 284 (1925).

<sup>20</sup> D. Coster and S. Goudsmit, *Naturwissenschaften* **1**, 11 (1925).

<sup>21</sup> Burger and Dorgelo, *Zeits. f. Physik* **23**, 258 (1924), etc.

sumably pass through the L group. Evidently in this case, at least, the relative intensity is not affected by the intervening group.

No quantitative theory has yet been devised to deal with the relative intensities of such lines as Mo  $K\beta$  and  $K\gamma$ .<sup>22</sup> Duane and Stenström<sup>23</sup> have measured the relative intensity of  $\beta$  to  $\gamma$  in the K series spectrum of tungsten and find 35/15 or 2.3/1. The observed ratio 7.7/1 in molybdenum shows that the  $\gamma$  line grows weaker as the atomic number of the radiating element decreases.

Our thanks are due Professor William Duane for his constant interest in this research.

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<sup>22</sup> *Note added to proof.* Our attention has been called to the paper of F. C. Hoyt (Phil. Mag. **46**, 135, 1923) (see also article in this issue) in which an attempt was made to calculate the relative intensity of lines of this type by application of Bohr's correspondence principle. A dependence of the relative intensity on the atomic number may be inferred from considerations advanced in this paper.

<sup>23</sup> Duane and Stenström, Proc. Nat. Acad. Sci. **6**, 477 (1920).