Second Series *December, 1925* *Vol. 26, No. 6*

THE

PHYSICAL REVIEW

A PRECISION MEASUREMENT OF THE CHANGE OF WAVE-LENGTH OF SCATTERED X-RAYS

By H. M. Sharp

Abstract

According to the quantum theory, the change of wave-length due to scattering is $\delta\lambda=(h/mc)\,(1-\cos\phi)$ where h is the radiation constant, m the mass of an electron, c the velocity of light and ϕ the angle of scattering. In the present experiments ϕ was made as large as possible (169°) by placing the slits and calcite crystal in a lead box fastened to the Coolidge tube. Mo $K\alpha$ and $K\beta$ radiation was scattered *from paraffin* back to the crystal and then to a photographic plate; Zr $K\alpha$ rays for comparison were obtained from a Zr radiator. The best plate required an exposure of 48 hr. The change of wave-length was measured 16 times by means of a special microphotometer. After certain corrections were made, the shift came out $(.04825\pm.0002)$A; the theoretical value is $.04798\pm.0001$. This agreement is an excellent confirmation of the theory. This effect may also be used to obtain a measurement of m which is independent of e/m. The *value of the mass of the electron* thus computed from these measurements is $(8.99\pm.034)\times10^{-28}$ gm in agreement with the mean result from deflection experiments, whereas the spectroscopic value of e/m gives 9.04.

According to the quantum theory, the change of wave-length of x-rays when scattered is given by the expression[1]

$$\delta\lambda=(h/mc)\,(1-\cos\phi) \tag{1}$$

where h is Planck's radiation constant, m is the mass of the electron, c the velocity of light and ϕ the angle which the scattered beam makes with the primary beam. Of the many published tests of this equation, the ones for which the greatest precision is claimed are perhaps those of A. H. Compton,[2] Becker and others[3] and Kallman and Mark.[4] Compton used an ionization spectrometer and a carbon radiator, and made tests at different angles of scattering. His probable error was about 2 percent. The others obtained photographic spectra of rays scattered at about

[1] A. H. Compton, Phys. Rev. **21,** 207 and 483 (1923).
[2] A. H. Compton, Phys. Rev. **22,** 409 (1923).
[3] J. A. Becker, E. C. Watson, W. R. Smythe, R. B. Brode and L. M. Mott-Smith, Phys. Rev. **23,** 763 (1924).
[4] H. Kallman and H. Mark, Naturwissenschaften **13,** 297 (1925).

90°, Becker using aluminum as radiator, and Kallman and Mark using graphite. In both of the latter investigations the estimated probable error was about 1 percent. Greater precision in testing the equation should be obtained if the angle of scattering is increased to nearly 180°, for then $\delta\lambda$ is greater, and, since it is the cosine of the angle which enters into the equation, the error in the calculated value of $\delta\lambda$ resulting from an error in measuring ϕ is greatly reduced.

Experimental procedure. A test in which a large angle of scattering is used has been made with the apparatus shown diagrammatically in Fig. 1. X-rays from the molybdenum target T of an x-ray tube fall on a paraffin radiator R, and some of the rays are scattered almost directly back to the calcite crystal C. The slit which determines the width of the spectrum lines is at S, and the photographic film is placed beyond the lead

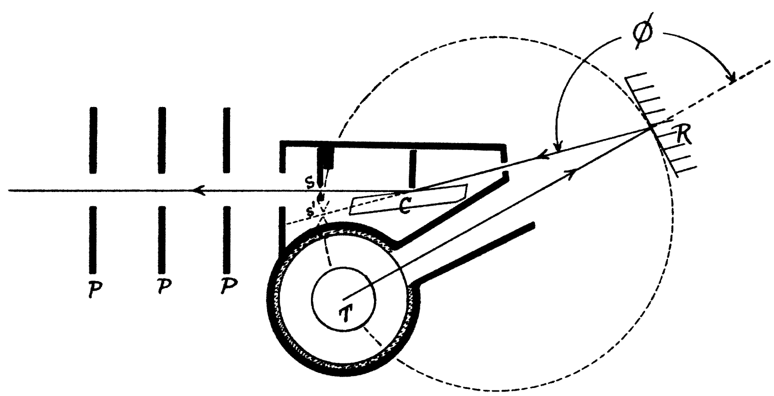

Fig. 1. Arrangement of apparatus.

screens P. The image S' of the slit S as reflected in the face of the crystal lies as close to the target T as is conveniently possible. It will be seen also from the figure that the angle of scattering TRS' varies but slightly as one alters the part of the radiator R from which the rays are scattered. This arrangement therefore affords a means of securing a spectrum of rays scattered at a nearly definite as well as a large angle.

The x-ray tube employed was of the narrow cylindrical, water-cooled type described by Compton.[1] It was allowed to rectify its own current of 18 to 20 m-amp., which was supplied by a transformer operating at about 40 peak kv. Surrounding the tube to eliminate stray x-rays was a cylindrical lead sleeve, insulated from the glass walls of the tube by about 1.2 mm of mica. In this sleeve there was an aperture to permit x-rays to proceed in the direction of the radiator. The crystal was set in plaster of Paris and the crystal and slit were supported in a frame rigidly attached to the lead sleeve and were enclosed by lead except as indicated in Fig. 1.

The distance from the target to the radiator was 10 cm. The slit was .4 mm wide and 2.2 cm long and the distance from the slit to the film was about 30 cm.

An enlargement of the best of several photographs thus obtained is reproduced in Fig. 2. The negative shows plainly the molybdenum $K\alpha$ and β lines. This represents an exposure, using an intensifying screen, of 32 hours with a paraffin radiator and 16 hours with a zirconium

Fig. 2. Enlargement of the spectrogram of which the density distribution is given in Fig. 3.

radiator. Attempts to secure photographs in which the molybdenum $K\beta$ modified and unmodified lines were strong enough to measure precisely were unsuccessful, due partly to the difficulty of making a tube operate under the conditions of the experiment with sufficient life to permit an exposure of the required length.

Determination of ϕ. The angle ϕ between a line joining the center of the focal spot and the radiator and a line from the radiator to the crystal, was determined by a geometrical method to be 169.7°. As we have seen, the variation in ϕ due to the fact that different portions of the scattered x-rays come from different portions of the radiator, is practically negligible; but there is a certain range of values of ϕ which results from the length of the slit and the size of the target. Rays which pass through the end of the slit are scattered at a larger angle than those which pass through the center. The range of ϕ due to the length of the slit was from 167.7 to 169.7°.

The target was photographed by allowing rays to pass through a pin-hole in a sheet of lead. From the size of the photograph and the distance of the film and target from the pin-hole, the size of the portion of the target from which x-rays proceeded was calculated. This was found to

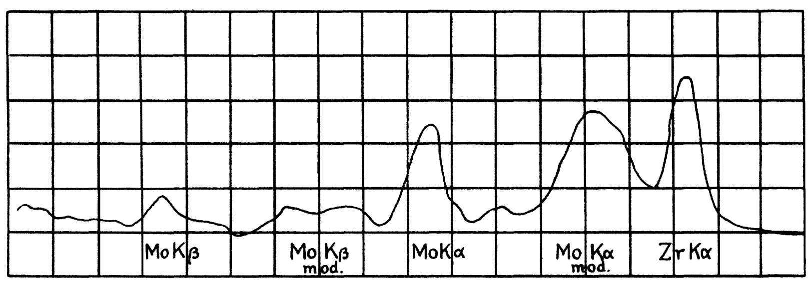

Fig. 3. Microphotometric measurement of the spectrogram reproduced in Fig.2.

be .5 cm in the vertical direction. The center of the target was 1.6 cm below the image of the slit. The smallest distance from the target to the image of the slit was thus 1.35 cm, and the largest distance 1.85 cm. Taking this into account the range of ϕ was from 166.3 to 171.5°.

Measurement of $\delta\lambda$. Measurements of $\delta\lambda$ were made on the film shown in Fig. 2. For this purpose 16 curves for different portions of the film were made with a microphotometer which consisted of a Coblentz thermopile of about 20 elements connected with a Leeds and Northrup high sensitivity galvanometer. One of these curves is shown in Fig. 3. The position of each peak was determined by making several measurements of the middle of the upper tenth of the curve representing the spectrum line. If b is the measured distance in centimeters between the Mo Kα peak and the Mo Kα modified peak, a the distance between the Mo Kα peak and the Zr Kα peak, and Δ the wave-length difference between the Mo Kα_1 and the Zr Kα_1 lines, the value of $\delta\lambda$ is approximately

$$\delta\lambda = (b/a)\Delta \tag{2}$$

The value of Δ according to the recent wave-length determinations of Leide,[5] is $0.78429-0.70780=0.07649$A.

In order to obtain a precise result the following small corrections were applied:

(a) On the Mo Kα peak: (1) For the effect of the α_2 line in shifting the peak, due mainly to the α_1 line. (2) For the effect of the Zr Kβ line in shifting the peak of the combined Mo Kα_1 and Mo Kα_2 line.

(b) On the Mo Kα modified peak: For the effect of the modified α_2 line.

(c) On the Zr Kα peak: (1) For the effect of the α_2 line in shifting the α_1 peak. (2) For the effect of the modified Mo Kα line in shifting the Zr Kα_1 and Kα_2 peak.

(d) A correction to change from the measurement of the angle to the measurement of the sine, since λ is proportional to $\sin\theta$.

The calculation of these corrections was facilitated by the observation that the photometer curve representing a simple spectrum line can be represented very approximately by an expression of the form $y=y_0e^{-x^2}$. When these corrections are taken into account, the change of wave-length can be expressed as

$$\delta\lambda=(b/a)\ (.07658\text{A})-.00040\text{A}. \qquad (3)$$

This differs from the approximate relation (2) by only 0.7 percent. The average value of $\delta\lambda$ thus obtained from 16 photometric curves is

$$\delta\lambda=(.04825\pm.00017)\text{A.}\quad \text{(expt.)}$$

In calculating $\delta\lambda$ from Eq. (1), we may take $1-\cos\phi=1.984\pm.001$. The value of h as determined by Duane, Palmer and Chi-Sun-Yeh[6] is $(6.556\pm.0009)\times10^{-27}$ erg sec. m is given by the expression $m=e/(e/m)$, where for e we may use Millikan's[7] value of $(4.774\pm.005)\times10^{-10}$ e.s.u., and for e/m Babcock's[8] value $(1.761\pm.001)\times10^{7}$ e.m.u. These values give

$$\delta\lambda=(.04798\pm.00009)\text{A.}\quad \text{(theory)}$$

This differs from the experimental value by an amount which is just within the combined probable error.

Discussion. This result is an excellent confirmation of the quantum formula for the change of wave-length of scattered x-rays.

[5] A. Leide, Comptes Rendus **180,** 1203 (1925).

[6] Duane, Palmer and Chi-Sun-Yeh, Jour. Opt. Soc. Am. **5,** 396 (1921).

[7] R. A. Millikan, Phil. Mag. (July, 1917).

[8] H. D. Babcock, Astrophysical Jour. **58,** 149 (1923).

If we write Eq. (1) in the form

$$m = h(1 - \cos \phi)/c\delta\lambda \ ,$$

we see that a measurement of $\delta\lambda$ affords a method of measuring the mass of the electron, which is independent of e/m. This is of especial interest in view of the fact that the spectroscopic values of e/m are consistently lower than those obtained by the deflection of streams of electrons.[9] Using for c the value 2.9986×10^{10} cm/sec and for h, $\delta\lambda$ and ϕ the values given above, m is thus found to be $(8.99 \pm .034) \times 10^{-28}$ gm. This value is probably not as accurate as the value $(9.04 \pm .01) \times 10^{-28}$ gm based on Babcock's spectroscopic determinations of e/m; but it is interesting to note that it lies closer to the value 8.98×10^{-28} gm calculated from Birge's average $e/m = 1.773 \times 10^7$ e.m.u. based chiefly on deflection experiments.[10]

In conclusion the writer desires to express his appreciation of the continued encouragement given by Professor A. H. Compton, who suggested the problem and has directed the work on it, and to acknowledge his indebtedness to his friend Mr. Y. H. Woo for assistance in evacuating the x-ray tubes.

RYERSON LABORATORY,
UNIVERSITY OF CHICAGO,
Sept. 22, 1925.

[9] Cf. Babcock, loc. cit.[8]

[10] R. T. Birge, Phys. Rev. **14**, 363 (1919).

Fig. 2. Enlargement of the spectrogram of which the density distribution is given in Fig. 3.