SCATTERING OF ELECTRONS IN IONIZED GASES

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Abstract

Velocity distribution of electrons in low pressure discharges from hot cathodes.—Analysis of the current received by a collector placed opposite a hot cathode indicates that there are three groups of electrons present; (1) *primary electrons* which retain practically all the momentum they acquired in passing through the positive ion sheath around the cathode; (2) *secondary electrons* moving in random directions with a Maxwellian velocity distribution corresponding to a temperature roughly proportional to the energy of the primaries (200,000° for 100 volt primaries) and approximately independent of the nature or pressure of the gas or the current density; (3) *ultimate electrons* having a Maxwellian distribution of velocities corresponding to a much lower temperature than that of the secondaries, a temperature which is independent of the current density or the voltage of the primary electrons and which varies with the gas used and decreases slowly as the pressure is raised. The number of ultimate electrons is roughly 1000 times that of the primaries and secondaries for mercury vapor, although relatively less numerous in the case of hydrogen. In the uniform positive column of arcs only ultimate electrons are present.

The velocity distribution of the primary electrons may be resolved into a drift velocity normal to the cathode surface and a random motion with a Maxwellian distribution corresponding to a temperature which varies approximately with the square of the current and is a maximum at a mercury vapor pressure of about 0.6 bar, falling to a low value at 8 bars, and is considerably higher with 35 volts accelerating potential than with 80 volts. Under favorable conditions with 40 mil-amp. this temperature may rise to 80000° and there are then appreciable numbers of these primaries which can reach a collector charged to a potential as much as 40 volts below that of the cathode. Experiments with two crossed electron beams prove that the secondary and ultimate electrons, directly or indirectly, are responsible for very little if any of the scattering.

A study of the ultimate electrons in the positive column of a low voltage mercury arc shows that the Maxwellian velocity distribution corresponding to a high temperature (30000°) is maintained in a small tube in spite of the fact that the negatively charged walls of the tube constantly tend to disturb this distribution by selective removal of the faster electrons. The number of collisions of the electrons with each other and with atoms were far too few to maintain the observed distribution. Measurements of the mobilities of electrons in arcs and of the electron concentration differences produced by a transverse magnetic field gave values for the mean free path only 1/10 as great as the values determined by a direct method. These results all indicate that the electrons in low pressure arcs suffer many changes of momentum during the time intervals between successive collisions between atoms, ions or electrons.

Mechanism of electron scattering.—After analyzing the effects to be expected when a beam of primary electrons encounters a cloud of stationary electrons or a cloud of particles of small mass but of high temperature, it appears that the experimental results for the ratio of the scattering to the average retardation of the beam was best accounted for by collisions with a cloud of radiation quanta, a kind of Compton effect. The observed scattering is, however, about 10^{16} times greater than a normal Compton effect would give It is suggested that the joint action of excited atoms and radiation may multiply the Compton effect sufficiently, or it may be that the presence of a free electron within a distance of a wave-length from an excited atom may greatly increase the probability of a quantum jump and that momentum is delivered to the electron in the process. In arcs the ultimate electrons may be in a kind of thermal equilibrium with radiation and excited atoms.

Scattered electrons in high vacuum.—In connection with a study of the Barkhausen-Kurz effect and in a magnetron with negatively charged end plates, evidence of the presence of electrons with abnormally high speeds has also been obtained. These have been explained as due to electric oscillations, but the effects have also been obtained when oscillations are not present. This seems to indicate that electron scattering similar to that observed in low pressure gases can also occur in high vacuum.

I. Electrons in the Positive Column of Low Voltage Arcs

Distribution of velocities. By studying the volt-ampere characteristics of small electrodes or collectors placed in the path of an arc in mercury vapor at low pressures, it has been found[1,2] that the free electrons in the arc are moving in nearly random directions with velocities which are distributed quite accurately in accordance with Maxwell's distribution law.

When the collector is at large negative potentials with respect to the surrounding gas, it receives only positive ions, but at higher potentials (algebraically) electrons also are collected. At first only electrons of the highest velocities are able to reach the collector against the retarding potential, but as the potential is raised, making the retarding field weaker, electrons of lower velocity can be collected. If the velocities of the electrons are distributed according to Maxwell's law, the logarithm of the electron current flowing to a negatively charged collector should be a linear function of the potential of the collector, with a slope (when plotted using natural logarithms) of e/kT, where T is the temperature corresponding to the electron velocities and e/k has the value 11600 degrees per volt.

When the collector is made positive with respect to the surrounding gas the electron current increases with the potential more slowly and according to entirely different laws. Thus the semi-logarithmic plot of the current-voltage characteristic should be a straight line only for negative voltages and a kink therefore should occur in the curve at a point at which the potential of the collector is the same as that of the

[1] Langmuir, Jour. Frank. Inst. **196**, 751 (1923).
[2] Langmuir and Mott-Smith, Gen. Elec. Review **27**, 449, 538, 616, 762, 810 (1924).

surrounding gas. By this method collectors may be used to measure the true space potentials.

When a small plane collector is at the space potential in an arc in mercury vapor, the electron current is about 400 times as great as the positive ion current which flows when large negative voltages, sufficient to repel all electrons, are used. Thus, if the collector is disconnected (or is allowed to "float") it charges itself negatively with respect to the surrounding gas until it is able to repel 399/400$^{\text{ths}}$ of all the electrons which move towards it, the electrons and positive ions then being collected in equal numbers. Experiments with mercury vapor, hydrogen, nitrogen, argon and neon at low pressures have shown that the semi-logarithmic plots for negative voltages are usually surprisingly straight over very wide ranges of current. The straight semi-logarithmic plot of the electron current extends to negative voltages far beyond that to which the collector charges itself when it is floating. For example, the straight plot was easily observed down to currents only 1/6000$^{\text{th}}$ of the total current.[3] A. F. Dittmer working with Karl Compton at Princeton has recently found, using an improved method by which positive ions and electrons are separated, that with a 40 milli-ampere arc in nitrogen at 8 bars pressure, the straight line plot extends down to current densities of less than 10^{-5} of the total current density. The electron temperature in this case was about 50000°.

The electron temperatures corresponding to these straight plots are practically independent of the current densities in the arcs, but increase as the pressure is lowered. For example, currents of from 0.1 to 5 amperes in a tube 3 cm in diameter gave electron temperatures of about 30000° with mercury vapor at 1 bar; 20000° at 5 bars, and 10000° at 30 bars. With other gases the electron temperatures are usually higher.

If thermal equilibrium existed in the ionized gas the electrons would of course have a Maxwellian distribution of velocities corresponding to the temperature of the gas. If this distribution is disturbed by any external cause it tends to re-establish itself through the agency of every mechanism by which electron velocities can be modified. For example, collisions of electrons with each other, with atoms or excited atoms, or the interaction of electrons with the black body radiation, will all tend to bring back the Maxwellian distribution of the electrons. The rate of re-establishment of this distribution would be different for each mechanism if that alone were allowed to act. Corresponding to each mechanism we may therefore conceive of a "time of relaxation" which is the time

[3] See, for example, Fig. 1 in Gen. Electric Rev. article (loc. cit.[2]), p. 451.

that would be needed for any disturbance in the Maxwellian distribution to fall to $1/e^{\text{th}}$ of its value, if only the one mechanism were effective.

We will consider first the interaction between atoms and electrons. The mercury atoms which strike the walls of the tube deliver their surplus energy and leave the glass with a kinetic energy corresponding to room temperature. For mercury vapor at 1 bar and a temperature of 300°K, there are 2.4×10^{13} atoms per cm^3. Measurements with collectors in a 2 amp. arc in a 3 cm tube have shown that there are 8×10^{10} free electrons per cm^3. In this case the average distance an atom will travel before colliding with an electron is 1.5 cm and in a tube 3 cm in diameter it will make only about four collisions with electrons between consecutive collisions with the wall. Taking the temperature of the electrons as 30,000°, the mercury atom gains on the average only 0.08° at each collision. Thus the temperature of the mercury vapor which strikes the walls will be only 0.3° above the temperature of the walls. This at least represents the temperature that can be acquired by direct interaction with the electrons. Of course there will be a minute fraction of mercury atoms which collide with excited mercury atoms and thus acquire very high kinetic energy, but we may ignore these at present. The time of relaxation corresponding to the interaction between electrons and atoms by collisions is thus at least 100,000 times too great to be effective in bringing about temperature equilibrium and thus affording a means of maintaining a Maxwellian distribution for the electrons.

Since the normal free path λ_e of electrons under the conditions considered, is 30 cm, it is clear that the average distance an electron would have to travel before colliding with an excited atom would be much greater than this and therefore collisions of the second kind, in the ordinary meaning of this term, cannot be responsible for bringing about the electron velocity distribution. The same may be said of collisions of electrons with each other. The concentration of electrons is only $1/300^{\text{th}}$ that of the atoms and if we assume the target area of an electron is the same as that of an atom, the average distance that an electron would move before colliding with another electron would be 6400 cm (allowing for the velocity of the electron being struck). A careful calculation has been made, based on Coulomb's law, of the effect of the electric force between the electrons in increasing the transfer of momentum by collisions. It is found that the effective free path varies with the electron velocity, but that the free path is still so great that this kind of interaction between electrons must be completely rejected as a possible mechanism for bringing about the observed Maxwellian distribution.

Effect of the charged glass walls. The glass walls of the discharge tube are negatively charged because of the fact that the electron current density in the discharge is greater than the positive ion current density. In accordance with observation we may take the electron current in mercury vapor to be 400 times the ion current. If the electron temperature is 30,000° it can then be calculated (in agreement with experiment) that the walls will be charged to -15.5 volts with respect to the ionized gas and will thus be covered by a positive ion sheath. All electrons which have energy components normal to the surface of the glass of less than 15.5 volts will be specularly reflected from the walls without loss of energy, but electrons having energy components greater than this will pass through the sheath and be taken up by the walls except for the few that are reflected back. Thus the walls are continually disturbing the Maxwellian distribution by removing the high speed electrons. We should therefore expect distinct departures from a straight line semi-logarithmic plot at collector voltages below that to which the glass walls become charged, unless there is a mechanism which effectively re-establishes the Maxwellian distribution within a distance less than that which separates the collector from the opposing glass walls.

In the experiments made by Harold Mott-Smith Jr. and the writer with mercury arcs in a tube 3 cm in diameter,[4] the collector which was designated H consisted of a square nickel plate 1.9 cm on a side which had been bent into a cylindrical surface which conformed to the inner surface of the tube. If electrons traverse the tube in straight paths with constant velocity and are specularly reflected from the cylindrical walls, the radial velocity components in successive collisions with the walls remain constant. If then the walls of the tube are negatively charged to say -15.5 volts *few electrons can reach a cylindrical collector such as H, with radial components greater than 15.5 volts unless these electrons have acquired this large radial velocity within the distance they have travelled since their last collision with the wall.*

The experiments showed in all cases that the straight semi-logarithmic plot extended to negative voltages far below those to which the collector H charged itself when it was allowed to float. This must be taken as proof that the higher velocity electrons were produced from low velocity electrons while these travelled a distance of less than 3 cm.

From the complete absence of a kink in the semi-logarithmic plot at the wall potential we must conclude that the time of relaxation corresponding to the mechanism by which the electrons acquire their Max-

[4] Loc. cit[2], p. 538.

wellian distribution is small compared to the time taken for the electrons to traverse the tube, in other words, that the free path of the electrons corresponding to this mechanism is small compared to 3 cm.

Effect of charged conducting walls. Recently Mott-Smith has carried out experiments with a mercury arc at low pressures passing through a metallic cylinder split lengthwise into two halves, which are insulated from one another. One half was used as a collector while the other was maintained at a series of different negative voltages to see if the velocity distribution of the electrons reaching one electrode was modified in any way as different groups of electrons were removed by means of the opposite electrode. It was found in fact that by charging one electrode so strongly negative that even the high speed electrons were reflected from it, the temperature of the electrons collected by the opposite half-cylinder was distinctly higher than if the electrode were at only −15 volts with respect to the gas so that all the higher speed electrons were removed. *But in both cases a perfectly straight semi-logarithmic plot was obtained.* The removal of the high speed electrons lowered the temperature, but the Maxwellian distribution was established while the electrons moved between the two collectors. Experiments are now in progress by A.F. Dittmer using plane collectors whose distance apart can be varied, to determine directly the rate at which the Maxwellian distribution is brought about.

Evidence as to the mean free path of electrons.[5] If a small potential gradient dV/dx is allowed to act on electrons having a Maxwellian velocity distribution, the electrons drift in the field with an average velocity v_x given by the mobility equation

$$v_x = 4\gamma\lambda/\pi v \tag{1}$$

where γ the acceleration of the electrons is

$$\gamma = \frac{e}{m}\frac{dV}{dx}, \tag{2}$$

λ is the mean free path of the electrons and v is the average velocity of the electrons (assumed to follow Maxwell's law). Expressing v in terms of T_e and inserting the value of e/m we get

$$v_x = 3.62 \times 10^9 \cdot \frac{\lambda}{\sqrt{T_e}}\frac{dV}{dx} \tag{3}$$

where v_x, λ and x are expressed in cm-second units and V is in volts.

[5] A brief note on the abnormally small values of the free paths of electrons obtained from the mobility and from the effects of transverse magnetic fields was given in the last paper by Langmuir and Mott-Smith, loc. cit.[2], p. 819.

The current density I_x of the arc is related to v_x by the equation

$$I_x = n_e e v_x$$

or if I_x is in amp. per cm²

$$I_x = 1.59 \times 10^{-19} \ n_e v_x. \tag{4}$$

Combining these equations

$$I_x = 5.76 \times 10^{-10} \frac{n_e \lambda}{\sqrt{T_e}} \frac{dV}{dx}. \tag{5}$$

With the 2 ampere arc in mercury vapor at 1 bar in a tube 3.2 cm in diameter the current density I_x was 0.25 amp. per cm², $n_e = 8 \times 10^{10}$, $dV/dx = 0.25$ volts per cm, and $T_e = 30{,}000°$. Substituting these into Eq. (5) and solving for λ we find that the free path λ of the electrons in the arc is 3.8 cm. Direct measurement of the number of 50 to 250 volt electrons which pass through ionized mercury vapor at 1 bar without collision with atoms, give a free path of at least 30 cm. The recent investigations of Ramsauer and others, show that for lower velocities (below those necessary to produce ionization), the free paths tend to increase. A large number of measurements of λ with arcs in mercury vapor at various pressures, in tubes of various diameters and with a wide range of current densities have been made in this laboratory. The free path calculated from the mobility always comes out to be 5 to 10 times lower than can be reconciled with direct determination or with values calculated by the usual methods of the kinetic theory.

The application of transverse magnetic fields of 50 to 100 gauss to a mercury arc at low pressures causes the arc to be deflected to one side. By measuring the *ratio of the electron concentrations* at two points along a diameter of the tube which is perpendicular to the magnetic field, it is possible to calculate the free path by a method entirely independent of that involving the mobility. This method also gives free paths about 10 times smaller than the distances which electrons travel before colliding with atoms as obtained by direct measurement.

Both of these results as well as the rapid establishment of the Maxwellian distribution of electron velocities, suggest that *the electrons in a mercury arc suffer many changes in momentum during the time that elapses between consecutive collisions with atoms.*

Summary. This discussion of the results obtained from a study of low pressure arcs shows (1) that the free electrons have velocities with a Maxwell distribution corresponding (in the case of mercury vapor at 1 bar) to a temperature of 30,000°; (2) that this distribution is maintained even when the walls are negatively charged and hence are constantly removing the faster electrons; (3) that the number of collisions with

atoms and electrons is far too small to maintain this distribution, the mean free path being of the order of the tube diameter; (4) that mobility experiments indicate that the electrons suffer at least ten changes of momentum between consecutive collisions with atoms. A possible explanation of these results will be presented and discussed in Part IV below.

II. Electrons Accelerated from a Hot Cathode

These Maxwellian distributions occur not only in ordinary low pressure arcs but are produced in gaseous discharges under many different conditions. We will now consider some experiments which have been made with spherical bulbs or cylindrical vessels containing a hot tungsten cathode and a disk-shaped anode. With gases at low pressures, currents of the order of ten milli-amperes and anode potentials of 50 to 250 volts, the space potential in the strongly ionized gas is nearly uniform and is somewhat above that of the anode. The cathode is surrounded by a positive ion sheath in which there is a sharp potential drop so that the electrons from the cathode are accelerated within a distance of a fraction of a millimeter, to a velocity exceeding that which corresponds to the potential difference between the anode and cathode.

Recently a very large number of experiments have been made by S. P. Sweetser, C. G. Found and H. A. Jones to investigate the production of these high speed electrons. A brief report of this work was made at the April meeting of the Physical Society[6] in Washington.

In one set of experiments (Exp. 548) a straight tungsten filament 1.1 cm long and 0.18 mm in diameter was mounted within about 2 cm of the center of a spherical bulb of 12 cm diameter. A disk-shaped collector 1.1 cm in diameter, backed by mica and movable in the direction of its axis, was mounted so that the axis passed through and was perpendicular to the filament at its center. The lead to the disk was insulated so that the collector could receive electrons only on the surface facing the filament. An anode was also placed in the bulb to one side of the line connecting the other electrodes.

By lighting the filament so that it emitted 5 milli-amperes and applying -50 volts to the cathode with respect to the anode (assumed to be at zero volts) the mercury vapor became strongly ionized and its potential was nearly uniform and equal to that of the anode. The electrons emitted by the filament acquired their full velocity within a few tenths mm and were therefore projected outwards in directions normal to the filament, thus forming a disk shaped "beam" of electrons. Those electrons

[6] Langmuir, Abstract in Physical Review **25**, 891 (1925).

which reach the collector with a large part of the momentum which they acquired in passing through the positive ion sheath, we shall call *primary electrons.* The primary electrons in this beam which were intercepted by the collector were moving perpendicularly to its surface. To prevent these electrons from reaching the collector it was necessary to apply a potential lower than that of the cathode.

Thus, as the collector passed through the potential of the cathode, the current-voltage curve of the collector showed a sudden change in current which was a measure of the number of primary electrons which reached the collector without collisions. This discontinuity was very sharp when the cathode emission was only 5 m-amp. taking place within a range of only about 2 volts, which was not greater than would be expected because of such effects as the initial velocities of the emitted electrons and the voltage drop along the filament. Thus no appreciable number of electrons were present in the gas with velocities higher than those of the primaries.

Scattering of primary electrons. When the electron emission was raised to 10 m-amp. however, there were found to be unmistakable indications of the presence of electrons having velocities higher than the original primaries. Fig. 1 shows some typical data taken with 10 m-amp., with a cathode potential of -50 volts, and a distance of 3 cm between filament and collector. The mercury vapor pressure was only 0.23 bar corresponding to saturated vapor at 0°C. The curve marked I_b gives the collector current in micro-amperes as a function of the collector potential. The curves 0.1 I_b and 0.01 I_b represent the same data on a scale 10 and 100 times smaller so as to bring the upper portions of the curve within the plot.

It is seen that at collector potentials below -60 only positive ions are collected. The currents vary with the potential because of an "edge correction" which causes[7] the area of the positive ion sheath to increase as the sheath thickness increases. The dotted line A shows how this current would vary at higher potentials if no electrons were collected. At potentials above -59 volts, at point M, electrons begin to be collected, showing the presence of appreciable numbers of electrons having energies of from 52 to 59 volts although the energy of the primaries was only 50 volts.

From the data of Fig. 1 we see that with the collector at -40 at point N practically all the primaries have been collected. With a cathode current of only 5 m-amp. on the other hand, the primaries were all collected with voltages of about -47. This shows that the effect of raising the primary current from 5 to 10 m-amp. was to scatter the primary electrons,

[7] See loc. cit.[2], p. 540.

causing not only an increase in the velocities of some of the primaries but a decrease in the velocity of others. The average energy of all the primaries was slightly lowered to about 47 volts. The total number of primary electrons collected was twice as great with 10 as with 5 m-amp. cathode current, so that the effective free path of the injected electrons remained

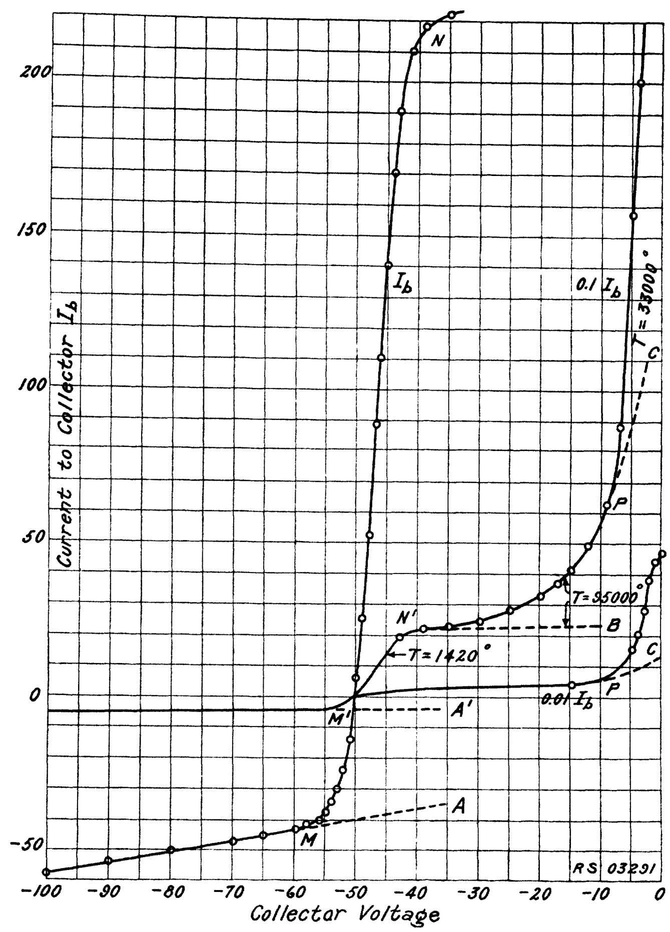

Fig. 1. Electron scattering in mercury vapor at 0.23 bar (0°C) with 10 m-amp. primary current and the cathode at −50 volts.

the same, if by free path we mean the distance that the electrons travel before losing the greater part of their momentum in their original direction.

By subtracting from the observed currents (Fig. 1) the positive ion currents corresponding to the dotted line A we obtain the total electron current ΔI_b to the collector. In Fig. 2 the logarithms of these currents in micro-amperes have been plotted against the collector voltage. In this way the data for from -55 to 0 volts are brought within a single convenient curve. The rise MN which occurs between -55 and -40 volts corresponds to the primary electrons. If these all had the original energy of 50 volts the rise should occur as a vertical line at an abscissa of -50 volts. The inclination of the line from the vertical thus measures the amount of electron scattering.

The line $N'B$ in Fig. 1, corresponding to the horizontal dotted line NB in Fig. 2, gives the current as it would be if no other electrons than the primaries were collected. The differences between the observed currents $N'P$ and those of line $N'B$ (Fig. 1) thus correspond to a new group of electrons which probably result from collisions of the primary electrons with gas molecules. We shall call these *secondary electrons*. This group includes primaries which have lost most of their energy and also electrons which are ejected from gas molecules by these collisions.

The distribution of velocities among the secondary electrons is found by taking the differences between the observed currents and the current corresponding to $N'B$ (Fig. 1), and by plotting the logarithms of these differences as indicated by the points along the line D (Fig. 2). Within the range from -30 to -12 volts the points lie on a straight line. This proves that the velocity distribution follows Maxwell's law. From the slope the temperature of this secondary electron group is found to be 95,000°. By extrapolation to higher voltages along the straight line D, the currents corresponding to the secondary electrons are calculated as indicated by the dotted line PC. Thus the line NPC is the curve of the secondary electrons.

The difference between the observed currents PR and the secondary currents PC (Fig. 1) gives another group of electrons which we will see have a Maxwellian velocity distribution and are in fact the same as the free electrons found in the positive column of the low voltage discharge. We shall call these the *ultimate electrons*. If the currents due to these are small compared to the current flowing to the anode the logarithms of the currents may be plotted and the slope of the resulting straight line taken to measure their temperature. But, it often happens that the concentration of the ultimate electrons is so high that the current taken by the collector becomes comparable with that flowing to the anode. The collector may, in fact, rob the anode of nearly all its current and thus raise the space potential along with the collector potential. In these cases, as mathematical analysis shows, we should plot the logarithm of the ratio of the electron current to the collector, to the current to the anode. The slope of this line as indicated by line E measures the temperature of the ultimate electrons. In view of the logarithmic scale of Fig. 2 it will be seen that the number of ultimate electrons greatly exceeds that of the primary or secondary electrons. The relative numbers of electrons in the various groups is seen more clearly in Fig. 1 because there the currents themselves are plotted as ordinates. The dotted lines $N'B$ and PC on Fig. 1 are drawn to correspond to those in Fig. 2.

In other experiments disk-shaped collectors were mounted close to the primary electron beam but in a plane perpendicular to the axis of the cathode so as to be parallel to the direction of the electron beam. The current to such a collector gives no indication whatever of the prim-

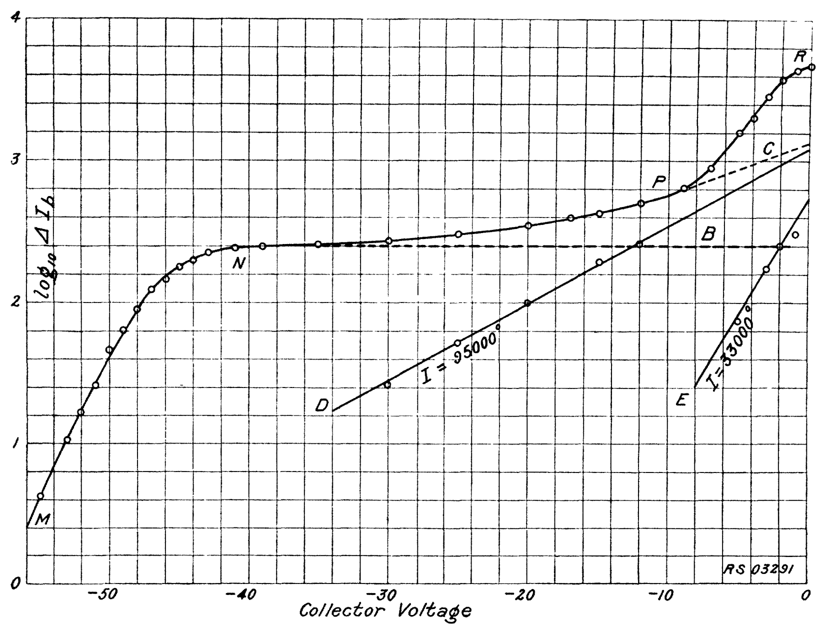

Fig. 2. Semi-logarithmic plot of the electron currents from the data of Fig. 1.

ary electrons corresponding to the rise MN, but the groups of secondary and ultimate electrons corresponding to the rises between NP and PR are exactly the same as those shown in Fig. 2 for a collector mounted perpendicular to the beam. Such experiments justify our separation of the electrons into groups of primaries and secondaries and establish the fact that the primaries move mainly in one direction while the secondaries and ultimate electrons move equally in all directions.

In our present discussion our interest centers on the *distribution of the velocities of the primary electrons* after they have passed through the ionized gas. The portions of the curves MN in Figs. 1 and 2 should give us this distribution. In order to analyze such curves mathematically the hypothesis was made that the scattering of the primaries takes place in such a way that the resultant velocity distribution can be analyzed into a uniform translational velocity and a superposed Maxwellian distribution corresponding to a temperature T—this distribution being analogous to that of the molecules of a gas which is moving with uniform velocity.

We can now calculate what fraction of the electrons in such a beam can move against a given retarding field like that in the sheath of a negatively charged plane collector. Assuming that the velocities due to temperature motion are small compared to those of translation it can be shown that the current I of electrons which reaches a collector is given by

$$I = \tfrac{1}{2} I_0 (1 + \operatorname{erf} \beta) \tag{6}$$

where erf denotes the error function

$$\operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-x^2} dx\,. \tag{7}$$

I_0 is the current when all the primary electrons are collected and β is defined by the equation

$$\beta = (\sqrt{-V} - \sqrt{V_1}) / \sqrt{V_T}. \tag{8}$$

Here V is the potential of the collector with respect to the ionized gas (or anode), V_1 is the potential corresponding to the translational velocity of the primary electrons and V_T is the potential corresponding to the energy kT, k being the Boltzmann constant. If V_T is expressed in volts

$$V_T = T/11600. \tag{9}$$

When the collector potential is not too far from that of the cathode the value of β from Eq. (8) may be expanded giving

$$\beta = \text{const} + V/(2\sqrt{V_1 V_T}). \tag{10}$$

Thus β is a linear function of the collector potential and from Eqs. (6) and (10) we then see that $\operatorname{erf}^{-1}[(2I - I_0)/I_0]$ is a linear function of V having a slope equal to $1/(2\sqrt{V_1 V_T})$. If this function of I is plotted against V and a straight line is obtained it proves the existence of the kind of velocity distribution we have assumed, and enables us to determine the temperature corresponding to the random part of the motion. These operations may be more conveniently carried out by using probability paper in which the vertical lines are uniformly spaced and the horizontal lines, which are denoted by various values of a parameter a, are spaced according to the relation

$$y = \mathrm{erf}^{-1}(2a-1)$$

Temperature of primary electrons. In Fig. 3 the primary electron currents corresponding to the data of Figs. 1 and 2 are plotted on probability paper. The abscissas are the collector voltages while the ordinates are $\Delta I_b/I_0$ where I_0 has been taken as 257 micro-amperes in accordance with the data of Fig. 1. It is seen that the observed points lie very close

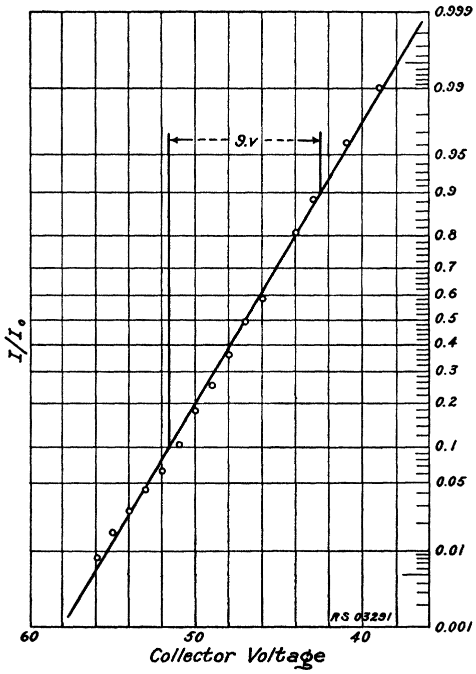

Fig. 3. Probability plot of the primary electron current from data of Fig. 1.

to a straight line. The difference in voltage corresponding to the 2 points $I/I_0 = 0.1$ and 0.9 is 9.0 volts. The increment of $\mathrm{erf}^{-1}(2a-1)$ corresponding to these values is 1.812. Thus by (10)

$$dV/d\beta = 9/1.812 = 2\sqrt{V_1 V_T}.$$

Since the energy of the primary electrons in the beam corresponds to 50 volts we place $V_1 = 50$ and thus find $V_T = 0.122$ volts or by (9) $T = 1420°$. The voltage on the probability plot (Fig. 3) which corresponds to the midpoint (i.e. where $I/I_0 = 0.5$) measures the average translational velocity of the electron beam. Thus from Fig. 3 the forward velocity of the beam corresponds to 47.2 volts although the original velocity of the primary electrons corresponded to 50 volts. The mechanism producing the scattering therefore does not act uniformly in all directions but produces the greatest changes in momentum in a direction opposite to that of the original beam.

The data from large numbers of experiments, of which the data of Fig. 3 are typical, confirm the correctness of the hypothesis that the primary electrons have random Maxwellian velocities superposed upon the translational velocity. The degree of scattering in this case is very small as the voltage corresponding to the random velocities is only 0.122 while the voltage of the translational velocity is 47.

Variation with current density. To illustrate the effects produced by modification in the experimental conditions we may consider another typical run made with the same apparatus. The pressure of the mercury vapor was 1.6 bars corresponding to saturated vapor at 20°C. The electron emission from the cathode was 30 m-amp. The cathode was at a potential of -50 volts with respect to the anode, and the distance from the collector to the cathode was 3 cm as in the run illustrated in Figs. 1 to 3. The data were plotted and analyzed as before. Appreciable electron currents were observed with collector potentials as low as -90 volts showing that some electrons (about 3 micro-amperes) having energies as high as 90 volts had been produced from the original beam of 50 volt electrons. The secondary electrons gave a straight semi-logarithmic plot corresponding to $T=95{,}000°$ which is the same temperature as found from the data of Fig. 1. The temperature of the group of ultimate electrons was 15,600°. The plot of the primary electron current to the collector, on probability paper, gave 17 consecutive points in the voltage range from -90 to -10 volts which came very close to a straight line (maximum departure 3 volts) whose slope corresponded to $T=56{,}000°$. The mid-point of this line was at -36 volts so that the beam had been retarded by an amount corresponding to 14 volts.

The very much higher temperature of the primary electrons in this experiment as compared to the previously described run is due mainly to the higher primary current density, but is partly due to the higher pressure of mercury vapor.

Fig. 4 illustrates the current-voltage curves of the collector for potentials below that of the cathode with a series of different primary current densities. The figures written near the curves are the electron emissions from the cathode in milli-amperes. Three different cathode potentials were used, -35, -50 and -70 volts. The actual experimental points are indicated in only one curve, which being typical, illustrates the accuracy with which the observations fit the curves.

The moderately sloping parts of the curves, which are continued as dotted lines to the right, give the positive ion currents. The departures from these lines measure, as before, the primary electron currents. It will be noted that with 5 m-amp. the electrons begin to be collected only when the collector comes within 1 or 2 volts of the cathode potential, showing that there are very few electrons having higher velocities than those of the original primaries. As the current density is increased to

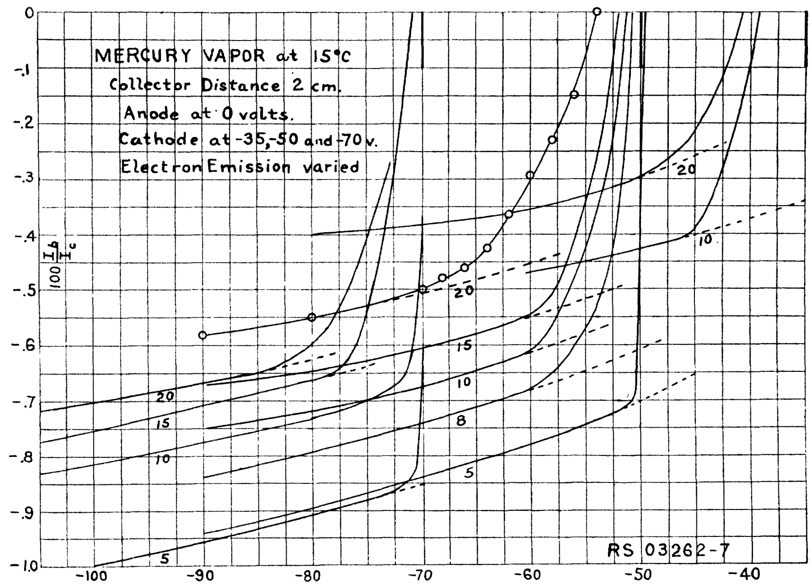

Fig. 4. The effects of cathode voltage and current density on the scattering of electrons. The ordinates give the ratio I_b/I_c where I_b is the negative current received by the collector and I_c is the electron emission from the cathode. The figures marking the curves are the values of I_c in milliamperes.

10 or 20 m-amp. however, electrons begin to be collected at potentials of 10 or even 20 volts below that of the cathode. The effect seems to be somewhat more marked at 35 and at 50 volts than at 70 volts.

In a series of later experiments the primary electron temperatures were determined for each of a series of primary current densities corresponding to electron emissions ranging from 3 to 30 m-amp. The mercury vapor pressure was 0.23 bars (0°C), the collector distance 3 cm, and the cathode potential -50 volts. The primary electron temperature was found, within the experimental error, to be proportional to the 2.2 ± 0.2 power of the current.

Variation with the distance of the collector. Experiments in which different distances between the cathode and the collector were used have shown that the primary electron temperatures increase roughly in proportion to the distance traversed by the electrons. Further experiments on this effect are in progress.

Variation with cathode voltage. In a series of experiments with mercury vapor at 0.23 bar (0°C) with a collector distance of 3 cm and a current of 10 m-amp. the primary electron temperatures varied with the voltage as follows:

voltage:	−80	−70	−50	−40	−35
T:	800°	900°	1300°	1700°	2200°

With 20 m-amp. current, the other conditions being the same, the temperatures were

at −80 volts	at −50 volts
2050°	4750°

The greatest scattering of the electrons thus occurs at the lowest voltages, but, in general it is difficult and unsatisfactory to work with voltages below 50 volts because the effects of the scattering then tend to be masked by the secondary and ultimate electrons. As the voltage difference between cathode and anode decreases not only does the velocity of the primary electrons decrease but the character of the excitation of the ionized gas may change. For example, with higher voltage, atoms may be excited to higher energy levels. These experiments do not enable us to determine which of these factors is responsible for the change in the amount of scattering.

TABLE I

Effect of mercury vapor pressure on electron scattering.

Primary current (m-amp.)	Mercury Temp.	Mercury Pressure (bars)	Electron temperature	Retardation (volts)	$\frac{\Delta E_l}{\Delta E_r}$
10	0°	0.23	1360°	3.1	17.6
	20	1.6	870	2.3	20.4
	30	3.7	240	1.8	58.
	40	8.0	140	1.4	77.
20	0	0.23	5500	5.0	7.0
	5	0.37	7100	4.5	4.9
	10	0.61	9800	4.8	3.8
	15	1.0	5900	3.5	4.6
	20	1.6	4000	3.2	6.2
	35	5.5	750	1.8	18.5

Variation with mercury vapor pressure. Table I shows the way that the electron scattering varied with the vapor pressure of mercury. The cathode was at −50 volts and the collector distance was 3 cm. The last column gives the retardation of the beam or the decrease in the translational energy of the beam as determined from the voltage of the mid-point of the straight line on the probability plot.

Experiments with crossed beams. The fact that the electron scattering is negligible at low current density and increases rapidly at higher currents suggests that it is caused by the action of radiation or by excited atoms or ions. To gain more information as to the effect of ionization and excitation of the gas a spherical bulb (12 cm in diameter) was provided with two straight cathode filaments, A and B, each 1.1 cm long, mounted in the same plane at right angles to each other. Facing these cathodes at distances of 4 cm there were disk-shaped collectors (A' and B') of 1.1 cm diameter, the axes of these disks being in the same plane as the two cathodes.

By lighting only one cathode, A, a single disk-shaped beam of primary electrons (about 1 cm thick) was produced. The two collectors A' and B' being about the same distance from the center of the bulb, received the same number of secondary and ultimate electrons, but since B' was outside of the beam (and its surface was parallel to the direction of the beam) only the collector A' received primary electrons.

When both cathodes were lighted the volt-ampere characteristics of the two collectors gave the amount of scattering of each of the two perpendicular electron beams, and thus made it possible to determine the influence of one electron beam on the other.

It was found that the temperature and the number of both the *secondary* and the *ultimate* electrons were always very nearly the same for both collectors whether or not the collector was in the beam of primaries. The temperatures of both these groups were independent of the currents from the cathodes. The number of electrons in each of the 2 groups was a function of the sum of the currents emitted by the two cathodes and was in fact approximately proportional to this sum. Thus it was immaterial whether one cathode emitted 20 m-amp. while the other emitted none, or each of the cathodes emitted 10 m-amp.

The behavior of the primary electrons was entirely different. The total number of primaries that could be received by each of the two collectors was of course proportional to the electron emission of the opposing cathode. If the scattering of the primary electrons were due to the free electrons, or ions, or to the kind of excited atoms and radiation that is produced by the secondary and the ultimate electrons, we should expect that the temperatures of the primary electrons should be determined by the temperatures and the concentrations of these two groups of electrons.

Thus the temperatures of the primaries should also be a function of the sum of the emissions from the two cathodes.

The experiments showed, however, that the primary electron temperature depended mainly on the emission of electrons from the cathode which opposed the collector and was usually only slightly influenced by the emission from the other cathode.

For example, in some runs with mercury vapor at 0.75 bar (12°C) with both cathodes at -80 volts, cathode A was heated to such a temperature that it emitted 5 m-amp. while B was adjusted to emit 20 m-amp. The temperature of the primaries reaching A' was 800° while those reaching B' had a temperature 5200°. When cathodes A and B were at the same temperature and each emitted 20 m-amp., the temperatures of the electrons received by each collector was still about 5000°. By then turning off the heating current to one of the cathodes, say A, so that its emission fell to zero, the temperature of the electrons received by the collector B' facing the other filament was practically unchanged notwithstanding the fact that the concentration of the secondary and ultimate electrons and of the positive ions throughout the whole space in the bulb, fell to about half value.

With mercury vapor pressures corresponding to 20° and especially to 30°C one beam of primary electrons seemed to be able to cause a distinct increase in the temperature of the other beam (which it intersected for about 1/4 of its length). For example, at 30° and with both cathodes at -80 volts but only one cathode lighted so as to emit 20 m-amp., the electron temperature observed at the opposing collector was 700°. By allowing the other cathode also to emit 20 m-amp. both beams of primaries gave a temperature of 2500° thus showing a marked increase in temperature caused by the other beam. However, if one cathode was heated to emit 40 m-amp. while the other was cold the temperature of the beam was 20,000° or 8 times as high as when each cathode emitted 20 m-amp., although the concentrations of the secondary, and ultimate electrons was the same in the two cases. Evidently with mercury at 30° the exponent of the power of the current with which the electron temperature increases is higher than it is with vapor from mercury at 0°C.

From these results we may draw the definite conclusion that the main cause of scattering of 80 volt electrons, especially at low pressures of mercury vapor, is not the ionization, excitation or radiation produced by, or characteristic of the secondary or the ultimate electrons, but the scattering must be due to a more direct effect of the primary electrons and one which is confined to the region actually traversed by these primary electrons. This indicates that the scattering cannot be due to electrons or ions per se. It leaves open, however, the question as to whether radiation or excited atoms may be the cause. Thus 80 volt primary electrons are capable of exciting mercury atoms to energy levels far above those obtainable from the secondary or the ultimate electrons, and radiations of correspondingly high frequency are similarly produced. Excitation and radiation of this kind might well be confined to the region traversed by the primaries and might thus explain the results of the experiments. The distinct but slight effect of one beam of electrons on the other would then be due to the action taking place within the region (about 25 percent of the total path) where the two beams intersect. Experiments are now in progress to study parallel beams and beams with intersect over nearly their whole lengths in order to clear up this question.*

In some other experiments an arc of 0.2 to 0.5 ampere was passed through mercury vapor in a 12 cm bulb from an external mercury cathode while the distribution of the velocities of 50 volt primary electrons from a hot cathode in the bulb was studied. It was found that the ionization due to the arc did not increase the primary electron temperatures.

III. Mechanism of Electron Scattering

Before taking up this general problem, let us enquire whether the scattering of the electron beam can be due to encounters with other electrons. Since the electrons in the beam start out from the cathode with the same velocity, they have no velocity with respect to one another and therefore cannot deliver energy to one another by collision. Space charge due to the electrons in the beam is neutralized by positive ions, but even if it were not, the electric field due to space charge could produce only transverse scattering and would not change the longitudinal velocities of the electrons.

Scattering by clouds of stationary electrons. If the scattering is to be produced by electrons, it must be due to the electrons in the ionized gas and these as we have seen have velocities much lower than those of the primaries. Let us therefore consider the scattering of a beam of electrons when it enters a cloud of stationary electrons. If we assume that the collisions follow the laws of elastic spheres we find that an electron which

* *Note added Sept. 2, 1925.* In some recent experiments two straight tungsten filaments 1 cm long were mounted parallel to one another at a distance of about 1 mm, so that two electron beams could be obtained that were practically coincident. In this case the primary electron temperature depended only on the *sum* of the electron emissions from the two cathodes, showing that the scattering did not occur within the cathode sheath but resulted from some effect taking place throughout the space travered by the primary electrons.

has made one collision with a stationary electron has on the average lost one half its momentum in the forward direction and has given half its energy to the other electron. Thus, if we have a beam of electrons which are moving with a homogeneous velocity corresponding to a voltage V_0 and we let this beam encounter a cloud of stationary electrons so that each electron in the beam makes one collision with an electron, we find that the translational velocity of the beam is reduced to a value corresponding to $\frac{1}{4}V_0$. An observer, moving along with this retarded beam, will see that the electrons in the beam are now moving in random directions with velocities corresponding to the energy $\frac{1}{4}V_0$. The effect of a single collision for each electron thus causes a decrease in the longitudinal energy of the beam of ΔE_l and an increase in the *random* energy ΔE_r given by

$$-\Delta E_l = \tfrac{3}{4}V_0 e \tag{11}$$

$$\Delta E_r = \tfrac{1}{4}V_0 e \tag{12}$$

the ratio being

$$-\Delta E_l/\Delta E_r = 3 \tag{13}$$

The electrons in the retarded beam cannot, however, in this case get a Maxwellian velocity distribution. There is no mechanism by which an electron can acquire velocities exceeding those in the original beam. If the number of electrons in the cloud is so small that the average decrease in the longitudinal energy is say 5 percent (about the average observed in our experiments) then 93.3 percent of the electrons have made no collisions at all and must retain all their original translational energy while 6.7 percent have lost 3/4 of their original energy. This would give a velocity distribution having no resemblance to a Maxwellian distribution. Thus we must conclude that the observed scattering cannot be due to the direct action of electrons upon those in the beam.

Scattering by clouds of entities of very small mass. If the particles in the cloud have masses m_p negligibly small in comparison to those of electrons, these difficulties disappear. In this case the fractional decrease in the translational velocity v_l of an electron in the beam is[8]

$$-\frac{\Delta v_l}{v_0} = \frac{4}{3}\,\frac{m_p}{m}\,n \tag{14}$$

where n is the number of collisions, and v_0 is the original velocity of the electrons in the beam. The decrease in longitudinal energy is thus

$$-\Delta E_l = \tfrac{4}{3}\, m_p v_0^2 n = \tfrac{8}{3}\,(m_p/m) n V_0 e\,. \tag{15}$$

[8] See Eq. (789) in Jeans' Dynamical Theory of Gases, 2nd Edition, p. 282. Cambridge, 1916.

A particle of velocity v_p colliding with an electron in the beam delivers its whole momentum $m_p v_p$ to the electron and the energy E thus acquired by the electron is

$$E=(m_p/m)E_p \tag{16}$$

where E_p is the kinetic energy of the particle. Since the energy of the particles which *strike* the electron is $2kT_p$ we have for the average random energy after n collisions

$$\Delta E_r=2nkT_p(m_p/m)\,. \tag{17}$$

This average energy corresponds to $\frac{3}{2}kT$ so that the temperature of the primary electrons after n collisions is

$$T=\tfrac{4}{3}\,nT_p(m_p/m). \tag{18}$$

By comparing Eqs. (15) and (17) we find for the ratio of the decrease in longitudinal energy to the increase in random energy

$$\frac{-\Delta E_l}{\Delta E_r}=\frac{4}{3}\,\frac{V_0 e}{kT_p}\,. \tag{19}$$

For convenience we may express T_p in terms of the equivalent voltage V_p defined by

$$V_p e=kT_p \tag{20}$$

so that (19) becomes

$$\frac{-\Delta E_l}{\Delta E_r}=\frac{4}{3}\,\frac{V_0}{V_p} \tag{21}$$

Scattering by quanta. The momentum of a quantum of energy $h\nu$ is $h\nu/c$. If this quantum acts on an electron it may deliver to the electron any momentum between 0 and $2h\nu/c$ according to the direction taken by the scattered quantum. If the quanta are scattered with equal probability in all directions, as if by elastic spheres, the average momentum transferred may be taken to be $h\nu/c$. The average energy delivered to the electron by n collisions is

$$E_r=\tfrac{2}{3}\,n(h\nu)^2/mc^2. \tag{22}$$

Thus after n collisions with quanta the electron temperature is

$$T=(4/9)\,n(h\nu)^2/kmc^2\,. \tag{23}$$

We may conveniently express the energy of the quanta $h\nu$ in terms of the equivalent voltage V_q according to the relation

$$h\nu=V_q e\,. \tag{24}$$

Introducing the numerical values of k, m, c and e and expressing V_q in volts, Eq. (23) thus reduces to

$$T=0.0102nV_q^2. \tag{25}$$

Thus, for example, with quanta of a frequency corresponding to 50 volts, the temperature of the electrons increases by 25° for each collision with quanta.

The fractional decrease in drift velocity of the electrons after n collisions with quanta is

$$-\Delta v_l/v_0=4nh\nu/3mc^2=2.62\times 10^{-6}nV_q \tag{26}$$

and the decrease in longitudinal energy is thus

$$-\Delta E_l=4v_0^2nh\nu/3c^2=\tfrac{4}{3}(v_0/c)^2nV_qe. \tag{27}$$

Comparing this with (22) we find

$$-\Delta E_l/\Delta E_r=4V_0/V_q. \tag{28}$$

Relation between the retardation and the scattering. Let us compare these calculated values of $\Delta E_l/\Delta E_r$ with the results of the experiments. In the experiment for which data were given in Fig. 1 the retardation corresponded to 2.8 volts and the electron temperature, due to scattering, was 1420°. The average energy in random direction E_r corresponds to $\frac{3}{2}kT$, so to express E_r in volts we divide the temperature by 2/3 of 11600, or 7730. Thus we find $-\Delta E_l/\Delta E_r=15.3$; i.e., the loss of energy by retardation is 15.3 times as great as the thermal energy of the scattered electrons.

Comparing this result with Eq. (28), we find $V_0/V_q=3.8$, and therefore $V_q=13$ volts since $V_0=50$ volts. Thus, quanta of a frequency corresponding to 13 volts would produce the ratio of retardation to scattering which was found by the experiments.

Rather variable results are obtained from other experiments. For example, the last column of Table I gives values of $-\Delta E_l/\Delta E_r$ ranging from 3.8 to 77, which would correspond to quanta having frequencies corresponding to voltages V_q from 53 down to 2.7. These values are all of reasonable order of magnitude, since radiation of such frequencies may well be present in gases excited by electrons of 50 volts energy.

If we assume that the scattering occurs according to the laws of the kinetic theory, by Eq. (21), the voltages corresponding to the energies of the particles will be 1/3 of those just found for V_q. These values also seem possible, although less probable, and we therefore cannot decide between the two theories from these data. Since, however, no entities other than quanta are known having masses less than those of electrons, we will confine our attention at present to scattering by radiation.

Magnitude of Compton effect. According to the classical electromagnetic theory[9] the scattering coefficient of light by free electrons is

$$\alpha = 6\pi a^2 n_e \tag{29}$$

where α is the fraction of energy scattered per unit length of path through a region occupied by n_e electrons per unit volume and a is the radius of the electron by Lorentz's theory ($a = 1.88 \times 10^{-13}$ cm). Thus we find

$$\alpha = 6.7 \times 10^{-25} n_e \text{ per cm.} \tag{30}$$

According to the correspondence principle, we should expect approximately this amount of scattering even if the scattering takes place by quanta. For metallic aluminum $n_e = 8 \times 10^{23}$ electrons per cm^3, so we obtain $\alpha = 0.53$ per cm, which gives a scattering of the order of magnitude found for the Compton effect.[10] For a mercury arc n_e is of the order of 10^{10} electrons per cm^3, so that α should be about 10^{-14}. This is in accord with the fact[11] that no Tyndall cone is observed when a strong beam of sunlight is focussed by a large lens into a nearly non-luminous 2-ampere arc in argon at a few millimeters pressure.

With a primary electron current of 10 m-amp. and 50 volts, the total energy escaping from the disk-shaped electron beam is about 0.02 watt per cm^2, or about 5×10^{15} quanta ($V_q = 25$ volts) per cm^2 per second. With a scattering coefficient of only 10^{-14} there would be 50 quanta scattered per cm^3 per second. To give the observed electron temperatures of 1420° each electron would have to make (according to Eq. 25) 230 collisions with quanta. Since there are 6×10^{16} electrons per second in the primary beam, the number of collisions per second would need to be 1.5×10^{19} or about 3×10^{17} per cm^3 per second, instead of the 50 collisions calculated above.

Thus, the observed scattering is about 10^{16} times greater than that to be expected from a normal Compton effect.

This difficulty seems to be very similar to that met with in connection with the theory of chemical reaction velocity[12] where it is found that a chemical molecule must become activated millions of times more frequently than it collides with other molecules. Yet the energy that it can

[9] Leigh Page, Astrophysical Jour. **52**, 67 (1920).

[10] Since the frequencies of x-rays are sufficient to remove most of the electrons in aluminium, we may consider the electrons as free electrons in regard to the scattering of this kind of radiation.

[11] According to rough experiments in this laboratory.

[12] Langmuir, Jour. Amer. Chem. Soc. **42**, 2190 (1920).

receive by radiation, according to the usual radiation laws, is millions of times too small to cause this activation.[13]

Scattering by the joint action of radiation and excited atoms. Because of the small magnitude of the Compton effect we must conclude that the scattering of electrons is not due to any direct interaction between them and radiation. It may well be, however, that the effect is due to an *interaction between radiation, excited atoms and electrons.*

Resonance radiation is enormously strongly scattered by excited atoms, practically all the radiation coming within a distance of about one-half wave-length from an excited atom being scattered, whereas the radiation scattered by an electron is only 6 times that falling on an area equal to the cross-section of the electron. For radiation corresponding to 25 volts the wave-length is 490A. The radiation scattered by an excited atom of a 25 volt energy level is thus about 10^{14} times greater than that scattered by an electron. This is about the order of magnitude of the ratio (10^{16}) of the observed scattering to that calculated for a normal Compton effect. If resonance radiation is trapped within the region traversed by the primary electrons, the ratio 10^{16} would be reduced, so that it might well be brought down to the value 10^{14} corresponding to the radiation scattered by excited atoms.

When a quantum of resonance radiation is scattered by an atom or excited atom the direction of the quantum is changed and the difference of momentum is available to deliver an impulse to some particle. Although we would expect in general that the momentum would be delivered to the atom or excited atom, it may well be that if an electron is close enough, the momentum may be delivered to the electron. In some such way as this we may suppose that the electrons can receive momenta whose magnitude is in accord with Compton's theory, although the probability that an electron may receive this momentum is 10^{14} times greater than according to the normal Compton effect.

Consider the excitation of a mercury atom by a quantum of resonance radiation (λ2537). The difference in the angular momentum of the atom before and after excitation must be a multiple of $h/2\pi$. Before the encounter, the two-body system, consisting of atom and quantum, will also contain an angular momentum equal to the product of the momentum $h\nu/c$ of the quantum, by the distance between the center of the atom and the path of the quantum. This angular momentum in general will not be a multiple of $h/2\pi$. After the collision we have a one-body system having a fixed angular momentum. Thus, it would seem that a

[13] See also a recent paper by R. C. Tolman, Jour. Amer. Chem. Soc. **47**, 1524 (1925).
[14] Barkhausen and Kurz, Phys. Zeits. **21**, 1 (1920).

mercury atom can be excited by a quantum only if a third body is present which can take up the residual angular momentum. An electron within a distance comparable to a wave-length may serve as such a third body and may thus increase the ease with which excited atoms are formed. Perhaps without such third bodies, resonance radiation cannot even be scattered, although it is not necessary to draw this conclusion, since scattering involves a two-body problem both before and after the scattering. Experiments to determine whether the scattering coefficient of resonance radiation is dependent on the presence of free electrons are in progress by Mr. Whitten in this laboratory. An attempt will also be made to determine whether low velocity electrons in mercury vapor or sodium vapor are scattered when resonance radiation is allowed to act on the vapor.

The loss of energy by retardation of a beam of primary electrons is always much greater than the energy that appears as thermal energy of the scattered electrons. A beam of primary electrons may thus deliver energy directly to the radiation, or to excited atoms, by the reverse of the process that causes scattering of the electrons. Thus the average frequency of the radiation in the primary beam probably increases as the current density of the beam is raised. This may be the reason that the primary electron temperature increases with the square or even with a higher power of the current density.

As an alternative hypothesis to account for the large amount of electron scattering, we may assume that the presence of an electron within a distance comparable to a wave-length from an excited atom, greatly increases the probability that the excited atom will emit a quantum of radiation. Thus, the average intensity of radiation near the electron would be much greater than that in the space as a whole and the magnitude of the Compton effect would thereby be increased. The momentum delivered to the electron in this case may, however, be very much greater than that due to a Compton effect. Of course, if an electron increases the probability of emission of radiation by an excited atom it must also increase the probability of the absorption of a quantum by an atom according to the reverse process. If scattering of resonance radiation is due to quantum jumps then, the scattering should be increased by the presence of electrons.

IV. Explanation of Maxwellian Distribution of Ultimate Electrons

We have seen that the primary electrons from a cathode at -50 or -80 volts are scattered mainly by some direct effect of the primary electrons and not by the action of the ionization or excitation caused by the secondary or the ultimate electrons. In the positive column of an arc primary and secondary electrons are absent. It might seem, therefore, that the mechanism by which the electrons in an arc acquire their Maxwellian distribution must be different from that which causes the scattering of primary electrons. It seems more probable, however, that the mechanism is essentially similar, the difference being merely one of degree.

For example, in an arc in mercury vapor at 5 bars pressure the temperature of the ultimate electrons is about 20,000°, corresponding to an average energy ($\frac{3}{2}kT$) of 2.6 volts. According to Eq. (25) the corresponding radiation (λ4700) would increase the temperature of electrons by only 0.08° for each quantum scattered, whereas 50 volt radiation which might exist in the primary beam could raise the temperature by 25° per collision.

In the case of the ultimate electrons in an arc it seems probable that there is a kind of thermal equilibrium between the electrons and the radiation and excited atoms. That is, the electrons lose as much energy to the radiation on the average, as they gain from it. Thus there is probably a definite relation between the temperature of the ultimate electrons and the frequency of the ultraviolet radiation or the average energy levels of the excited atoms.

The effective electron free-path determined from the mobility of electrons in an arc is not the average distance the electron moves before being acted on by a quantum, but is the distance it must move before losing most of its momentum in a given direction. By Eq. (26) we see that an electron will lose about half its forward momentum when it has made 190,000 collisions with quanta of a frequency corresponding to 2.6 volts. The experiments have given an effective free path of about 3 cm at one bar so the distance between successive interactions with quanta would average 1.5×10^{-5} cm.

It will be understood that the foregoing theoretical considerations are purely tentative. The fundamental cause of electron scattering can be established only after further experimental studies of the laws governing it.

V. Scattered Electrons in High Vacuum Tubes.

One of the most striking characteristics of electron scattering is the ability of some electrons to pass to an electrode which is more negatively charged than the cathode from which the electrons originate. A similar phenomenon is observed in vacuum tubes having apparently the highest vacuum if the anode consists of one or more wires of small size at high positive potential while a surrounding cylindrical electrode is negatively charged with respect to the cathode. A current of electrons flows to this cylinder. This effect, discovered by Barkhausen and Kurz[14] was explained by them as being due to electrical oscillations of electrons in the evacuated space. These oscillations were actually detected by Barkhausen and Kurz. Gill and Morell[15] have outlined a theory which suggests a cause of the oscillations.

Experiments in this laboratory, however, have shown that although oscillations frequently exist in such a tube they do not seem to be the sole cause of the currents to the negative cylinder. For example, in some experiments the logarithm of the current to the cylinder varied linearly with voltage on the cylinder suggesting a Maxwellian distribution of the high speed electrons in the space. The temperature of these electrons was found to be proportional to the potential on the anode. With cylinder potentials close to those of the cathode electrical oscillations were observed, but when the cylinder was made more strongly negative the oscillations stopped abruptly and completely as far as could be determined, and simultaneously the current to the negative cylinder *increased* several fold.

Similar effects are observed in a magnetron having end plates charged negatively with respect to the cathode. A longitudinal magnetic field tends to prevent electrons from reaching the cylindrical anode and the negative potentials on the end plates tend to prevent their escaping at the ends of the cylinder. Under these conditions, small currents of electrons do flow to the cylinder in spite of the magnetic field and flow to the end plates even if these are 20 to 50 volts negative with respect to the cathode. Yet oscillations can very rarely be detected. These experiments have been repeated by Mr. C. G. Found with the very best vacuum conditions using an anode of deposited magnesium on the glass wall and immersing the whole apparatus in liquid air, but the currents to the negative electrodes were not decreased.

It will be noted that these effects might be explained if the trapped electrons in the Barkhausen-Kurz tube or in the magnetron were scattered so as to obtain a Maxwellian distribution. However, it is hard to see how this distribution can be brought about with so few excited atoms and so little radiation as must be present in these tubes.

[15] Gill and Morell, Phil. Mag. **44**, 161 (1922).

VI. Oscillations as a Possible Cause of Scattering in Ionized Gases

Since oscillations occur so often in tubes of this character and have been suggested as a cause of the currents to negative electrodes, it has been felt throughout the experiments on the scattering of electrons in ionized gases, that it is important to determine whether oscillations are present. For this reason we have frequently used radio detectors capable of detecting oscillations of 0.5 meter wave-length and have introduced small capacities and inductances in various parts of the circuits to see if any of the currents to collectors could be modified in this way. In none of the experiments on electron scattering in low pressure gases using hot cathodes have oscillations been observed. In some experiments with mercury arcs oscillations occurred at first until high resistances or choke coils were used in the anode circuits. The effects produced by the oscillations were not at all like those characteristic of electron scattering.

Research Laboratory,
General Electric Co.,
Schenectady, New York,
July 16, 1925.