THE DISTRIBUTION OF INITIAL VELOCITIES AMONG THERMIONIC ELECTRONS

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ABSTRACT

The method used was to measure the number of electrons from a straight tungsten filament which were able to arrive at a co-axial cylindrical electrode against various retarding potentials. In order to eliminate certain disturbing factors, particularly photo-electric effects, this electrode was made in the form of a very fine grid and those electrons passing between the grid wires were collected upon an outside electrode and there measured. A rather complicated intermittent heating current arrangement allowed emission from the filament only when its surface was at uniform potential, and insured that the retarding potential had exactly the desired value. A current regulator kept the heating current constant to 1/30 percent. (1) *Electrons from tungsten.* Measurements of the variation of electron current with voltage were made at eight different temperatures ranging from 1440°K to 2475°K. Correction was made for the contact potential difference between filament and grid. At each temperature it was found that, except in the range of voltage where the current was limited by the space charge phenomenon, the current varied with voltage in just the manner calculated upon the assumption that the electrons leave the filament with velocity components distributed according to Maxwell's law for an electron atmosphere in temperature equilibrium with the hot filament. At 2475°K the assumed Maxwell distribution was verified up to a retarding potential so great that only one electron out of 10^{10} emitted electrons was able to reach the collector. It is believed that the present results are more reliable and extensive than any hitherto obtained, and that they are conclusive for electron emission from tungsten in a high vacuum. (2) *Electrons from oxide coated platinum.* Subsequent measurements by Dr. C. Davisson have shown that the electrons emitted from Wehnelt cathodes also have velocity components distributed according to Maxwell's law.

THE pioneer experiments of O. W. Richardson and F. C. Brown[1,2] upon the initial velocity distribution of thermionic electrons have been followed by the work of Schottky[3] and the recent experiments of Ting,[4] Jones,[5] Potter,[6] Rössiger[7] and Congdon.[8] The experiments tend to establish the conclusion that thermionic electrons are emitted into a high vacuum with velocity components distributed according to Maxwell's law.

[1] Richardson and Brown, Phil. Mag. **16,** 353 (1908)
[2] Richardson, Phil. Mag. **16,** 890 (1908); **18,** 681 (1909)
[3] Schottky, Ann. der Phys. **44,** 1011 (1914)
[4] Ting, Roy. Soc. Proc. **98,** 374 (1920-21)
[5] Jones, Roy. Soc. Proc. **102,** 734 (1923)
[6] Potter, Phil. Mag. **46,** 768 (1923)
[7] Rössiger, Zeits. f. Phys. **19,** 167 (1923)
[8] Congdon, Phil. Mag. **47,** 458 (1924)

With the aid of experimental refinements the present investigation[9] of this subject has greatly extended the range covered by previous measurements. The results are entirely in agreement with the view that the initial velocities of thermionic electrons are distributed according to Maxwell's law for the velocity components of a gas of molecular weight equal to that of the electrons and having the temperature of the emitting filament. It is believed that these results possess sufficient reliability to be entirely conclusive for the case of electron emission from tungsten in a high vacuum.

The experiment consists essentially in measuring the electron current flowing from a tungsten filament to a coaxial cylindrical electrode against various retarding voltages. This electron current is determined as a function of the retarding potential and the filament temperature.

For an infinitely long cylindrical arrangement of this kind Schottky has shown that, *if the emitted electrons leave the filament at the temperature* T *with velocities distributed according to Maxwell's law*, the current reaching the surrounding cylinder at a potential V negative to the filament is given by the expression,

$$i = i_0 \frac{2}{\sqrt{\pi}} \left[\sqrt{\frac{Ve}{kT}} \epsilon^{-Ve/kT} + \int_{\sqrt{Ve/kT}}^{\infty} \epsilon^{-x^2} \, dx \right], \tag{1}$$

in which i_0 represents the saturation emission, e is the numerical value of the electronic charge, k is Boltzmann's gas constant and the sign of V is chosen positive for the filament positive to the cylinder. The calculation assumes the diameter of the filament small in comparison with the diameter of the cylinder. It also assumes that the current is not limited by the charge of the electrons in the space between filament and cylinder.

TABLE I

Ve/kT	$\log_{10}(i_0/i)$	Ve/kT	$\log_{10}(i_0/i)$
1	.2423	10	3.7698
2	.5827	11	4.185–
3	.9523	12	4.6024
4	1.3371	14	5.4398
5	1.7312	16	6.2812
6	2.1318	18	7.1245
7	2.5369	20	7.9714
8	2.9455	25	10.0978
9	3.3567		

The first term on the right hand side of Eq. (1) is predominant except for small values of Ve/kT. The relation between log (i_0/i) and Ve/kT should therefore be approximately linear, and it is convenient to test

[9] A brief report of the results was published in Science **42,** 392 (March 30, 1923).

Eq. (1) by plotting these variables. As corresponding values of log (i_0/i) and Ve/kT obtained from Eq. (1) are of some general interest and are not readily written down from inspection of the equation, a table of such values is given above. The values of the definite integral were obtained from Czuber's Wahrscheinlichkeitsrechnung Vol. 1 and Pearson's Tables for Statisticians and Biometricians.

Most previous experimenters have tested their results by plotting the above quantities and determining the average slope. Ting, Jones and Potter have stated in this connection that the slope should approach the value .405 as Ve/kT becomes large. The actual limiting value, as can be seen from Eq. (1), is $\log_{10} e$ or .4343. The theoretical value in the range in which these observers worked is, however, about .40. In the present experiment observations were taken over a much greater range and at the largest values of Ve/kT the slope should be greater than .42.

The experimental work to be described below consisted of measurements upon pure tungsten in a high vacuum. These measurements yielded values of current, retarding voltage and temperature which are in excellent agreement with the calculated quantities of Table I. The assumed Maxwell distribution of initial velocities is thus verified for pure tungsten in a good vacuum. Since the completion of this work similar measurements have been made by Dr. C. Davisson of this laboratory upon a number of platinum filaments coated with the oxides of barium and strontium. In every case the measurements gave results in agreement with the calculated values of Table I, showing that the same distribution law holds for the velocities of the electrons from these coated filaments.

Part I. Experimental Arrangements

A. The Experimental Tube

The method employed was similar to that used by previous experimenters in that the essential measurements consisted in taking readings of the current to a cylinder from a co-axial filament while the cylinder was maintained at a negative potential with respect to the filament. It was the object of the experiment, however, to obtain greater accuracy than had been previously obtained and to extend greatly the range over which observations were taken. For these reasons it was necessary to consider various disturbing factors which had not been of importance in the work of previous experimenters.

With the simple arrangement of an emitting filament surrounded by a collecting cylinder the measured current includes, in addition to the electrons reaching the cylinder against the retarding potential V, photo-electrons emitted from the cylinder under the influence of light from the filament, reflected electrons leaving the cylinder, and a possible emission of positive ions from the filament. Preliminary tests showed the existence of at least the first of these disturbing currents and its great importance at large values of V.

All of these extra currents were eliminated by the interposition of a fine grid between the filament and the collecting cylinder, the retarding potential V being maintained between filament and grid and the cylinder being always positive to the grid by about 10 volts. With this arrangement all electrons which are emitted from the filament with velocities sufficient to carry them past the retarding potential of the grid can reach the collector. The photo-electrons produced at the cylinder and the reflected electrons cannot escape on account of the potential between grid and cylinder. The collector is sufficiently positive to the filament so that no positive ions emitted from the latter can reach the former. There is still the possibility of photo-emission from the grid reaching the cylinder. Currents arising from this cause were observed and were balanced out by a method described in a later section.

A cross-section of the experimental tube is shown in Fig. 1. The filament is of pure tungsten 6.960×10^{-3} cm in diameter and 4.75 cm long. It is supported horizontally from lavite blocks and held taut by a molybdenum spring. It lies along the axis of a nickel cylinder A of diameter 0.95 cm and length 6.2 cm. Except for two small bracing pieces the central portion of this cylinder, for a distance of 1.3 cm, is cut away and replaced by a number of nickel wires running parallel to the filament and welded to the halves of the cylinder. These wires are .013 cm in diameter and are placed as carefully as possible .023 cm between centers, forming a very fine grid separating the filament from the outside collecting cylinder. This outside cylinder B, 1.8 cm in diameter and 2.6 cm long, is supported only by a stiff lead wire sealed through a special press at O. For extreme insulation this press is separated from other leads by a grounded copper disk sealed completely through the glass of the tube at P. The distance between O and P is about 4.5 cm. A nickel shield, connected to the grid cylinder A, completely surrounds the cylinder B. This shield covers the stiff support wire and extends nearly down to the seal at O. The leads to the filament and to the grid cylinder are taken out through the top of the tube, and are removed as far as possible from the lead going to the press at O.

This experimental tube was sealed to a small tube containing cocoanut charcoal. Both were thoroughly exhausted and proper heat treatment given to the metal parts, the glass bulb and the charcoal. After being sealed from the pumps the charcoal tube was kept immersed in liquid air. Ionization measurements showed that the gas pressure was not of a higher order than 10^{-8} mm of mercury during the time the tube was in use.

B. Heating Current Supply

The rectifying arrangement. One of the most important experimental requirements was an arrangement for maintaining the emitting filament at uniform potential during the time currents were measured. This

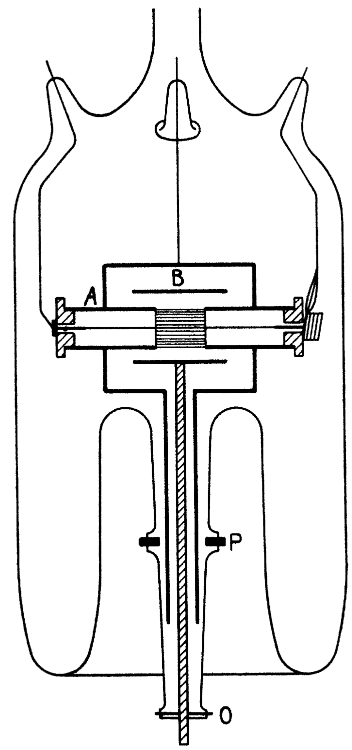

Fig. 1. The experimental tube.

was accomplished by using an intermittent heating current and allowing emission from the filament to reach the cylinder only during the intervals between current pulses. The intermittent current was obtained by the rectification of 500 cycle current by means of a tungar bulb (T_1 in Fig. 2). Emission was prevented during the time heating current flowed by applying the measured retarding potential V between the grid cylinder and the point a (Fig. 2) which is the negative end of a non-inductive resistance of 73 ohms. During a heating current pulse the retarding potential between filament and grid was increased by the IR voltage across this resistance. This reached a peak value of about 120 volts and cut off the emission current for all but a negligible fraction of the duration of the pulse. Between current pulses there was no voltage across the resistance and the potential difference between filament and grid cylinder had exactly the desired value V.

The requirement that the potential between filament and grid cylinder should have exactly the value V for all of the time during which any electrons could reach the grid cylinder placed the two following rather

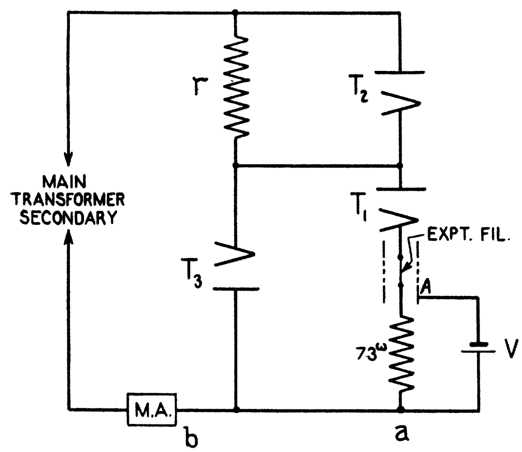

Fig. 2. The current rectifying mechanism.

stringent limitations upon the wave form of the pulsating heating current. During the interval between current pulses no current must flow through the 73 ohm resistance; and the heating current pulses must begin and end very sharply. The wave form was made to meet these requirements by supplying a voltage of 500 cycle frequency from which all harmonics had been removed, and by the use of the rectification system shown in Fig. 2. This employs three tungar tubes, T_1, T_2 and T_3. T_1, as previously described, prevents current from flowing in the filament during one-half of the cycle. To accomplish this entirely it is necessary to prevent the reverse voltage across T_1 from rising to the point where a slight current passes. It is one of the functions of T_3 to prevent this reverse voltage from rising above about 15 volts. The non-inductive resistance r is necessary in order that the supply voltage shall not be short-circuited by T_3 during one-half of each cycle.

To satisfy the condition that the heating current pulse shall begin and end sharply it is necessary that a symmetrical current shall flow through all parts of the circuit which offer an inductive reactance (e.g., the transformer whose secondary is indicated in Fig. 2). This symmetry necessitates the use of T_2 to by-pass the non-inductive resistance r during one-half of the cycle, and T_3 is again necessary to by-pass the 73 ohm resistance during the other half cycle. In making an experiment the resistance r was adjusted until the output current from the main transformer was approximately symmetrical. This condition was recognized by the zero reading of a sensitive direct current measuring instrument located in the circuit at the point b.

A thorough quantitative consideration of the heating current wave form, based upon the characteristics of the tungar tubes, the insulation of the circuit of Fig. 2 from ground (which was better than 10^9 ohms) and oscillograph figures, showed that the severe requirements had been fully satisfied.

The current regulator. The 500 cycle voltage supplied from the generator fluctuated constantly by one or two percent. Since a variation of this amount in the heating current of the filament caused an objectionably large change in the thermionic emission, it was necessary to employ some form of current regulating device. For this purpose there was developed a fairly satisfactory current regulator, of which a large vacuum tube was the essential element. The fluctuations of the supply voltage were applied to the grid of this tube and the resulting variations of the plate current produced the regulating effect. This apparatus operated upon one-half of the a.c. wave and maintained the effective value of the filament heating current very nearly constant.

The constancy of the heating current was observed by means of the emission from a small filament heated by this current. This so-called "tester tube" filament in shunt with a variable non-inductive resistance was located in series with the experimental filament. The sensitivity of the milliammeter which measured the emission from this filament ($MA2$ in Fig. 3) was such that a change of 1/100 of one percent in the effective value of the intermittent current was easily detected. The regulating arrangement was sufficiently good to maintain the effective heating current constant to about 1/30 percent.

C. The Essential Circuit

A diagram of the essential circuit is given in Fig. 3. In this figure the letters A, B, P and O refer to the same parts of the experimental tube as in Fig. 1. The switch S_1 is used to connect the filament of the tube to the intermittent current supply or to the storage battery V_5. The value of the storage battery current can be accurately measured on a Leeds and Northrup potentiometer used with a standard ohm. Switch S_2 is used to apply the voltage V_3 (90 volts) between the filament and the grid cylinder A when it is desired to measure the saturation emission. The retarding potential V is supplied by the battery V_1 and is measured by the accurate voltmeter VM. When desired, the value of the potential V can be increased suddenly by a large amount by opening the switch K_2. V_2 applies the constant potential of 10 volts between grid cylinder A and collecting cylinder B.

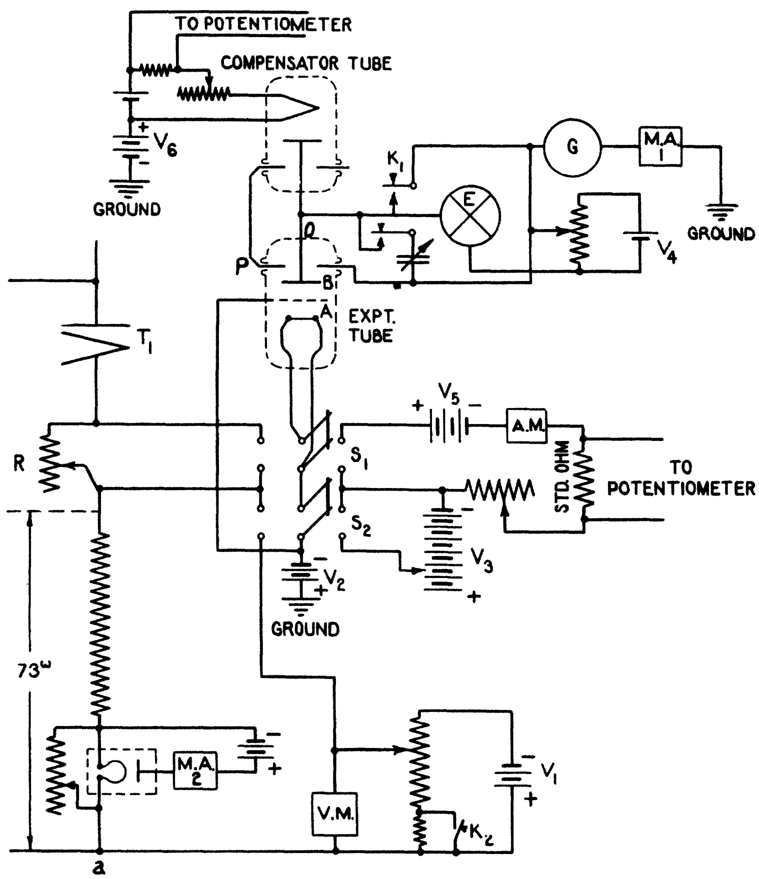

Fig. 3. The essential measuring circuit.

The current measuring system attached to the collecting cylinder consists of the Compton electrometer E, the Leeds and Northrup galvanometer G, and the calibrated milliammeter $MA\,1$. The electrometer was equipped with a series of shunt capacities making it suitable for measuring currents from 10^{-15} to 10^{-8} ampere and the galvanometer with a series of shunt resistances making it suitable for currents from 10^{-8} to 10^{-4} ampere. The experimental tube and the electrometer together with its first two shunt capacities were enclosed in a case lined with tin and sealed as tightly as possible by means of a heavy lid clamped over strips of sponge rubber. The tube of charcoal, which was attached to the experimental tube, projected through a hole in the case, and the cracks were sealed with paraffin. Dishes of drying material were always kept in the case to prevent electrical leaks arising from moisture. A telegraph sounder within the case was used to insulate or ground the active pair of electrometer quadrants. A small glass window in the lid permitted movements of the electrometer mirror to be observed. An oil seal in the top of the case allowed the entrance of a movable rod by means of which the first two shunt capacities could be brought into use. The electrometer case with the electrometer and its various shunt capacities constituted a measuring device of wide range. For the use of this apparatus the writer is indebted to Dr. H. A. Pidgeon of this laboratory.

In Fig. 3 is shown a tube marked "compensator tube" which was mounted in the case near the experimental tube. A small disk within this tube was connected to the collecting cylinder of the experimental tube, and this disk was insulated with the same extreme care as was used for the collecting cylinder. The light from the tungsten filament contained in this tube was used to liberate photo-electrons from the disk. The direction of this photo-electric current was opposite to the direction of flow of current to the collecting cylinder B, and this photoelectric current could be used to neutralize the current due to photoemission from the grid cylinder A. This current amounted to about 1×10^{-12} ampere at the highest temperature of the experimental filament.

PART II. EXPERIMENTAL RESULTS

The temperature of the filament was determined from the diameter and the heating current using the temperature characteristics of tungsten given by Worthing and Forsythe.[10] The effective value of the interrupted heating current could be accurately found by measuring the d.c. heating current which gave the same saturation emission. (Switches S_1 and S_2 of Fig. 3 are used in this connection.) The essential part of the experiment consisted in obtaining data giving the current i to the collecting cylinder as a function of the voltage between filament and grid cylinder at a series of different heating currents. The value of the heating current was determined at the beginning and at the end of each of these sets of measurements.

[10] Worthing and Forsythe, Phys. Rev. **18,** 144 (1921)

The points given by one such set of measurements are plotted in Fig. 4. (The former convention regarding the sign of V is adhered to.) This is for a filament heating current of 0.493 ampere corresponding to a temperature of 1830°K. Before attempting to fit the observed points by the calculated values of Table 1 it is necessary to estimate the value of the contact difference of potential between the hot tungsten filament and the nickel grid cylinder and also the value of i_0. It appears that the observed points give a curve breaking at about $V = -52$ volt, $\log i = 7.8 - 15$. Then taking this value as $\log i_0$ and the contact potential of the filament relative to the cylinder as $+.52$ volt the theoretical curve as calculated from the figures of Table I is given by the solid line.

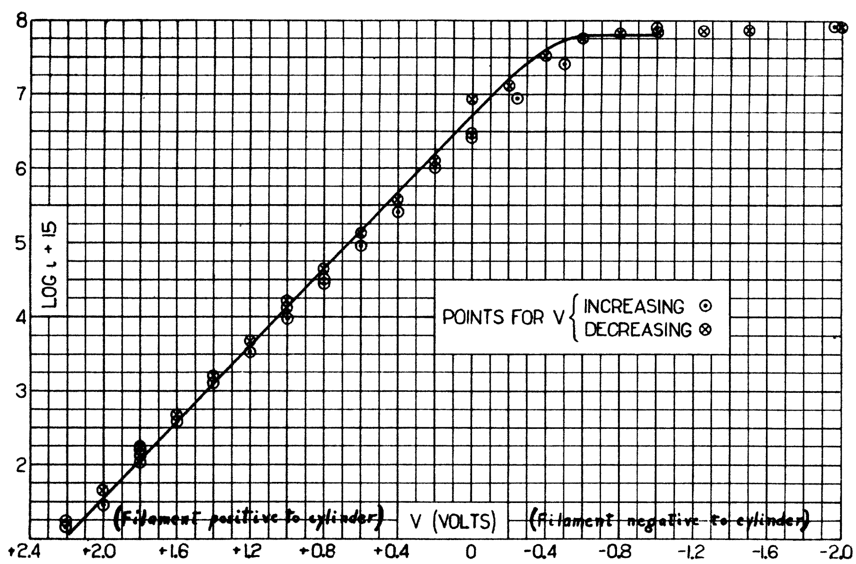

Fig. 4. Current-voltage data at 1830°K.

In taking the points of Fig. 4 the curve was traced twice, once for increasing values of retarding potential and immediately afterwards for decreasing values of retarding potential, ending at -10 volts. The two sets of points do not coincide very well, and it seems that more reliance should be placed upon the latter. The behavior is as if the contact potential between filament and grid cylinder had been changed slightly by the initial flow of saturation emission from the filament, and that this change disappeared only slowly as retarding potential was applied. At each temperature data were taken in this way. The same discrepancy shown in Fig. 4 was generally found, and little reliance was ever placed upon the points corresponding to increasing values of retarding voltage.

At temperatures below 1830°K the curves of log i against V had quite sharp break points allowing the values of contact potential and log i_0 to be found and the data to be tested directly against the calculated values of Table I. As the temperature was raised above 1830°K the break point of the log i-V curve disappeared, because of the increased importance of space charge. It then became impossible to locate the values of the contact potential and of log i_0 by direct observation.

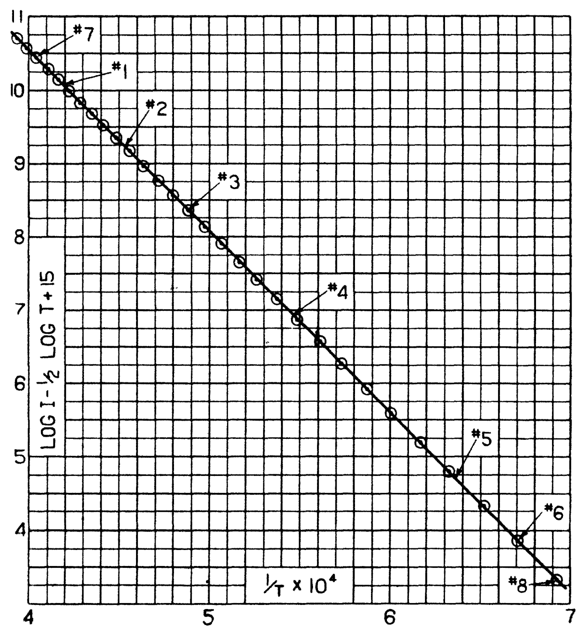

Fig. 5. Emission-temperature data. (The arrows indicate points at which current-voltage data were taken.)

The procedure in these cases consisted in calculating the values of log i_0 from Richardson's equation,

$$\log_{10} i_0 = C + \tfrac{1}{2}\log_{10} T - (b/T)\log_{10} e \ , \tag{2}$$

and evaluating the theoretical curves therefrom. The curves thus calculated should have the same form as the experimentally determined curves but will be displaced from them by an amount corresponding to the contact difference of potential. The amount of this displacement can easily be determined by inspection. When this correction is made the theoretical curves coincide very accurately with the experimental points (as shown in Fig. 6) except, of course, in the region where the space charge phenomena are predominant.

The constant b of Eq. (2) was obtained in the ordinary way. The experimental filament was heated by a storage battery, V_5 of Fig. 3, and saturation emission measured as a function of temperature. Fig. 5 gives the resulting plot of $\log I-\frac{1}{2}\log T$ and $1/T$. In this plot I is the saturation current to the collecting cylinder, which is about one-tenth of the grid cylinder current.

From Fig. 5 the constant b was found to be 56,800°K, corresponding to $\varphi=4.90$ volts.* Although this value of b is slightly less than the value which corresponds to emission in zero field (i.e. i_0), the difference is not

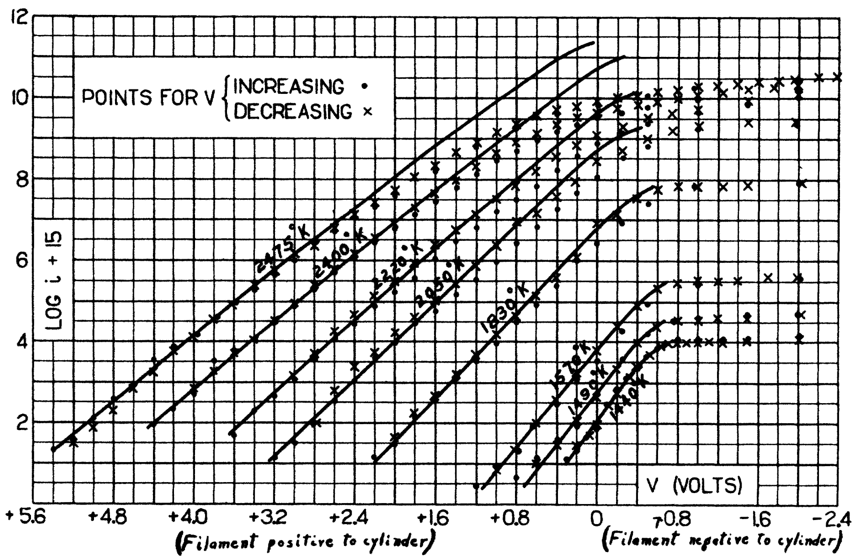

Fig. 6. Current-voltage data at different filament temperatures.

important for the present purpose. The constant C of Eq. (2) was obtained by using this value of b together with the known value of $\log i_0$ at 1830°K. The constant C was thus found to be 4.65 and Eq. (2) becomes,

$$\log_{10} i_0 = 4.65+\tfrac{1}{2}\log_{10} T - 24{,}670/T\ . \tag{3}$$

In Fig. 6 are shown the plots of $\log i$ against V for eight different temperatures. The solid curves in the figure are obtained from the calculated values of Table 1 and are fitted to the experimental points by the methods which have just been described. At the four lowest temperatures the experimental data agree entirely with the values cal-

* This measurement was, of course, not designed as an accurate determination of the work function of the present filament.

culated from Eq. (1) within the limits of error of the measurements. At the higher temperatures the data deviate from the theoretical relationship only for values of V near zero and in the way to be anticipated as the effect of space charge. A consideration of the experimental points of Fig. 6 together with the theoretical values of Table I and the method by which the fit must be accomplished shows that no one of the eight sets of experimental data can be so satisfactorily represented by Eq. (1) if the value of the temperature T is taken 10 percent in error. Some of the sets of data cannot be so well represented by Eq. (1) even for a temperature only 5 percent in error.

A summary of the data is given in Table II to show clearly the range of the measurements. The numerals in the first column indicate the order in which the sets of observations were made.

TABLE II

Summary of data

Order	Heating current	T	$\log i_0+15$ (from Eq. 3)	Contact pot. diff. (from Fig. 6)
8	.309 amp	1440°K	4.13	+.68 volt
6	.330	1490	4.68	+.68
5	.365	1570	5.50	+.67
4	.493	1830	7.80	+.52
3	.610	2050	9.28	+.42
2	.705	2220	10.16	+.38
1	.806	2400	11.06	+.27
7	.852	2475	11.38	−.03

CONCLUSION

The foregoing experiment shows conclusively that the thermionic electrons emitted from tungsten into a high vacuum have velocity components distributed according to Maxwell's law for the distribution of velocities in an electron atmosphere in temperature equilibrium with the hot filament.

The author is very glad to thank Dr. W. Wilson of this laboratory for his interest in this experiment, and especially Dr. C. Davisson for initiating the work and for his constant interest and assistance.

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