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QUANTUM THEORY OF THE INTENSITY OF THE MODIFIED BAND IN THE COMPTON EFFECT

By G. E. M. Jauncey

Abstract

The theory of a previous paper (Phys. Rev. **25**, 314, 1925), which dealt with the scattering of x-ray quanta by electrons in circular Bohr orbits, is extended to the scattering by L and M electrons and by electrons in elliptic orbits. For a certain range of positions of the electron in each orbit the mass of the whole atom is added to that of the electron and the quantum is scattered without change of wave-length. For a given type of K, L or M orbit, the ratio of the number of modified scattered quanta to that of the unmodified quanta is calculated. From this the intensity of the modified band relative to that of the unmodified line when all the K, L, and M electrons are operative in the scattering process is determined. Curves are given showing the *intensity of the modified bands when* Mo Kα *x-rays are scattered by carbon and sulfur* at 30° and 90°. The theory gives for the ratio of intensity of modified band relative to unmodified .90 and 1.74 for carbon at 30° and 90°, and .42 and .62 for sulfur at 30° and 90°, the ratio increasing with the scattering angle and decreasing with increasing atomic number. This result tends to explain the difficulty which has been experienced in obtaining the modified band when Mo Kα x-rays are scattered by sulfur.

1. Introduction

THE simple quantum theory of the scattering of x-rays as developed by Compton[1] has been extended by the writer in a previous paper[2] so as to take account of the motions of the electrons in their Bohr orbits. When these motions are taken into account it was shown that the existence of the unmodified line may be explained since there are for a given angle of scattering certain positions of the electron in its Bohr orbit from which the impinging primary quantum is unable to eject the electron. For these positions the mass of the whole atom is added to that of the electron and the scattering takes place with negligible change of wave-length. In Fig. 1, the momentum of the primary quantum h/λ_0 is repre-

[1] A. H. Compton, Phys. Rev. **21**, 483 (1923)

[2] G. E. M. Jauncey, Phys. Rev., **25**, 314 (1925); see also, A. H. Compton, Phys. Rev. **24**, 168 (1924)

sented by OA, the momentum of the scattered quantum h/λ_ϕ by OB, that of the electron in its Bohr orbit before scattering by AC and that of the recoil electron immediately after it has been ejected from the Bohr orbit by the scattering process is represented by BC.[3] Assuming a circular Bohr orbit the point C may be at any point on the sphere shown. As C takes different positions on the sphere the length of OB varies. Excepting for Assumption III of the previous paper, OB would have a maximum length when C is at H and a minimum length when C is at G. Assumption III however makes the restriction that the difference between the energies of the primary and scattered quantum must be at least equal to the

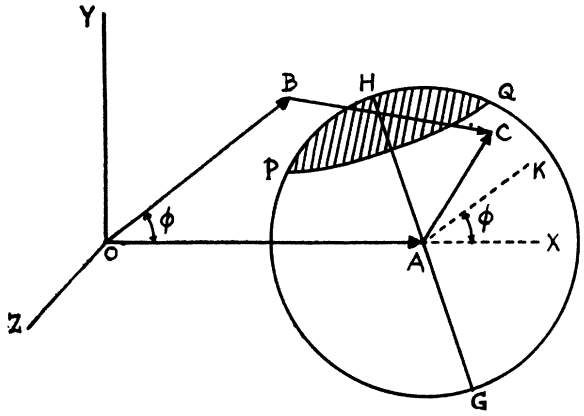

Fig. 1

binding energy of the electron in its orbit. This occurs when C is on the circle PQ. For positions of C within the shaded area the electron cannot be ejected and for these positions of C there is no change of wave-length on scattering. Due to the variation of OB for varying positions of C in the unshaded area, there is a wave-length width of the modified line for a given angle of scattering ϕ. Further, since the ratio of the probability of C being in the shaded area to that for C being in the unshaded area is equal to the ratio of these areas, the number of x-ray corpuscles scattered at a given angle ϕ without change of wave-length to the number scattered with change of wave-length is as the shaded area to the unshaded area on the sphere. This gives a correlation between the existence of the unmodified line, the width of the modified line and the respective energies of the unmodified and modified lines. In the present paper the theory of the previous paper is extended to the case of scattering by the L and M electrons as well as by the K electrons and to the case of electrons in elliptic orbits as well as circular orbits.

[3] For further details, see the previous paper[2].

It is perhaps interesting to note that in the case of circular Bohr orbits where the binding energy of the electrons is much smaller than the energy of the primary quantum the line GAH (Fig. 1) bisects the angle OAK, the line AK as well as the line OB being in the plane XOY.

2. Theory

The equation of an ellipse in polar coordinates referred to one of its foci as origin is

$$\frac{1}{r} = \frac{1}{a} \times \frac{1+\epsilon \cos A}{1-\epsilon^2} , \tag{1}$$

where r is the radius vector from the origin to a point on the ellipse, A is the azimuth of the line joining the origin to the point relative to the direction of the major axis, a is the semi-major axis and ϵ is the eccentricity. We shall consider only the case where an electron is moving in an orbit about a massive nucleus so that the nucleus is at the focus at which the origin is taken. Also we shall consider only the simple case where there is no relativity change of the mass of the electron due to its velocity as it moves in its elliptic orbit so that there will be no advance of the perihelion of the elliptic orbit. Sommerfeld[4] gives for an elliptic Bohr orbit

$$1 - \epsilon^2 = n_a^2/n^2 , \tag{2}$$

$$a = h^2n^2/4\pi^2meE , \tag{3}$$

and

$$-W = hc/\lambda_s = 2\pi^2me^2E^2/h^2n^2 , \tag{4}$$

where n_a is the azimuthal quantum number, n the total quantum number, e the electronic charge, E the effective nuclear charge, $-W$ the binding energy of the electron, and λ_s a critical absorption wave-length of the scattering substance. Introducing the quantity $\alpha_s \equiv h/mc\lambda_s$ of the previous paper Eqs. (3) and (4) above give

$$a = hn/2\pi mc\sqrt{2\alpha_s} . \tag{5}$$

We have from the Bohr theory that the angular momentum $mr^2(dA/dt)$ is equal to $n_a h/2\pi$ and hence from Eqs. (1) and (3)

$$\frac{dA}{dt} = \frac{4\pi mc^2n^2\alpha_s(1+\epsilon \cos A)^2}{hn_a^3} . \tag{6}$$

The velocity of the electron in its orbit is given by

$$(\text{velocity})^2 = r^2(dA/dt)^2 + (dr/dt)^2 ; \tag{7}$$

therefore by differentiating Eq. (1) and from Eqs. (2) and (6)

$$p/mc \equiv b = (n/n_a) \times \sqrt{1+2\epsilon \cos A+\epsilon^2} \times \sqrt{2\alpha_s} , \tag{8}$$

[4] A. Sommerfeld, Atomic Structure, English ed., p. 235.

where p is the momentum of the electron in its orbit when at the azimuthal angle A. It is seen that the electron has its greatest momentum when at perihelion ($A=0$) and its least momentum when at aphelion ($A=\pi$). For a circular orbit when $n_a=n$ and $\epsilon=0$ we have

$$p/mc \equiv b = \sqrt{2\alpha_s} . \tag{9}$$

In the previous paper the value of p/mc for a circular orbit is given as $\sqrt{2\alpha_s-\alpha_s^2}/(1-\alpha_s)$. This value however was obtained when the relativity change of mass at high velocities was taken into account and approximates to that in Eq. (9) when α_s is small. As α_s is small when L and M electrons do the scattering we may take Eq. (8) as being sufficiently accurate for our immediate purposes.

For a given value of A, b is a constant. Furthermore since the axes of the elliptic orbits for a large number of electrons are orientated equally in all directions, the problem for $A=$constant becomes similar to the problem for circular orbits as discussed in the previous paper. In that paper it is shown that for small values of α_s the extreme values of $l \equiv \lambda_\phi/\lambda_0$ are given by

$$l = 1+\alpha_0 \operatorname{vers}\phi \pm 2\sin\tfrac{1}{2}\phi \times \sqrt{2\alpha_s} , \tag{10}$$

where $\alpha_0 = h/mc\lambda_0$, provided that the restriction of Assumption III of the previous paper is for the moment considered as inoperative. However since in Eq. (10) $\sqrt{2\alpha_s}$ represents p/mc for a circular orbit, we may use the value of p/mc as given by Eq. (8) for a given or constant value of A. The value of b for a constant value of A we shall represent by b_A. The extreme values of l due to scattering by a large number of electrons in elliptic orbits whose axes are orientated equally in all directions for a constant value of A are therefore given by

$$l = 1+\alpha_0 \operatorname{vers}\phi \pm 2b_A \times \sin\tfrac{1}{2}\phi , \tag{11}$$

hence

$$l_1 - l_2 = 4b_A \sin\tfrac{1}{2}\phi , \tag{12}$$

where l_1 and l_2 are the maximum and minimum values of l as given by Eq. (11).

The ratio of the number of electrons between the azimuths A and $A+\delta A$ to the whole number of electrons which are moving in elliptic orbits of a given eccentricity ϵ is equal to the ratio of the time δt for the electron to move from A to $A+\delta A$ to the time for the electron to move from aphelion to perihelion. That is,

$$\delta N_E/N_E = 2\delta t/T \tag{13}$$

where δN_E is the number of electrons with azimuths between A and $A+\delta A$, N_E is the total number of electrons and T is the period of the electron in its orbit. By integrating Eq. (6) between the limits $A=0$ and $A=2\pi$ we have[5]

$$T=hn/2mc^2\alpha_s, \tag{14}$$

and therefore from Eqs. (6), (13), and (14)

$$dN_E/dA=N_E(n_a/n)^3/\pi(1+\epsilon\cos A)^2\,. \tag{15}$$

In the previous paper it is shown that for circular orbits (that is, for orbits in which $b=$constant) the number of x-ray corpuscles scattered in

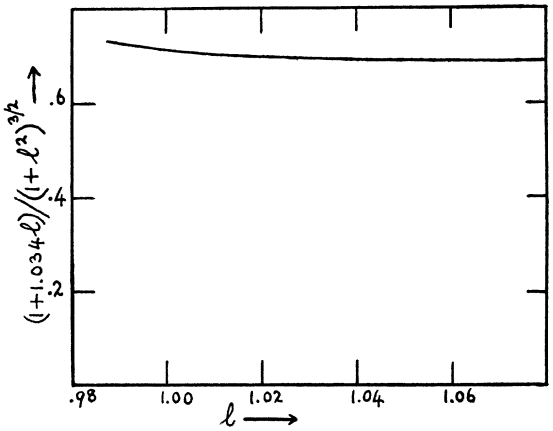

Fig. 2. Number intensity of x-rays scattered by electrons in circular orbits.

a given direction ϕ for which l is between l and $l+\delta l$ is proportional to δP, where according to Eq. (13) of the previous paper

$$P=(l-u)/\sqrt{1+2l\cos\phi+l^2}\,, \tag{16}$$

and

$$u=1+\alpha_0(1-\alpha_s)\operatorname{vers}\phi\,.$$

The number intensity may be defined as the ratio of the number of x-ray corpuscles between l and $l+\delta l$ to δl. Hence we have

$$\text{number intensity}\quad\propto\frac{dP}{dl}=\frac{1-(l+u)\cos\phi+lu}{(1-2l\cos\phi+l^2)^{3/2}}\,. \tag{17}$$

For the scattering of Mo Kα rays by the K electrons of carbon we have (see previous paper) $l_1=1.080$ and $l_2=0.988$. Plotting the curve of the number intensity against l between these two values of l we obtain approximately a straight line parallel to the axis of l as shown in Fig. 2.

[5] B. O. Peirce, Short Table of Integrals, pp. 41, 43.

We may therefore obtain approximately the number intensity by dividing the total number of x-ray corpuscles between l_2 and l_1 by (l_1-l_2).

Now the number of x-ray corpuscles δN_x scattered by δN_E electrons per unit of time is proportional to δN_E, if we assume that the chance of an electron scattering an x-ray corpuscle is independent of the velocity of the electron. We may therefore replace δN_E by δN_x in Eq. (15) if we multiply the right hand member by a proportionality constant k. This number δN_x is spread over a wave-length range represented by (l_1-l_2) as given by Eq. (12). Hence the number intensity of the x-ray corpuscles scattered at an angle ϕ by electrons between the azimuths A and $A+dA$ is dI_N where

$$dI_N = kN_E f'(A)dA/(4\pi\sqrt{2a_s}\times\sin\tfrac{1}{2}\phi)\ , \tag{18}$$

and

$$f'(A)=(n_a^4/n^4)(1+\epsilon\cos A)^2\sqrt{1+2\epsilon\cos A+\epsilon^2}\ . \tag{19}$$

Since according to Eq. (12) the wave-length range (l_1-l_2) decreases with increase of A, the number intensity of the x-ray corpuscles at the edges of the band of wave-lengths produced by electrons in azimuth A is equal to the sum of the number intensities produced by electrons between the azimuths 0 and A when all the electrons of the total azimuthal range 0 to π are operative in the scattering process. Hence the number intensity at the edges of the wave-length band produced by electrons in azimuth A is

$$I_n=\frac{kN_E}{4\pi\sqrt{2a_s}\sin\frac{1}{2}\phi}\times\int_0^A f'(A)dA\ . \tag{20}$$

I_n is a function of the upper limit A and we therefore introduce the function

$$f(A)\equiv\int_0^A f'(A)dA\ . \tag{21}$$

This function is independent of the scattering angle ϕ and also of the binding energy a_s, and depends only upon the total and azimuthal quantum numbers. These are the same for a given type of L or M orbit irrespective of the particular atom in which the orbit occurs. $f(A)$ has been obtained by graphical integration for the orbits shown in the following table.

TABLE I

Values of function $f(A)$

Curve	Orbit	Total quantum number, n	Azimuthal quantum number, n_a	Eccentricity
I	L	2	1	0.866
II	M	3	2	0.745
III	M	3	1	0.943

In order to facilitate comparison with experimental data, the quantity

$$F(A) = \pm (n/n_a)\sqrt{1+2\epsilon \cos A+\epsilon^2} \tag{22}$$

is introduced. Instead of plotting $f(A)$ against A, it is plotted against $F(A)$ and the curves I, II, and III of Fig. 3 obtained. The curves are shown for values of $F(A)$ for the range of -1.4 to $+1.4$. Outside of this range the ordinates are very small. The curves are all drawn to the same scale. The curved parts are obtained from the appropriate values of A, the highest point of the curved portions being obtained when

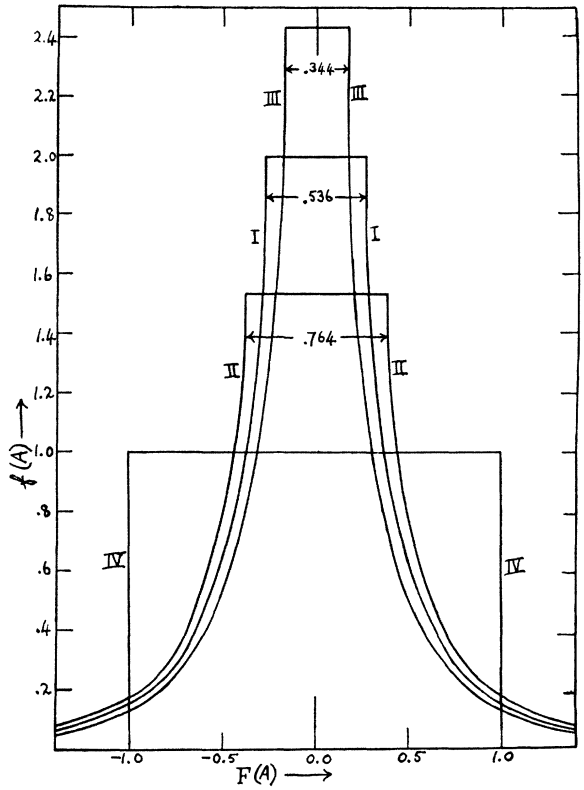

Fig. 3

$A = 180°$. It is seen that these highest points are joined by a horizontal line. This is because within the wave-length range of the x-ray corpuscles scattered by the electrons of azimuth $A = 180°$ the number intensity is constant. Curve IV of Fig. 3 is for scattering by electrons in a circular orbit when $n_a = n$ and $\epsilon = 0$. This curve is drawn to the same scale as curves I, II, and III.

3. Application to Experiment

Scattering from carbon. We shall consider the scattering of MoKα x-rays at 30° and 90°. Taking $\phi = 30°$ we have $\lambda_0 = 0.71$ A, which gives $a_0 = 0.034$. The values of a_s for the electrons in the various orbits in carbon are shown in Table II. The values of λ_s are obtained from curves given by K. T. Compton and F. L. Mohler.[6]

TABLE II

Scattering from carbon

Orbit	n	n_a	λ_s	a_s	Number of electrons per atom
K	1	1	47A	.00051	2
L_{III}	2	2	1200	.000020	2
L_I	2	1	350	.000069	2

From Eq. (11) the value of $l-1$ for the center of the modified line at $\phi = 30°$ is 0.00456, and the value of the wave-length shift of the center of the modified line from the unmodified line is $\lambda_0(l-1)$ or 0.0032 A. The wave-length width for a circular orbit is $4\lambda_0\sqrt{2a_s}\times\sin\frac{1}{2}\phi$ or $0.735\sqrt{2a_s}\,A$. For the K electrons this is 0.0235 A. Hence the modified band extends to .0117 A on either side of the center of the band. The number intensity of the x-ray corpuscles scattered by the K electrons will be taken as unity as shown by curve K of Fig. 4, where the abscissas are now values of the wave-length shift $\lambda_\phi - \lambda_0$ in angstroms. For the L_{III} electrons the wave-length width is 0.0046 A, or a range of 0.0023 A on either side of the center. It has been assumed that the chance of an x-ray corpuscle being scattered by an electron is independent of the velocity of the electron in its orbit and hence the number intensity of the corpuscles scattered by a given number of electrons varies inversely as the wave-length width. The intensity is therefore .0235/.0046 times the intensity for scattering by the K electrons since there are as many K electrons as L_{III} electrons in an atom of carbon. Hence the horizontal portion of the L_{III} curve of Fig. 4 is 5.1 units above the wave-length axis.

In order to obtain the intensity curve for the scattering by the L_I electrons we proceed as follows. The value of $4\lambda_0\sqrt{2a_s}\times\sin\frac{1}{2}\phi$ for these electrons is .0086 A. If the orbits were circular the wave-length band would extend to 0.0043 A on either side of the center of the modified band and the intensity would be .0235/.0086 or 2.7 times the intensity produced by the K electrons, since there are as many K electrons as L_I electrons. The broken curve of Fig. 4 is for the L_I electrons on the assumption that the electrons move in circular orbits. The electrons how-

[6] K. T. Compton and F. L. Mohler, Nat. Res. Council Bull. No. **48**, pp. 106-109

ever move in elliptic orbits ($n=2$, $n_a=1$) and we make use of curve I of Fig. 3 to obtain the true curve for the L_I electrons. Let us imagine curve IV, Fig. 3 to be distorted so as to fit the broken curve of Fig. 4 by multiplying the ordinates and abscissas of curve IV Fig. 3 each by the appropriate factor, then if the same distortion applies to curve I Fig. 3, this distorted curve will be the true curve for the L_I electrons. This curve is shown as curve L I in Fig. 4. The ordinate of the horizontal portion of curve I Fig. 3 is 1.99 times as great as the ordinate of the horizontal portion of curve IV Fig. 3. Hence the horizontal portion of curve L I Fig. 4 is 1.99×2.7 or 5.37 units above the wave-length axis.

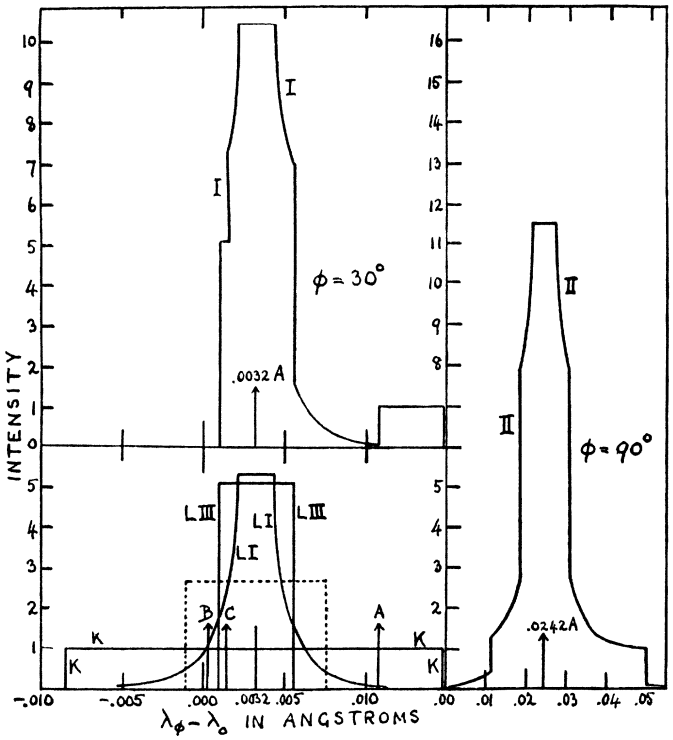

Fig. 4

So far we have made no use of Assumption III of the previous paper. This assumption requires that the least possible change of wave-length on scattering is $\lambda_0^2/(\lambda_s-\lambda_0)$. For the K electrons, this least value is .0108 A and is represented by the vertical line *A* in Fig. 4. This means that the area to the right of line *A* and under curve K, Fig. 4, represents the number of x-ray corpuscles scattered by the K electrons with change of wave-length, while the area to the left represents the number of x-ray

corpuscles scattered without change of wave-length. The ratio of the two areas is .0041/.0193 or 0.21. This means that, of the x-ray corpuscles scattered by the K electrons at 30°, 17.3 per cent are scattered with change of wave-length and 82.7 percent without change of wave-length. For the L_{III} electrons the least possible change of wave-length is 0.0004 A. This is represented by the line *B*. In this case, however, the whole of the area under curve L III, Fig. 4, is to the right of line *B*. Hence all the x-ray corpuscles scattered by L_{III} electrons are scattered with change of wave-length. For the L_I electrons the least possible change of wave-length is .0014 A, which is represented by the line *C*. The ratio of the area under curve L I which is to the right of line *C* to that which is to the left of line *C* is 3.8. Hence, of the x-ray corpuscles scattered by the L_I electrons, 79 percent are scattered with change of wave-length and 21 percent without change of wave-length. Hence when all of the K, L_I and L_{III} electrons are operative in the scattering process we have, since there are equal numbers of each group, that of the scattered x-ray corpuscles 65.4 per cent are scattered with change of wave-length and 34.6 percent without change of wave-length.

The ordinates of the portions of the curves K, L III, and L I Fig. 4 which are to the right of the lines *A*, *B*, and *C* respectively are added together, obtaining curve I Fig. 4. This curve represents the intensity of the x-ray corpuscles scattered with different changes of wave-lengths. The experimental curve of the energy intensity against the change of wave-length will only approximate curve I Fig. 4. The reasons are (1) this curve is a number intensity curve and not an energy intensity curve, and even as a number intensity curve the curve is an approximation, as will be seen by referring to section 2 of this paper. The energy intensity curve may be obtained by dividing each ordinate of curve I Fig. 4 by the appropriate value of $l \equiv \lambda_\phi/\lambda_0$. The greatest value of l is 1.02 and the least value 1.001 so that the energy intensity curve only differs slightly from curve I. (2) An experimental apparatus always has a finite resolving power; hence the sharp angular discontinuities of curve I Fig. 4 cannot be observed experimentally. The experimental curve however should show a width of the modified band and an intensity at the center of the band (neglecting the band due to the K electrons) in approximate agreement with curve I Fig. 4.

We next consider the relative intensities of the modified band and of the unmodified line. In Compton's experiments[7] the primary $K\alpha_1$ and $K\alpha_2$ lines of molybdenum are not separated. Since the distance between

[7] A. H. Compton, Phys. Rev. **22**, 408 (1923)

these two lines is .004 A, the spectrometer arrangement has a resolving power not greater than that corresponding to a wave-length width of .002 A. Hence we may take the primary x-ray corpuscles and also the unmodified scattered x-ray corpuscles as being spread over a wave-length range of .002 A. We have seen that of the x-ray corpuscles scattered by the K electrons 82.7 percent are scattered without change of wave-length. Before applying Assumption III of the previous paper we had 100 percent of the corpuscles scattered by the K electrons and these were distributed over a wave-length range of .0235 A giving an intensity which we have represented as unity in Fig. 4. The intensity produced by 82.7 percent of the corpuscles spread over a range of 0.002 A is therefore $.0235\times.827/.002$ or 9.2 units. For the L_I electrons, 21 percent of the corpuscles are scattered without change of wave-length, so that the intensity of the unmodified corpuscles scattered by the L_I electrons is $.0235\times.21/.002$ or 2.5 units. Hence the total intensity of the unmodified line as scattered by all the electrons is $9.2+2.5=11.7$ units. The intensity of the center of the modified band is seen to be 10.47 from curve I Fig. 4. Hence the ratio of the intensity of the modified to that of the unmodified line is $10.47/11.7=.90$. Due to the resolving power represented by a wave-length width of 0.002 A, the short wave-length side of the modified band will overlap the long wave-length side of the unmodified line so that the modified line will appear to be very indistinct.

For $\phi=90°$, the value of $l-1$ for the center of the modified band is 0.034, and the wave-length shift from the unmodified line is 0.0242 A. The wave-length width of the modified band for a circular orbit is $(2.01\sqrt{2a_s})$ A. Hence if we shift the center of each of the curves K, L III and L I, Fig. 4, to $\lambda_\phi-\lambda_0=0.0242$ A from $\lambda_\phi-\lambda_0=0.0032$ A, and if we multiply the horizontal distance of each point on these curves from the ordinate through $\lambda_\phi-\lambda_0=0.0032$ A, Fig. 4, by 2.01/0.735, we obtain the curves for $\phi=90°$. The positions of the lines *A*, *B* and *C* however are independent of the angle of scattering and therefore remain the same as before. The ordinates of each of the curves to the right of the lines *A*, *B* and *C* are added, thus obtaining the intensity curve for the modified band when the K, L_{III} and L_I electrons are all operative in the scattering process. This curve is shown in curve II Fig. 4. Again assuming a resolving power represented by 0.002 A we can calculate the relative intensities of the modified band and the unmodified line. For $\phi=90°$ all of the L_{III} electrons scatter with change of wave-length while practically all of the L_I electrons do likewise. It is only 25 percent of the K electrons which scatter without change of wave-length. Hence the intensity of the unmodified line is $0.25\times0.0526/0.002$ since the value of $(l_1-l_2)\lambda_0$ for

$\phi = 90°$ is 0.0526 A. Here we take the unit of scattered intensity as being the intensity due to scattering by the K electrons alone at 90°. In our units therefore the intensity of the unmodified line is 6.58. The intensity of the most intense part of the modified band is seen by reference to curve II Fig. 4 to be 11.47. The ratio of the intensities of the modified and unmodified lines is therefore 1.74. In the case of scattering at 30° this ratio was 0.90. The modified band therefore stands out much more prominently for $\phi = 90°$ than for $\phi = 30°$. Furthermore there is no overlapping of the short wave-length edge of the modified band with the long wave-length edge of the unmodified line. Taking into account these two facts the modified band should be much more distinct on a photographic plate taken at $\phi = 90°$ than on a plate taken at $\phi = 30°$. Ross[8] has found this to be so.

Scattering from sulfur. Compton and Mohler[6] give the following values for sulfur

TABLE III

Scattering from sulfur

Orbit	n	n_a	λ_s	α_s	Number of electrons per atom
K	1	1	5.1A	.00474	2
L_{III}	2	2	83.2	.00029	4
L_I	2	1	55.1	.00044	4
M_{III}	3	2	1190.0	.00002	3
M_I	3	1	345.0	.00007	3

The curve for the intensity of the modified band at $\phi = 30°$ when all of the K, L and M electrons are operative in the scattering process is shown in curve I Fig. 5. This is a composite curve similar to curve I Fig. 4. In arriving at this curve use is made of curves II and III Fig. 3 in the calculation of the intensities of the modified bands scattered by the M_{III} and M_I electrons respectively. The values of $\lambda_0^2/(\lambda_s - \lambda_0)$ for the K, L_{III}, L_I, M_{III} and M_I electrons are 0.114, 0.0061, 0.0093, 0.00042, 0.00145 A respectively and the percentage of unmodified corpuscles scattered by each of these types of electrons is 100, 66, 90, 1, 13 respectively. The intensity at $\lambda_\phi - \lambda_0 = h$ vers $\phi/mc = 0.0032$ A is 65 units, the unit being the scattered intensity due to the K electrons if Assumption III of the previous paper is inoperative. The unit is similar to the unit in the curves of Fig. 4. Assuming a resolving power represented by 0.002 A, the intensity of the unmodified line is 154 units and the ratio of the intensity of the modified to that of the unmodified line is .42. Hence for scattering at $\phi = 30°$ the intensity of the modified band for sulfur is

[8] P. A. Ross, Phys. Soc. Meeting, Dec. 29-31, 1924

much less for the same intensity of the unmodified line than for carbon where the ratio is .90. As in the case of carbon there will be overlapping of the modified and unmodified lines due to a finite resolving power and the unmodified line will be very indistinct. In fact, for sulfur at $\phi = 30°$ a photographic plate would very likely give little or no evidence of the existence of a modified line at $\lambda_\phi - \lambda_0 = .0032$ A.

The curve for the intensity of the modified band at $\phi = 90°$ is shown in curve II Fig. 5. The percentage of unmodified corpuscles scattered by each of the K, L_{III}, L_I, M_{III}, and M_I types of electrons is 96, 12, 12, 0 and

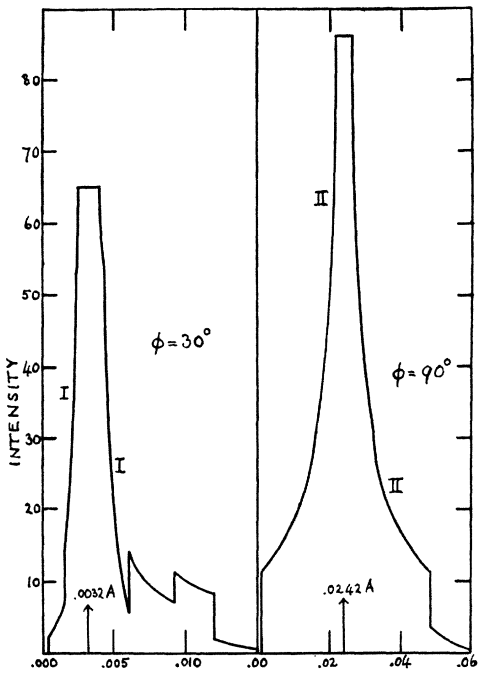

Fig. 5. Intensity of modified scattered rays from sulfur.

0 respectively. The intensity at $\lambda - \lambda_0 = h$ vers $\phi/mc = 0.0242$ A is 86 units, while the intensity of the unmodified line assuming a resolving power represented by .002 A is 140 units. The ratio of the two intensities is .62. For carbon at 90° the ratio is 1.74. Hence at this angle the modified line for carbon stands out much more distinctly on a photographic plate than when sulfur is used. This may explain the difficulty which several experimenters (including the writer) have experienced in obtaining the modified line for sulfur.

4. Discussion

A theory has been developed which takes into account the effect of the momentum of the electrons in their Bohr orbits on the intensity of the modified band in the Compton effect. The theory requires an increase of the intensity of the modified band relative to that of the unmodified band as the angle of scattering is increased. This is in qualitative agreement with experiment. Also the theory requires for a given angle of scattering a decrease of the intensity of the modified band relative to that of the unmodified line as the atomic number of the scatterer increases.* This agrees qualitatively with experiment. The theory predicts a finite width of the modified line and this finite width is shown in the experimental curves of Compton[7] and Compton and Woo.[9] It is for this reason that the writer has used the phrase "modified band" in this paper. Due to this finite width it seems that the modified $K\alpha_1$ and $K\alpha_2$ lines cannot be separated since the two modified bands overlap even when the $K\alpha_1$ and $K\alpha_2$ lines are separated in the unmodified line. However it may be possible in the case of scattering by paraffin to separate the molybdenum $K\alpha_1$ and $K\alpha_2$ lines in the modified band. It may be that the binding energy of some of the electrons in this compound is so small that the width due to this fact is less than the separation of the two spectrum lines.

In conclusion, it should be noted that the numerical values of λ_s given in Tables II and III are only approximate, in fact they have been interpolated in some cases. The values are in some cases calculated from the ionization potentials and there is some doubt as to whether these potentials may not be excitation rather than ionization potentials. The curves given in Figs. 4 and 5 can therefore be considered as only approximate.

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* Note added April 29. In addition the theory requires that the relative intensity of the modified band increase as the wave-length of the primary x-rays decreases.

[9] Compton and Woo, Proc. Nat. Acad. Science **10**, 271 (1924)