MEASUREMENTS OF β-RAYS ASSOCIATED WITH SCATTERED X-RAYS

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Abstract

Stereoscopic photographs of beta-ray tracks excited by strongly filtered x-rays in moist air have been taken by the Wilson cloud expansion method. In accord with earlier observations by Wilson and Bothe, two distinct types of tracks are found, a longer and a shorter type, which we call P and R tracks, respectively. Using x-rays varying in effective wave-length from about 0.7 to 0.13 A, the ratio of the observed number of R to that of P tracks varies with decreasing wave-length from 0.10 to 72, while the ratio of the x-ray energy dissipated by scattering to that absorbed (photo-electrically) varies from 0.27 to 32. This correspondence indicates that about 1 R track is produced for every quantum of scattered x-radiation, assuming one P track is produced by each quantum of absorbed x-radiation. The *ranges* of the observed R tracks increase roughly as the 4th power of the frequency, the maximum length for 0.13 A being 2.4 cm at atmospheric pressure. About half of the tracks, however, had less than 0.2 of the maximum range. As to *angular distribution,* of 40 R tracks produced by very hard x-rays (111 kv), 13 were ejected at between 0 and 30° with the incident beam, 16 at between 30° and 60°, 11 at between 60° and 90° and none at a greater angle than 90°. The R electrons ejected at small angles were on the average of much greater range than those ejected at larger angles. These results agree closely in every detail with the theoretical predictions made by Compton and Hubbard, and the fact that in comparing observed and calculated values, no arbitrary constant is assumed, makes this evidence particularly strong that the assumptions of the theory are correct, and that whenever a quantum of x-radiation is scattered, an R electron is ejected which possesses a momentum which is the vector difference between that of the incident and that of the scattered x-ray quantum.

In recently published papers, C. T. R. Wilson[1] and W. Bothe[2] have shown the existence of a new type of β-ray excited by hard x-rays. The range of these new rays is much shorter than that of those which have been identified with photo-electrons. Moreover, they are found to move in the direction of the primary x-ray beam, whereas the photo-electrons move nearly at right angles to this beam.[3] Wilson, and later Bothe,[4] have both ascribed these new β-rays to electrons which recoil from scattered x-ray quanta in accordance with the predictions of the quantum theory

[1] C. T. R. Wilson, Proc. Roy. Soc. A **104,** 1 (1923)
[2] W. Bothe, Zeits. f. Phys. **16,** 319 (1923)
[3] See, e.g., F. W. Bubb, Phys. Rev. **23,** 137 (1924)
[4] W. Bothe, Zeits. f. Phys. **20,** 237 (1923)

of x-ray scattering.[5] In support of this view, they have shown that the direction of these rays is right, and that their range is of the proper order of magnitude. The present paper describes stereoscopic photographs of these new rays which we have recently made by Wilson's cloud expansion method. In taking the pictures, sufficiently hard x-rays were used to make possible a more quantitative study of the properties of these rays.

The cloud expansion apparatus used in our work was patterned closely after Wilson's well-known instrument except that all parts other than the glass cloud chamber itself were made of brass. The timing was done by a single pendulum, which carried a slit past the primary beam and actuated the various levers through electric contacts. The Coolidge x-ray tube, enclosed in a heavy lead box, was excited by a transformer and kenotron rectifiers capable of supplying 280 peak kilovolts. For illumination we used a mercury spark, similar to that of Wilson, through which discharged a 0.1 microfarad condenser charged by a separate transformer and kenotron to about 40 kv. The photographs were made by an "Ontoscope" stereoscopic camera, equipped with Zeiss Tessar $f/4.5$ lenses of 5.5 cm. focal length. Eastman "Speedway" plates (45×107 mm) were found satisfactory.

A typical series of the photographs[6] obtained are reproduced in Plate I, (a) to (f), which show the progressive change in appearance of the tracks as the potential across the x-ray tube is increased from about 21 to about 111 kv.

Especially in view of the fact that the original photographs are stereoscopic, the negatives of course show much more detail than do the reproductions. These suffice to show, however, the two types of tracks, the growth of the short tracks with potential, and the fact that while the long tracks are most numerous for the soft x-rays, the short tracks are most in evidence when hard rays are used. These results are in complete accord with Wilson's observations.

Number of tracks. It has been shown[7] that if the above interpretation of the origin of the two classes of β rays is correct, the ratio of the number of short tracks (type R) to that of long tracks (type P) should be

$$N_R/N_P=\sigma/\tau \tag{1}$$

where σ is the scattering coefficient, and τ the true absorption coefficient of the x-rays in air; for σ is proportional to the number of scattered

[5] A. H. Compton, Bulletin Nat. Res. Council, No. 20, p. 19 (1922); and P. Debye, Phys. Zeits. (Apr. 15, 1923)

[6] These photographs were shown at the Toronto meeting of the British Association in August 1924.

[7] A. H. Compton and J. C. Hubbard, Phys. Rev. **23**, 448 (1924)

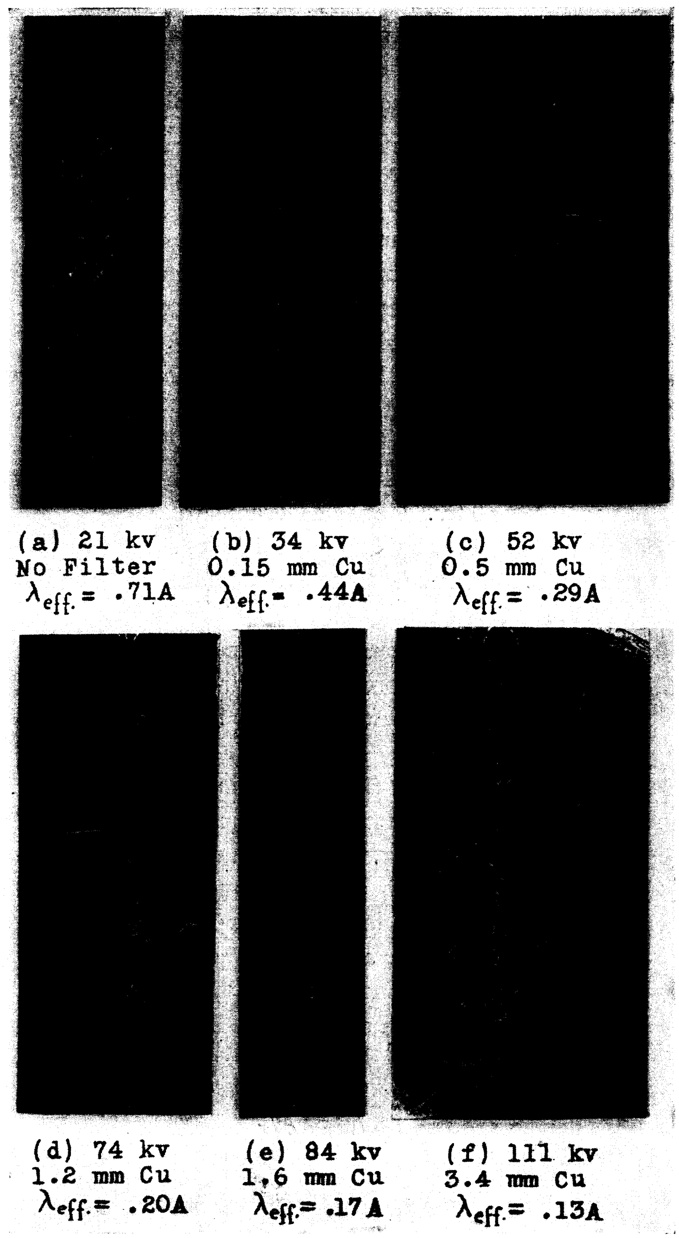

Plate I. The x-rays pass from top to bottom. In addition to the copper filter, they traverse glass walls 4 mm thick. For the short waves the shorter (R) tracks increase rapidly in length and number. Thus while in (a) nearly all are P tracks, in (f) nearly all are R tracks.

quanta, and τ to the number of quanta spent in exciting photo-electrons, per centimeter path of the x-rays through the air.

In Table I we have recorded the results of the examination of the best 14 of a series of 30 plates taken at different potentials. The potentials given in column 1 of this table are based on measurements with a sphere gap. The potential measurements required corrections due to a slight warping of the frame holding the spheres, and to the lowering of the line voltage when the condenser was charged for the illuminating spark. The latter error was eliminated in the later photographs, at 34, 21, and 74 kv, and the former error was corrected by a subsequent measurement of the sphere gap distances, checked by a measurement of the lengths of the P tracks obtained at the lowest potential. The probable errors of potential measurements are thus unfortunately large, amounting to perhaps 10 percent in every case except that of 74 kv, which is probably accurate to within 5 per cent.

TABLE I

Number of tracks of types R and P.

Potential	Effective wave-length	Total tracks N	R tracks N_R	P tracks N_P	N_R/N_P	σ/τ
21kv	.71A	58	5	49	0.10	0.27
34	.44	24	10	11	0.9	1.2
52	.29	46	33	12	2.7	3.8
74	.20	84	74	8	9	10
84	.17	73	68	4	17	17
111	.13	79	72	1	72	32

The effective wave-lengths as given in column 2 are the centers of gravity of the spectral energy distribution curves after taking into account the effect of the filters employed. Because of the strong filtering, the band of wave-lengths present in each case is narrow, and the effective wave-length is known nearly as closely as the applied potential.

All the tracks originating in the path of the primary beam are recorded in column 3. Of these, the nature of some was uncertain. At the lower voltages it was difficult to distinguish the R tracks from the "sphere" tracks which Wilson has shown are often produced near the origin of a β-ray track by the fluorescent K rays from the oxygen or nitrogen atoms from which the ray is ejected. At the highest voltage the length of some of the R tracks is so great as to make it difficult to distinguish them from the P tracks. The numbers of R and P tracks shown in columns 4 and 5 are those of the tracks whose nature could be recognized with considerable certainty, the uncertain ones not being counted. This procedure probably makes the values of N_R/N_P in column 6 somewhat too small for the lower potentials and somewhat too great for the higher potentials.

The values of σ and τ given in column 7 are calculated from Hewlett's measurements[8] of the absorption of x-rays in oxygen and nitrogen. We have taken from his data the value of τ for 1 A to be 1.93 for air, and to vary as λ^3. The difference between the observed value of $\mu=\tau+\sigma$ and this value of τ gives the value of σ which we used.

The surprisingly close agreement between the observed values of N_P/N_R and the values of σ/τ we believe establishes the fact that the *R tracks are associated with the scattering of x-rays.* In view of the evidence that each truly absorbed quantum liberates a photo-electron or P track,[9] the equality of these ratios indicates that *for each quantum of scattered x-rays about one R track is produced.*

The fact that for the greater wave-lengths the ratio N_R/N_P seems to be smaller than σ/τ may mean that not all of the scattered quanta have R tracks associated with them. This would be in accord with the interpretation which has been given of the spectrum of scattered x-rays. The modified line has been explained by assuming the existence of a recoil electron, and the unmodified line as occurring when the scattering of a quantum results in no recoil electron. On this view the fact that the unmodified line is relatively stronger for the greater wave-lengths goes hand in hand with the observation that N_R/N_P is less than σ/τ for the greater wave-lengths. In view, however, of the meager data as yet available on this point, we do not wish to emphasize this correspondence too strongly.

Ranges of the R tracks. The range of the recoil electrons has been calculated on the basis of two alternative assumptions.[10] First, assuming that the electron recoils from a quantum scattered at a definite angle, its energy is found to be

$$E=h\nu\frac{2\alpha\cos^2\theta}{(1+\alpha)^2-\alpha^2\cos^2\theta}, \tag{2}$$

where $\alpha=h\nu/mc^2$, and θ is the angle between the primary x-ray beam and the direction of the electron's motion. This energy is a maximum when $\theta=0$, and is then,

$$E_m=h\nu\frac{2\alpha}{1+2\alpha}. \tag{3}$$

[8] C. W. Hewlett, Phys. Rev. **17**, 284 (1921)

[9] See, e. g., A. H. Compton, Bull. Nat. Res. Council No. 20, p. 29, 1922

[10] See Compton and Hubbard, loc. cit.[7]

The second assumption is that the R electron moves forward with the momentum of the incident x-ray quantum. In this case the energy acquired is

$$E' = h\nu \cdot \tfrac{1}{2}\frac{\alpha}{1+2\alpha}(1-\tfrac{1}{4}\alpha^2+\cdot\cdot\cdot) \,. \tag{4}$$

Eq. (3) was found to agree considerably better than Eq. (4) with Wilson's experimental results.

The lengths of the tracks shown on our photographs could be estimated probably within 10 or 20 per cent. These measured values, reduced to a final pressure of 1 atmosphere, are summarized in Table II. In column 2 are recorded the lengths of the longest tracks observed at each potential. S_m is the range calculated from Eq. 3, using C. T. R. Wilson's result[1] that the range of a β-particle in air is $V^2/44$ mm, where V is the potential in kilovolts required to give the particle its initial velocity, and the frequency ν employed is the maximum frequency excited by the voltage applied to the x-ray tube. S' is similarly calculated from Eq. (4).

TABLE II

Maximum lengths of R tracks.

Potential	Observed	Calc. (S_m)	Calc. (S')
21kv	0mm	0.06mm	0.004mm
34	0	0.3	0.02
52	2.5	1.8	0.1
74	6	6	0.4
88	9	12	0.7
111	24	25	1.5

It is evident that the observed lengths of the R tracks are not in accord with the quantity S' calculated from Eq. (4). They are, however, in very satisfactory agreement with the values of S_m given by Eq. (3). This result agrees with the conclusion drawn from Wilson's data,[11] but is now based upon more precise measurements. It follows that *the momentum acquired by an R particle* is not merely that of the incident quantum, but *is the vector difference between the momentum of the incident and that of the scattered quanta.*[12]

This conclusion is supported by a study of the relative number of tracks having different ranges. If the maximum range of the recoil electrons is S_m, Compton and Hubbard find[7] that the probability that the length of a given track will be S is proportional to

$$(2\sqrt{S/S_m}+\sqrt{S_m/S}-2) \,. \tag{5}$$

[11] Compton and Hubbard, loc. cit.,[7] p. 449.

[12] That this is true for the β-rays excited by γ-rays has been shown in a similar manner by D. Skobeltzyn, Zeits. f. Phys. **28**, 278 (1924).

This expression assumes that the exciting primary beam has a definite wave-length. To calculate the relative number of tracks for different relative lengths to be expected, we have averaged this expression by a rough graphical method over the range of wave-lengths used in our experiments. These calculated values are given in the last column of Table III, for the relative ranges designated in column 1. A comparison of these

TABLE III

Relative lengths of R tracks.

Range of S/S_M	Per cent of R tracks within this range 52kv	74kv	88kv	111kv	Mean	Calc.
0- .2	44	66	60	54	56	53
.2- .4	34	20	26	32	28	22
.4- .6	19	8	4	8	10	14
.6- .8	0	3	5	3	3	8
.8-1.0	3	3	5	3	3	3

calculated values with the observed relative ranges shows a rather satisfactory agreement throughout. It will be noted further that the probabilities of tracks of different relative ranges is found to be about the same for x-rays excited at different potentials. This is in accord with the theoretical expression (5) for the probability, which is independent of the wave-length of the x-rays employed.

Angles of ejection of R tracks. On the view that the initial momentum of an R electron is the vector difference between the momenta of the incident and the scattered quantum, it is clear that these electrons should start at some angle between 0 and 90° with the primary beam. The probability that a given track will start between the angles θ_1 and θ_2 is on this hypothesis,[13]

$$\int_{\theta_1}^{\theta_2} P_\theta d\theta = 3ab\int_{\theta_1}^{\theta_2} \frac{a^2 \tan^4\theta + b^2}{(a \tan^2\theta + b)^4} \frac{\sin\theta}{\cos^3\theta} d\theta \,, \qquad (6)$$

where $a = (1 + h\nu/mc^2)^2$, and $b = (1 + 2h\nu/mc^2)$.

In our photographs only those taken at 111 kilovolts have tracks long enough to determine the initial direction with sufficient accuracy to make a reliable test of this expression. In all, the directions of 40 tracks were estimated, with the results tabulated in the second column of Table IV. In view of the fact that the photographs were stereoscopic, it was possible to estimate the angles in a vertical plane roughly, though not closer perhaps than within 10 or 15°. The values in the third column are calculated from Eq. (6). It is especially to be noted that, in accord with the

[13] See Compton and Hubbard, loc. cit.,[7] Eq. (14).

theory, no R tracks are found which start at an angle greater than 90° with the primary x-ray beam. In view of the small number of tracks observed and the approximate character of the angular estimates, the agreement between the two sets of values is as close as could be expected.

A more searching test of the assumption that the R tracks are electrons which have recoiled from scattered quanta is a study of the relative ranges of the tracks starting at different angles. (See columns 4 and 5 of Table IV.) The calculated ranges in column 5 are based on Eq. (2) for

TABLE IV

Number and range of R tracks at different angles, for 111 kv x-rays.

Angle of emission	Per cent of total number		Average range	
	(obs.)	(calc.)	(obs.)	(calc.)
0°-30°	34	28	9 mm	11 mm
30°-60°	39	50	4	4
60°-90°	27	22	0.9	0.3

the energy at different angles. In this calculation the effective wave-length, as estimated in connection with Table I, is employed. It will be seen that the observed ranges of the tracks ejected at small angles are much greater than that of those ejected at large angles, in substantial agreement with the theory.

It is worth calling particular attention to the fact that in comparing the theoretical and experimental values in these tables, no arbitrary constants have been employed. The complete accord between the predictions of the theory and the observed number, range, and angles of emission of the R tracks is thus of especial significance.

The evidence is thus very strong that there is about one R track or recoil electron associated with each quantum of scattered radiation, and that this electron possesses, both in direction and magnitude, the vector difference of momentum between the incident and the scattered x-ray quantum. Our results therefore afford a strong confirmation of the assumptions used to explain the change in wave-length of x-rays due to scattering, on the basis of the quantum theory.

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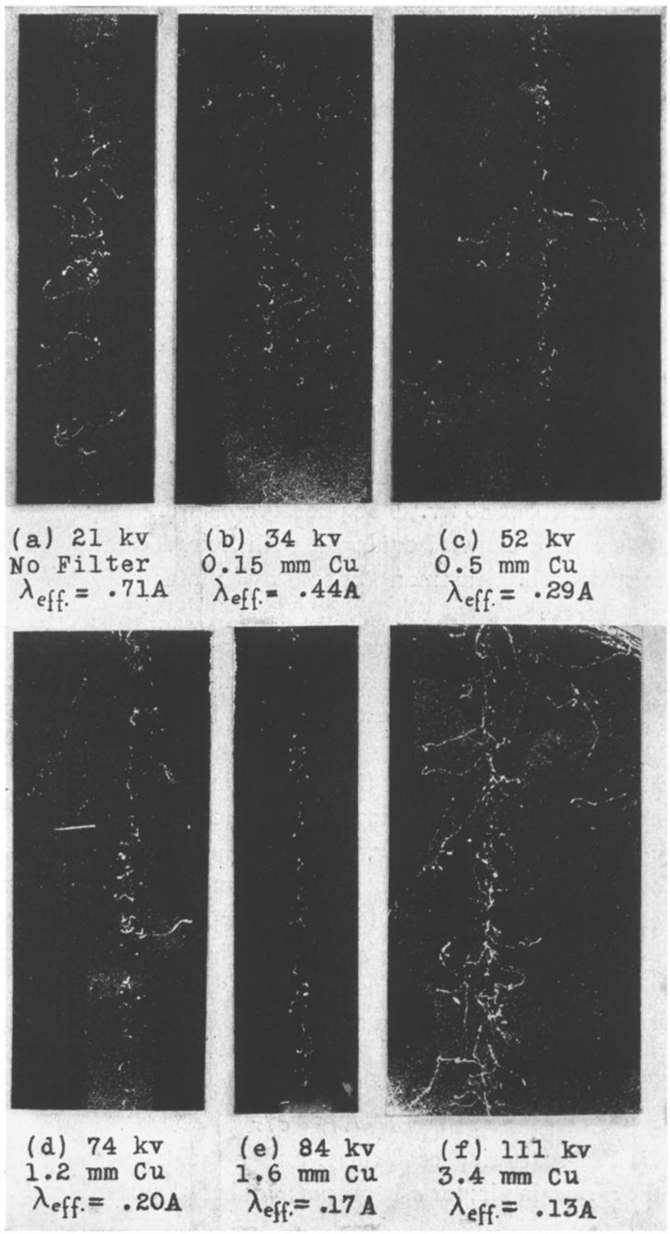

Plate I. The x-rays pass from top to bottom. In addition to the copper filter, they traverse glass walls 4 mm thick. For the short waves the shorter (R) tracks increase rapidly in length and number. Thus while in (a) nearly all are P tracks, in (f) nearly all are R tracks.