ON THE MOMENTUM IMPARTED TO ELECTRONS BY RADIATION

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ABSTRACT

Theoretical deflection of a beam of electrons by electromagnetic radiation.—Assuming (a) the theorems of conservation of energy and momentum and (b) the agreement of the light quantum theory with the wave theory in their estimates of the radiant energy scattered in various directions from a beam of electromagnetic radiation by a group of electrons, it is shown that the total momentum transferred to the electrons is the same on both theories and that $\Delta v/v=(8\pi/3)(e^4/m^3c^4v^2)l\rho$, where l is the distance traveled through radiation of density ρ, and e, m are in e.s.u. Even under very favorable conditions $\Delta v/v$ is only about $10^{-14}l$, hence the deflection of the beam of electrons would be too small to detect experimentally. In case only certain electrons are deflected, experimental arrangements such as are used to observe fish tracks may make these visible.

CONSIDERABLE attention has been drawn recently to the interaction between radiation and free electrons by the experiments of A. H. Compton[1] on the softening of x-rays due to scattering and by the experiments of C. T. R. Wilson[2] and Bothe[3] on "fish tracks." It is our purpose to discuss below the relation to Compton's views of an experiment which Sir J. J. Thomson suggested many years ago and also to point out some general conclusions as to the applicability of the Correspondence Principle to the problem here considered which follow from theorems of conservation. The problem proposed by J. J. Thomson was that of determining whether a beam of electromagnetic radiation produced an observable effect upon a stream of electrons. The experiment was tried by H. A. Wilson but the results were not published. Professor Wilson has kindly stated in a letter that the experiment indicated that the radiation probably produced no noticeable action on the electron stream. More recently C. J. Lapp[4] carried out a similar experiment and obtained what we regard as a similar negative result, although the experiment is a troublesome one, the elimination of spurious and secondary effects being difficult.

[1] A. H. Compton, Bull. Nat. Res. Council, No. 20, p. 19 (1922); Phys. Rev. **21**, 207 and 483, (1923); **24**, 168 (1924); A. H. Compton and J. C. Hubbard, Phys. Rev. **23**, 439 (1924); cf. also P. Debye, Phys. Zeits., April 15, 1923.

[2] C. T. R. Wilson, Proc. Roy. Soc. A **104**, (Aug. 1, 1923)

[3] W. Bothe, Zeits. f. Phys. **16**, 319 (July 19, 1923)

[4] C. J. Lapp, Phys. Rev. **20**, 104 (1922)

In an approximate calculation based on classical electrodynamics one of us[5] found that the deflection of a moving electron subjected to intense electromagnetic radiation is too small to permit of probable experimental detection. In the analysis radiation from the accelerated electron was neglected, and further consideration has shown that effects similar to those of radiation pressure are likely to be present. It is the purpose of the present paper to treat the problem in a more complete fashion from both a classical and a "light quantum" standpoint.

By the expression classical theory we mean the usual theory which assumes that the effects of a number of monochromatic wave trains on an electron are superposable. A more general theory would question this. Such a generalized view is suggested by considerations on the kinetic energy of free electrons in black-body radiation.[6] Since it is very likely that the total momentum transferred to the electron is the same on both views in the region of long wave-lengths, only the restricted theory will be dealt with in this paper.

In order to facilitate an exact calculation which takes into account relativity effects, since no matter how large the velocity of the original electron stream may be, the velocity of light with respect to it is always the universal constant c, it is advantageous to refer phenomena to axes fixed in the moving electron stream. Thus we consider a stationary collection of electrons and the scattering produced in them by a beam of radiation.

In some treatments of Compton's theory definite pictures of electrons and quanta are used.[7] So far as the question discussed here is concerned, however, no specific assumption as to shape need be made. It is necessary to use only the laws of conservation of energy and momentum.

Let the direction of the incident radiation be that of the axis of X. In the region of long wave-lengths the direction of the scattered radiation must be symmetrical with respect to a plane normal to the incident beam as this is required by the classical wave theory for a weak incident beam. Therefore on the light quantum theory also the same must be true. By the law of conservation of momentum the momentum $\mathbf{M}$ imparted to the electrons is given by

$$\mathbf{M} = \mathbf{G}_1 - \mathbf{G}_2 \tag{1}$$

where $\mathbf{G}_1$ and $\mathbf{G}_2$ are the momenta of the quanta before and after incidence,

[5] E. O. Hulburt, Phys. Rev. **21**, 650 (1923)

[6] Paper by G. Breit to appear in the Phil. Mag.

[7] Frank W. Bubb, Phys. Rev. **24**, 177 (1924); G. E. M. Jauncey, Phys. Rev. **22**, 233 (1923); L. S. Ornstein and H. C. Burger, Zeits. f. Phys. **20**, p. 345.

respectively. But as we have just explained, in the region of long wavelengths

$$\mathbf{G}_2 = 0, \tag{2}$$

hence

$$\mathbf{M} = \mathbf{G}_1. \tag{3}$$

Therefore, since

$$|G_1| = E_1/c, \tag{4}$$

where E_1 is the total energy of the quanta before incidence, we have by (3)

$$|M|_q = E_1/c \quad \text{(light quantum theory)}. \tag{5}$$

It is easily shown, however, that on the classical theory the relation of the mechanical momentum imparted to the electron to the energy scattered is

$$|M|_c = E/c \quad \text{(wave theory)}. \tag{6}$$

Since, by assumption, the Correspondence Principle holds for the total amount of energy scattered

$$E_1 = E, \tag{7}$$

and therefore by (5) and (6)

$$|M|_q = |M|_c, \tag{8}$$

which proves the statement made.

Considering the action of a plane wave on an electron and taking into account first order effects alone, it will be shown[8] by working out the equations of motion that if the electron should be traveling with a speed v through a distance l in a direction perpendicular to that of the propagation of the wave, the change in its velocity Δv is given by $\Delta v/v = 8\pi e^4 \rho l/3m^3c^4v^2$ where ρ is the energy density of the wave and $m,e,$ are respectively the mass and charge of the electron (in electrostatic units).

Denoting the position of the electron by x, y, z, the equations of motion are:

$$\begin{cases} m\ddot{x} - (2/3)\ (e^2/c^3)\dddot{x} = e\{E_x + (1/c)\ (\dot{y}H_z - \dot{x}H_y)\} \\ m\ddot{y} - (2/3)\ (e^2/c^3)\dddot{y} = (e/c)(zH_x - \dot{x}H_z) \\ m\ddot{z} - (2/3)\ (e^2/c^3)\dddot{z} = (e/c)(\dot{x}H_y - \dot{y}H_x) \end{cases} \tag{9}$$

where E and H are the electric and magnetic intensities respectively. With a sufficient approximation it follows from the third of these that

$$m\dot{z}\Big|_{t_1}^{t_2} = (e/c)\int_{t_1}^{t_2} \dot{x}H_y dt\ ,$$

which may also be written

$$m\dot{z}\Big|_{t_1}^{t_2} = (e/c)\int_{t_1}^{t_2} \dot{x}E_x dt\ , \tag{10}$$

[8] Let the wave be propagated in the direction of the axis of Z, the direction of the electric intensity being along OX.

because in a plane wave $E_x = H_y$. Multiplying the first of the three equations (9) by x and neglecting the second term on the right, we find by partial integration for the case of a periodic solution that

$$\int (e/c)\dot{x}\, E_x dt = (2/3)\ (e^2/c^4) \int_{t_1}^{t_2} \ddot{x}^2 dt \cong (2/3)\ (e^4/m^2c^4)\overline{E_x^2}(t_2 - t_1)\ .$$

But $\overline{E_x^2} = 4\pi\rho$. Therefore by (10)

$$m\Delta v = m\dot{z}\Big|_{t_1}^{t_2} = (8\pi/3)\ (e^4\rho/m^2c^4)\ (t_2 - t_1)\ .$$

Since $t_2 - t_1 = l/v$, it follows that

$$\Delta v/v = (8\pi/3)\ (e^4\rho/m^3c^4)l/v^2\ . \tag{11}$$

Assuming the very favorable conditions $\rho c = 1000$ watts/cm^2 and $v = 10^8$ cm/sec and taking the usual values for e, m, c, it is found that $\Delta v/v = 2\times 10^{-14}l$.

Now, we may assume that an agreement of classical and light quantum theories for the amount and direction of scattered energy implies also an agreement between the two theories for the total amount of momentum transferred to the electrons, since it may readily be shown that this is actually the case in the region of long wave-lengths, i.e., in the region in which an agreement between the two theories can be expected. Therefore experimental detection of the deflected beam must also be difficult if the phenomenon is governed by light quanta even though the individual deflections of the electrons may be large, because in this case the number of deflected electrons must be very small.

If however an experiment is devised in such a manner as to have under observation a very large number of electrons there may be a fair theoretical chance of observing deflections. Such conditions are realized in experiments on fish tracks, the number of electrons per cm^3 being of the order 10^{20}.

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