DETERMINATION OF ELEMENTARY CHARGE E FROM MEASUREMENTS OF SHOT-EFFECT.

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Abstract

Probability fluctuations in thermionic emission (shot effect).—Schottky first pointed out that if the electrons evaporate independently of each other, probability fluctuations of current are to be expected. These fluctuations were observed and roughly measured by Hartmann. Measurements have now been made, using a much higher frequency (750×10^3) and a reliable arrangement for measuring the small r.m.s. voltages (of order 10^{-4}) due to the effect. The method involves the measurement of the alternating current excited in a tuned circuit by the probability variations in the electron current through a vacuum tube (radiotron, UV 199), the Schottky equation for the mean square current being $\overline{J^2} = i_0 e/2RC$ where i_0 is the thermionic current, R and C the resistance and capacity of the tuned circuit and e the electronic charge. To measure the current, it is amplified by a known amount (using a special 4 stage amplifier) and is rectified so that its r.m.s. value may be determined with a d.c. meter. It is important that the rectifier be used only in the range in which it gives a current proportional to the square of the impressed voltage. The tuned circuit picks out a narrow band of frequencies present in the fluctuations, and since the amplification is not the same for frequencies slightly different from the resonance frequency, correction is made for this by a factor F, for which a mathematical expression is derived and values are obtained by integration or summation. Calibration of the amplifier is avoided by substituting, after each reading of the amplified and rectified shot-current, a measured pure sine voltage across the terminals of the tuned circuit, which is adjusted to give the same rectified current as the effect. Calling this v_1, the actual mean square shot voltage $\overline{v_0^2} = v_1^2/F$ and, $\overline{J^2} = Cv_1^2/LF$, where L is the inductance of tuned circuit. It was found that with currents limited by space charge, the observed effect might be only 20 per cent of the theoretical; but with currents limited only by temperature, the agreement was within one per cent. The effect may therefore be used to study the effect of space charge on electronic evaporation.

Determination of elementary charge e, by the shot effect.—Using temperature limited thermionic current, values of e were obtained which lie within two per cent of the mean, and this mean, 4.76×10^{-10} e.s.u., agrees closely with Millikan's value. The precision of these measurements can be greatly increased, so as perhaps to exceed that of the oil-drop measurements.

Study of secondary electronic current from an electrode by means of the shot effect.—Using a special tube with an extra grid, conditions were adjusted so that the primary and secondary currents balanced, yet a shot voltage of 79×10^{-6} volt was observed. This is exactly the value to be expected if both primary and secondary currents gave a full effect independently of each other, as would be the case if secondary electron emission is due, not to reflection or splashing, but to excitation and subsequent evaporation. This effect promises to furnish a powerful instrument for the study of such electronic phenomena.

Measurement of microvoltages of radio frequency.—A method was developed which enables a voltage of 20×10^{-6} to be measured within one per cent, and which is capable of extension to much smaller voltages.

1. Introduction

IN two noteworthy papers[1] W. Schottky has predicted the effects which are to be expected in vacuum tube circuits from probability fluctuations in thermionic emission, and has shown that the magnitude of these effects depends on the charge e carried by a single electron. These fluctuations, which may best be pictured as shock-excitation of the circuit by the impacts of individual electrons, were designated by Schottky as "schrot- (shot) effect." He was bold enough to suggest that, with the aid of modern amplifiers, these fluctuations might be measured and used for the determination of e.

Such measurements were undertaken by C. A. Hartmann,[2] who succeeded without difficulty in demonstrating the fluctuations, and obtained values of e of the right order of magnitude, though varying, in different tests, from about .07 to 3 times the correct value.[3]

The measurements here reported were undertaken for the purpose of determining what fraction of the noise in modern radio receivers is due to this cause. The results, as regards amplifiers, point to the shot-effect as the only internal source of noise in well-made radio-frequency amplifiers. As regards e, the measurements constitute a beautiful verification of Schottky's theory, and the method appears capable of great accuracy, fully comparable with, and perhaps surpassing, that of the oil-drop method.

2. Outline of Experimental Procedure

The circuit arrangements are shown in Figs. 1 and 2. The problem is to measure the alternating current excited in the tuned circuit L, C, R (Fig. 1) by the spontaneous variations in the electron current through the vacuum tube VT. From the value of this current e can be calculated.

The procedure is to amplify this alternating current by a known amount, rectify it, and measure it with a direct current meter. The meter must register the root mean square value of the current, since this is the only value that has any meaning. Fortunately, ordinary detectors give a rectified current proportional to the square of the impressed voltage

[1] W. Schottky, Ann. der Phys. **57**, 541-67 (1918); **68**, 157-76 (1922)

[2] C. A. Hartmann, Ann. der Phys. **65**, 51-78 (1921); Phys. Zeits. **23**, 436 (1922)

[3] After approximate corrections by R. Fürth for telephone and physiological distortion (Phys. Zeits **23**, 354, 1922), the divergence of these values is reduced to approximately ± 50 percent.

(for small voltages), and an ordinary ammeter reads the average of this current. Hence the ammeter in the plate circuit of the detector gives the mean square amplified voltage, and from this the mean square voltage

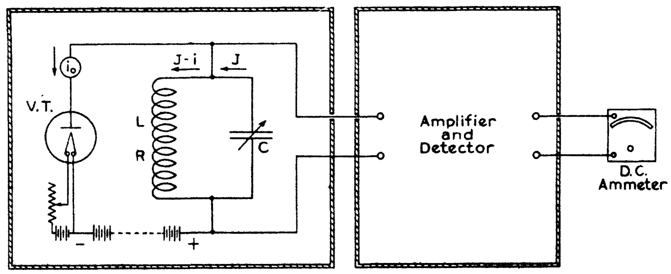

Fig. 1. Outline of experimental conditions for measuring shot-effect.

at the terminals of the tuned circuit L, C, R, can be calculated, and so the mean square current in this circuit, when the degree of amplification is known.

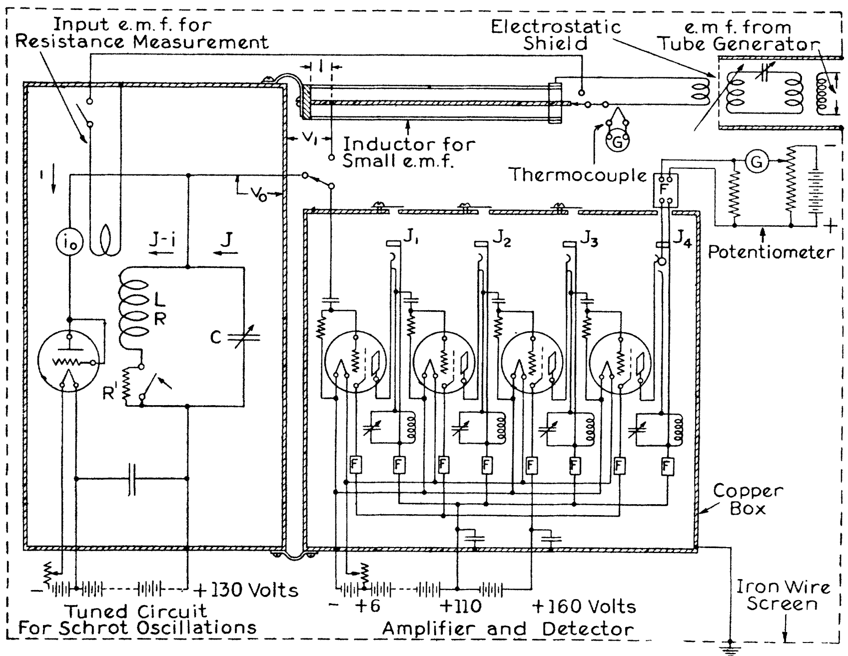

Fig. 2. Complete arrangement of apparatus for measuring shot-effect.

Finally, from the value thus obtained of the mean square current $\overline{J^2}$ in the circuit L, C, R, due to a given thermionic current i_0, the value of e can be calculated at once by Schottky's Equation

$$e = 2RC\overline{J^2}/i_0 \,. \tag{1}$$

The amplification is measured, for any given frequency, by impressing a small measured voltage from a tube generator upon the amplifier, and observing the reading of the output d.c. meter. It was found that the transmission band of the amplifier was so narrow, and the range of frequency of the complex current excited in the tuned circuit L, C, R, by the shot-effect so wide, that the different component frequencies of this current were amplified to very different degrees, some of them scarcely at all. It was necessary, therefore, after widening the transmission band of the amplifier as much as possible, to measure the amplification throughout this band, calculate the frequency spectrum of the shot-effect, and multiply the calculated energy of each component frequency by the amplification factor for that frequency.

These experiments were carried out at a frequency of approximately 750 kilocycles (400 meters). External disturbances were avoided by working in a closed cage of galvanized iron wire netting, which proved to be an excellent screen, and by housing the sensitive parts of the apparatus in copper boxes. Internal disturbances, such as arise from residual gas or mechanical shock in the tubes, were entirely absent, so that the tests could be made upon standard tubes of any type.

I. Theory

3. *Outline.* After Schottky's original derivation of the shot equation (1), simpler derivations, by very different methods, were given by Fürth[4] and by Ornstein and Burger.[5] Neither of these simple derivations, however, seems capable of giving the energy distribution in the frequency spectrum, which is required for the present experiments. Schottky's derivation, on the other hand, leads directly to this distribution, although Schottky has not carried the analysis that far.

The method of calculating this distribution, although simple, is not obvious. It seems desirable, therefore, to outline enough of Schottky's derivation to make our procedure intelligible; especially since Schottky's original derivation was amended in his subsequent papers, and is not available in complete, correct form.

Schottky's procedure is as follows:

Mean square current variation. Consider a hot filament containing a large number of electrons, each of which has the same probability of escaping into vacuum. Let the average number $\bar{n}$ escaping per unit time be small enough so that the escape of one does not affect the probability

[4] R. Fürth, Phys. Zeits. **23**, 354 (1922)

[5] L. S. Ornstein and H. C. Burger, Ann. der Phys. **70**, 622 (1923)

of escape of any other. The average number $\bar{n}$ represents the average over a long period of time and is a measure of the thermionic current as read by a d.c. meter.

The actual number $n_{\Delta t}$ escaping in successive single intervals Δt will not, in general, be $\bar{n}\Delta t$, but will show random positive and negative deviations, whose algebraic average is zero but whose mean square value is given by the well-known probability theorem

$$\overline{(n_{\Delta t}-\bar{n}\Delta t)^2}=\bar{n}\Delta t. \tag{2}$$

Multiplying both sides of this equation by $(e/\Delta t)^2$ one obtains

$$\left[\frac{\overline{(n_{\Delta t}-\bar{n}\Delta t)^2 e^2}}{\Delta t^2}\right]=\bar{n}e\cdot\frac{e}{\Delta t}.$$

or

$$\text{r.m.s. } i_{\Delta t}=\sqrt{i_0 e/\Delta t}. \tag{3}$$

The left-hand side of Eq. (3) represents the root-mean-square value of the variations of the average thermionic current during short periods Δt, that is, of the deviations of the short period average from the long time average i_0. It differs from r.m.s. currents used in engineering in only one respect, viz., that the quantity whose mean square is taken is not the instantaneous current, but the average current over a short period.

The right-hand side of Eq. (3) shows that this variable part of the thermionic current i is proportional to the square root of the charge e of a single electron. It would be 1.4 times as great if the evaporating unit were a doubly-charged ion, and would be zero, that is, there would be no fluctuations, if electricity were an infinitely fine-grained fluid. Hence the existence of shot-fluctuations is a necessary consequence of the finiteness of charge of the electron. This conclusion is independent of all assumptions except that the evaporation is a random process, governed by probability.

4. Effect of Current-Variations on a Tuned Circuit

Equation (3) is not suitable for measurements, since the current $i_{\Delta t}$ depends on the length of the interval Δt which is used to define the "short time average." It might seem at first sight that a true instantaneous value of i could be defined as the limit of this short time average as the time Δt approaches zero; that is, limit $(\Delta t=0)[e\sqrt{\overline{(n_{\Delta t}-\bar{n}\Delta t)^2}}/\Delta t]$. But in approaching this limit one would soon come to intervals containing single electrons, and the theory of probability would obviously cease to apply long before this, viz., as soon as $n_{\Delta t}$ ceased to be a very large number.

The difficulty disappears, as Schottky has shown, when one considers the effect of these current variations upon a tuned circuit (Fig. 1), since the period Δt can then be identified successively with the periods of each of the Fourier components of the current variations. Each Fourier component is a pure sine wave, whose effect on the tuned circuit can be calculated by routine methods; and the sum of the energies of these partial oscillations gives the total energy of oscillation of the circuit.

The Fourier representation of the thermionic current i during any long period T may be written

$$i=\sum_{k=0}^{\infty} C_k \sin(\omega_k t+\varphi_k) .$$

The term of zero frequency ($k=0$) represents the constant part i_0 (registered by the d.c. meter), whose effect upon both the tuned circuit and the amplifier is nil.

Each of the other components

$$i_k=C_k \sin(\omega_k t+\varphi_k) \tag{4}$$

will cause (a) a transient current in the tuned circuit, of frequency $1/(2\pi\sqrt{LC})$, whose effect is negligible because i_k represents a long train (of length T sec.) of pure sine waves; and (b) a forced oscillation

$$J_k=C_k\sqrt{\frac{1}{(1-LC\omega^2)^2+R^2C^2\omega^2}}\sin(\omega_k t+\varphi_k)$$

$$=C_k\sqrt{\frac{1}{(1-x^2)^2+r^2x^2}}\sin(\omega_k t+\varphi_k) \tag{5}$$

where $\omega_0=\sqrt{1/LC}$ = natural frequency of the circuit;

$r=R/L\omega$ = power factor of circuit;

$x=\omega/\omega_0$;

R = equivalent series resistance of the circuit, including losses in condenser and tube.

The mean square value of each current component J_k is $\overline{J_k^2}=\frac{1}{2}C_k^2\{1/[(1-x^2)^2+r^2x^2]\}$; and its energy $W_k=L\overline{J_k^2}$.

The total energy of the circuit is the sum of these components

$$W=L\sum_{k=0}^{\infty}\overline{J_k^2}=\tfrac{1}{2}L\sum_{k=0}^{\infty}C_k^2\left(\frac{1}{(1-x^2)^2+r^2x^2}\right) .$$

5. Mean Square Amplified Voltage

At this point we shall depart slightly from Schottky's procedure, in order to introduce into the calculation the different amplifications of the component frequencies. Instead of calculating the energy as Schottky did, we shall calculate directly the quantity that is to be measured, which is the mean square voltage at the terminals of the condenser C. This is proportional to the energy and hence follows the same additive law, viz., the total mean square voltage equals the sum of the component mean square voltages.

The mean square voltage of a single component is

$$\overline{v_k^2}=L^2\omega_k^2\overline{J_k^2}=\tfrac{1}{2}L^2\omega_0^2C_k^2\frac{x^2}{(1-x^2)^2+r^2x^2} \qquad (6)$$

The alternating voltage v_k is impressed directly upon the grid of the first tube of the amplifier (Fig. 2). Let the square of the voltage amplification for sine waves of frequency ω_k be represented by

$$\overline{E^2}/\overline{v^2}=A_0^2f(x) \quad (x=\omega_k/\omega_0) .$$

A_0 represents the maximum amplification, which is obtained at frequency ω_0, and $f(x)$, which varies from 0 to 1, represents the relative amplification for frequencies other than ω_0 (cf. Figs. 4 and 5).

The mean square voltage *after amplification* due to any component k will then be

$$\overline{E_k^2}=\tfrac{1}{2}L^2\omega_0^2A_0^2C_k^2\frac{x^2f(x)}{(1-x^2)^2+r^2x^2} \qquad (7)$$

and the total mean square amplified voltage will be

$$\overline{E^2}=\Sigma\overline{E_k^2}=\tfrac{1}{2}L^2\omega_0^2A_0^2\sum_{k=0}^{\infty}\frac{C_k^2x^2f(x)}{(1-x^2)^2+r^2x^2} . \qquad (8)$$

6. Evaluation of Fourier Coefficients

In order to find the value of C_k^2, Schottky proceeds as follows. The individual Fourier coefficients C_k, which may vary greatly from term to term, are not specified by the laws of probability, but only their means over periods containing many individual values. It is these mean values however, over finite periods Δt, which are required. Eq. (8) requires the sum of products of C_k^2 times a function $F(x)$. If the fundamental period T is taken large enough, an interval Δt large enough to satisfy Eq. (2) (upon which Eq. 8 depends) will correspond to a negligible change Δx in x, and hence a negligible change in $F(x)$. Hence each value of $F(x)$ in Eq. (8) may be multiplied by the mean square value of C_k over the period Δt, and this value may be obtained, in accordance with probability, by application of Eq. (2). Schottky[6] has shown that this mean square value $\overline{C_k^2}$ is the same for all frequencies ω_k which are near enough to the natural frequency ω_0 of the tuned circuit to have any appreciable effect upon it, and that this value is given by the equation

$$\overline{C_k^2} = 4i_0e/T\ . \tag{9}$$

where T is the fundamental period of the Fourier series. Also, if the number of components k is very large (T very large),

$$dx = d\left(\frac{\omega_k}{\omega_0}\right) = \frac{1}{\omega_0}d\left(\frac{2\pi k}{T}\right) = \frac{2\pi}{\omega_0 T}dk = \frac{2\pi}{\omega_0 T}\ .$$

Substituting these values of $\overline{C_k^2}$ and of dx in Eq. (8), we obtain

$$\overline{E^2} = \frac{L^2\omega_0^3A_0^2i_0e}{\pi}\int_0^\infty \frac{x^2f(x)dx}{(1-x^2)^2+r^2x^2}\ . \tag{10}$$

If all frequencies ω_k for which the amplitude is appreciable were amplified to the same degree so that $f(x) = 1$, Eq. (10) would reduce to Schottky's equation. The value of the integral[7] is then the same as calculated by Johnson[8] and Schottky,[9] viz., $\pi/2r$. Eq. (10) then becomes, after substituting $r = R/L\omega_0$, and $\omega_0^2 = 1/LC$,

$$\overline{E^2} = A_0^2Lei_0/2C^2R. \tag{11}$$

This is the equation used by Hartmann. It was subsequently derived in a different and much simpler manner by Fürth.[10]

[6] Schottky, Ann. der Phys. **57**, 556-9, 1918; **68**, 159-60, 1922. In the first derivation a factor 2 is omitted, which is corrected in the second reference.

[7] The quantity under the integral sign differs from that calculated by Johnson and Schottky by the factor x^2, which, however, does not change the value of the definite integral, as Schottky has pointed out (Ann. der Phys. **68**, 169, footnote).

[8] J. B. Johnson, Ann. der Phys. **67**, 154-6 (1922)

[9] W. Schottky, Ann. der Phys. **68**, 157-8 (1922). Schottky's original value of this integral (Ann. der Phys. **57**, 560) was wrong. The correct value was first given by Johnson[8] and later a very elegant solution, due to F. Schottky, was given by W. Schottky (l. c. p. 157).

[10] R. Fürth, Phys. Zeits. **23**, 354 (1922). Fürth adds up the transient oscillations of frequency ω_0 caused by the impact of individual electrons, with due regard for damping and phase. He thus obtains a sine wave oscillation of frequency ω_0 (natural frequency of the circuit) *of variable amplitude*, whereas Schottky's method gives a series of sine waves, each of *constant amplitude, of frequencies* ω_k *different from* ω_0. The two results are identical, since a modulated sine wave is equivalent, both mathematically and physically, to a constant "carrier wave" plus a series of "side bands," each of constant amplitude (cf. papers by Colpitts and Blackwell, A.I.E.E. Proc. **38**, 360, 1919; and R. V. Hartley, I.R.E. Proc. **11**, 34, 1923.) There is also no inconsistency in the methods

7. Amplification Factor

In the present experiments, preliminary calculations showed that the range of effective frequencies ω_k was comparable with, and in some cases much wider than, the transmission band of the amplifier; that is, the relative amplification $f(x)$ was considerably different for the different component frequencies of the shot voltage (cf. Fig. 6). The value of the integral in Eq. (10) will therefore be less than $\pi/2r$ (since $0<f(x)<1$). The ratio of the actual value of this integral to its maximum value $\pi/2r$ will be called the *amplification factor F.*

$$F=\int_0^\infty \frac{x^2 f(x)dx}{(1-x^2)^2+r^2x^2} \Big/ \frac{\pi}{2r} \, . \qquad (12)$$

In place of Eq. (11) we shall then have, for the mean square amplified voltage due to a thermionic current i_0,

$$\overline{E^2}=(A_0{}^2 \cdot Lei_0/2C^2R) \cdot F \, . \qquad (13)$$

In order to determine F it is necessary to evaluate the integral in Eq. (12). One observes, first, that only a small range of values of x in the neighborhood of $x=1$ contribute appreciably to the value of the integral. This is true of the total shot integral, Eq. (10) (see curve I, Fig. 6);[11] and still more so when each ordinate is multiplied by the relative amplification $f(x)$ (curve III, Fig. 6). Hence, with sufficient approximation

$$\int_0^\infty \frac{x^2 f(x)dx}{(1-x^2)^2+r^2x^2}=\int_{1-a}^{1+a} \frac{x^2 f(x)dx}{(1-x^2)^2+r^2x^2} \; ; \; a<<1 \, .$$

This limited integral may be simplified, by the method used by Schottky[9] (due to F. Schottky) for evaluating the complete shot-integral, as follows:

(1) Add to the given integral another integral

$$\int_{1-a}^{1+a} \frac{f(x)dx}{(1-x^2)^2+r^2x^2} \, ,$$

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of treatment, one of which considers only transient oscillations, and the other only forced oscillations.

Fürth's derivation gives the mean square voltage very simply. In the present experiments, however, the *relative intensity* of the different component frequencies is also required, since they are amplified by different amounts. This is not given by Fürth's method, but can easily be obtained, as shown below, by the Fourier analysis which Schottky has used.

[11] The degree of approximation can be determined by comparing the area under this curve, between any given values of x, with $\pi/2r$. In the experiments here reported the approximation was always sufficient with the x limits required by the methods of evaluation of the integral.

formed by substituting a new variable $z=1/x$ in the original integral, expressing the new limits in terms of z, and finally writing x for z. The limits of the new integral will be the same as for the original one provided $a^2<<1$, since $1/(1-a)=1+a$; and the value of $f(x)$ will also be the same for each value of the variable, viz. $f[1/(1-\delta)]=f(1+\delta)=f(1-\delta)$, on account of the symmetry of the function $f(x)$ with respect to $x=1$ (curve II, Fig. 6). The value of the original integral will therefore be correctly represented by half the sum of the two,

$$\int_{1-a}^{1+a}\frac{x^2f(x)dx}{(1-x^2)^2+r^2x^2}=\frac{1}{2}\int_{1-a}^{1+a}\frac{(1+x^2)f(x)dx}{(1-x^2)^2+r^2x^2}\,.$$

(2) Introduce a new variable, $y=(1/r)(x-1/x)$. The integrand becomes $dy/[2r(1+y^2)]$, and the new limits $-2a/r$ and $+2a/r$ respectively. The amplification factor then takes the simple form (cf. Eq. 12)

$$F=\frac{1}{\pi}\int_{-2a/r}^{2a/r}\frac{\varphi(y)dy}{1+y^2} \tag{14}$$

where $\phi(y)=f(x)=$relative energy amplification.

By means of Eq. (14) the amplification factor F may be obtained very simply, either by graphical integration, by multiplying each ordinate of the experimentally determined curve $\phi(y)$ by $[1/(1+y^2)]$ and measuring the area under the resulting curve (curve III, Fig. 4); or as a summation

$$\begin{aligned}F&=(1/\pi)\Sigma\varphi(y)\delta y/(1+y^2)\\&=(1/\pi)\sum_{-2a/r}^{2a/r}\varphi(y)\delta(\text{arc tan }y)\\&=(2/\pi)\sum_{0}^{2a/r}\varphi(y_n)\ (\text{arc tan }y_n-\text{arc tan }y_{n-1})\end{aligned} \tag{15}$$

The steps $y_n=2n\delta x/r$ were taken at intervals $\delta x=1/400$. Ten such steps were found sufficient in all cases, so that $a^2=(n\delta x)^2_{max}=1/1600$. This is sufficiently negligible compared with 1 to justify the approximation used in transforming the integral. This latter method was the one actually used for most of the computations.

When F has been determined in this way, and $\overline{E^2}$ measured as described in the next section, Eq. (13) may be used for calculating the charge of the electron, or, substituting for $\overline{E^2}$ its equivalent $\overline{E^2}=A_0{}^2v_1{}^2$ (see section 9),

$$e=2C^2Rv_1{}^2/i_0LF \tag{16}$$

where L, C, $R=$inductance, capacity and resistance of tuned circuit; $i_0=$thermionic current; $F=$amplification factor, defined by Eqs. (12)

and (15); v_1 = sine wave of frequency ω_0 which produces the same detected current as the shot-effect of the thermionic emission (see section 9).

II. Methods of Measurement

8. Measurement of $\overline{E^2}$

Hartmann measured the mean square voltage of the shot disturbance by comparing it with a pure sine wave which, after the same amplification, gave an equal audible sensation. Fürth[4] has justly criticized this method, on the ground that the response of the ear is proportional, not to the square of the sound amplitude, but to its logarithm.

Very fortunately, the vacuum tube detector as it is used in radio reception has exactly the characteristic required; that is, the rectified current is proportional, within proper limits, to the square of the alternating voltage impressed upon the grid. The conditions under which this "square law" holds can be found by direct calibration under operating conditions. Theoretically any detector that utilizes the curvature of a volt-ampere characteristic should give a sufficiently close approximation to square detection over a small range. Using grid-current curvature, we found that this range was generally less than 1/10 of a volt. This is too small a range for convenient accurate measurement of the change in plate current. The deviation from the voltage-square law was found, however, to be sometimes positive and sometimes negative, that is, the detected current was proportional to a higher or lower power than the square, according to the value of grid resistance and bias potential. By successive trials values of these variables were eventually found which gave a detected current proportional to the square of the impressed voltage, within the limits of accuracy of our measurements, over a range of 1.5 r.m.s. impressed volts.[12] These values, which apply only to the particular tube and plate voltage used, were 93,000 ohms grid resistance, connected to a bias potential of +5.0 volts with respect to the negative end of the filament.

[12] Observation of these limits of square detection is very important. Values of e as small as 1/50 the correct value were obtained in preliminary experiments, where the amplification was much too great and hence the signals impressed upon the grid of the detector tube too strong. The error was due to the fact that the occasional very strong peaks in the variable shot voltage, which contain a large part of the energy, go far beyond the saturation limit of the detector, and produce no greater individual rectified currents than much weaker peaks; while on account of their *intermittent* character their *integrated* effect is equalled by that of continuous waves only 1/50 as strong. Errors of the order of 2 fold may be produced with signals which do not run into the saturation range but do go beyond the square range into that of linear detection. In this case the rectified current is proportional to the square of the voltage for weak components, but to only the first power for strong ones.

The change in plate current was measured by a null potentiometer method, as indicated in Fig. 2, using the galvanometer only for detecting zero current.

9. Calibration of Amplifier and Detector

Measurements of $\overline{E^2}$ (Eq. 13) requires, in addition to the certainty that the detector current is proportional to the square of the voltage, a knowledge of the constant of proportionality, the "coefficient of detection." Eq. (13) also contains, as an unknown, the amplification constant A_0 of the amplifier. Both of these factors were determined simultaneously by impressing upon the grid of the first amplifier tube, immediately after each shot reading, a measured sine voltage of frequency ω_0 which gave the same detected current as the shot voltage had given; that is, which gave zero deflection of the galvanometer with the potentiometer setting unchanged. These readings could be made in close sequence (5 to 10 sec.) so that changes in battery voltage caused no annoyance. If v_1 is the value of this comparison voltage which is impressed upon the amplifier, its value after amplification will be $A_0 v_1$, and the square of this must be equal to the mean square amplified shot voltage $\overline{E_0^2}$ since both produce the same detected current.

Eq. (13) then becomes

$$A_0^2 v_1^2 = A_0^2 L e i_0 F / 2C^2 R \ ,$$

whence

$$e = 2C^2 R v_1^2 / i_0 L F \tag{16}$$

10. Measurement of i_0, L and C

Equation (16) is suitable for calculation of e, since all the quantities on the right hand side are directly measurable. The method of determining F has already been described. The thermionic current i_0, whose value ranged from 1/4 to 20 milliamperes, was measured with a calibrated d.c. milliameter. L and C were measured *in position* by comparison with Bureau of Standards standards. For this purpose the coil, condenser and vacuum tube were rigidly mounted in a large copper box, with connections complete, and this box was carried to our standards laboratory for calibration. Uncertainties due to distributed capacity were minimized by using a coil of small wire (.020 inch diam.) with 1/4 inch spacing between wires and with small total inductance (80 microhenries), and a correspondingly large value of condenser capacity[13] (610×10^{-12} farads).

[13] Hartmann (Ann. der Phys. **65**, p. 58) observed what he considered an additional tube capacity, dependent upon electron emission, which was approximately proportional to the emission, and reached the enormous value of 180 micro-microfarads for an emission of 20 milliamperes. We found no evidence of capacities of this magnitude, which would

The quantities R and v_1^2 required special methods of measurement, which merit brief discussion.

11. Measurement of Resistance

The effective series resistance of the input circuit, including vacuum tube, condenser, and connections, was measured in the usual way by inserting an additional known resistance. Instead of observing the ratio of deflections, we observed the ratio of impressed e.m.f. required to produce the same voltage across the tuned circuit, with and without the added resistance respectively. This voltage was measured by the first tube of the amplifier acting as a tube voltmeter. The circuit arrangements were such that this measurement could be made immediately before and after each shot reading, without changing any internal connections or opening the box, so that the resistance measured was that which was actually effective in the shot determinations. The measurement *in situ* was made possible by the fact that the amplifier had capacity coupling between tubes (see Fig. 2) so that any tube could act as a detector. The plate-circuit of each tube was provided with a jack ($J_1 - J_4$, Fig. 2) normally short-circuited, but capable of being opened by insertion of the plug P, which was connected through the filter[14] F to the potentiometer. For the resistance measurements, the plug P was inserted in the plate circuit of the first tube, and the voltage impressed upon the grid of this tube, with and without the added resistance respectively, was measured with the same instruments and by the same null method as the shot voltage. The input voltage was also measured with the same thermocouple as the shot calibration voltage v_0 (see next section) but with a larger inductor (viz. a magnetically coupled coil of one turn) than that described in the next section, since an input e.m.f. 1000 times greater was required. The throw over from the measurement of shot voltage to the measurement of resistance required, therefore, but two changes, (1) to plug into the first stage instead of the fourth stage of the amplifier, and (2) to switch from the shot inductor to the resistance inductor.

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have required a change of 30 percent in the tuning of the condenser C, in our resistance measurements, with tube hot and tube cold respectively (see next section). The actual retuning required was too small to measure, but was of the order of 1/10 micro-microfarad, which is the amount to be expected from the change of natural frequency of the circuit due to increased resistance.

[14] The filter F was built into a small box rigidly connected to the plug P. This box fitted tightly against the amplifier box and was grounded to it when P was in position. Hence P could be plugged into any stage without danger of feed-back due to high frequency current in the output lead-wires or ground-wire.

Hence these measurements could be made in close sequence, and changes in the tube resistance avoided.

12. Method of Measuring Small Calibrating Voltage v_1

The effective r.m.s. shot-voltage varied from 5 to 150 micro-volts in these tests. Accurately measured sine voltages of this magnitude at 750 kilocycles have never been used, and a reliable method had to be developed. We believe that the method finally adopted is perfectly reliable, and capable of application to still smaller voltages and higher frequencies.

Preliminary measurements, as well as calculations, showed that a resistance-potentiometer is out of the question for these small voltages, on account of the inductive drop in the wire.[15] Better and apparently reliable results were obtained by using magnetic coupling between two

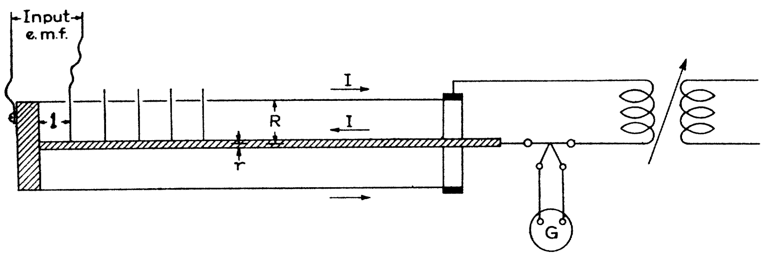

Fig. 3. Arrangement for producing small high-frequency e.m.f. for calibration. The inductance per unit length of the wire can be calculated, and hence the e.m.f. due to a measured current through it.

widely separated single-layer coils, one carrying a measurable current, and the other being in series with the test circuit.[16] The mutual inductance of such a pair of coils can be calculated with considerable accuracy. The necessity for careful screening of the coils, however, and the possibility of change of inductance due to the presence of the screening conductors, led us to abandon this method.

The method finally adopted utilized the inductive drop in a short section of a long straight copper wire, surrounded by a concentric return conductor (Fig. 3). The inductance per unit length of such a wire (assumed infinitely long) is[17]

$$L = 2 \log_e(R/r). \qquad (17)$$

[15] In order to reduce the inductive drop to 1 percent of the resistance drop, it would be necessary to work with wires of the order of 1/10 mm in length.

[16] We are indebted to Mr. H. B. Marvin, of the General Engineering Laboratory of the General Electric Co., for suggesting this method, and for the calculated values of mutual inductance.

[17] Rosa and Graver, Bull. Bur. Stds. **8**, 159, 1912.

r and R are, respectively, the outer radius of the wire and the inner radius of the cylinder and L is in c.g.s. electromagnetic units. In practical units,

$$L = .00460 \log_{10}(R/r) \text{ microhenries/cm.}$$

In our experiments, the diameters of wire and cylinder (brass tube) were .25 inch and 1.5 inches respectively, so that $R/r = 6$. Fine copper wires soldered to the rod at accurate distances of 3, 6, 9, 12 and 15 cm from the end, were brought out at right angles to the rod through small holes in the tube. In the first experiments the potential difference between pairs of these wires was impressed upon the amplifier. This has the disadvantage of a large ground-capacity error. It was later proved, both theoretically and experimentally, that the magnetic field is uniform up to the very end of the cylinder,[18] so that the inner surface of the end disk, carefully sweated into the tube so as to make contact at its inner edge, could be considered as the point of zero e.m.f., and the end of the cylinder used as one terminal. The closed end of the cylinder was therefore pressed directly against the amplifier box and grounded to it, and no further trouble was experienced with ground capacity or inductive drop in grounding wires.

This arrangement enabled us to measure voltages down to 20 microvolts, at a frequency of 750 kilocycles, with 1 per cent accuracy, and 5 microvolts with 5 per cent accuracy, using a thermocouple with 700 ohm heater and a portable Leeds and Northrup galvanometer. Voltages 10 times smaller could be measured with the same accuracy by making the radii R and r more nearly equal; and a further 10 fold reduction can be accomplished, at the sacrifice of convenience, by using a high-sensitivity galvanometer, or by replacing the thermocouple by a crystal detector.

13. The Amplifier

The amplifier (Fig. 2) was of a special type, which will be fully described elsewhere.[19] Only its external characteristics need be mentioned here. It consisted of 4 special pliotrons, constructed so as to be free from internal capacity feedback and thoroughly screened externally, in cascade arrangement as shown in Fig. 2. The individual tubes amplified from 40 to 45 fold under the given circuit conditions. Three tubes in

[18] This is nearly obvious from symmetry. It is proved formally as follows. Let P be any point between the cylinders (Fig. 3) at distance r from the axis. Let H be the field intensity at P. To find H, carry a unit pole around the axis in a circle of radius r (H being constant, by symmetry). The work done, viz., $2\pi rH$ must equal $4\pi i$, hence $H = 2i/r$. This holds for all points, both near to and far from the end.

[19] Hull and Williams, Abstract in Phys. Rev. **23**, 299 (1924). The full paper will appear soon.

series, the fourth being used as detector, gave a voltage amplification of 73,000. This is the product of the individual amplifications, viz. $(42)^4$, within the limits of our knowledge of the individual tube characteristics, and checks other evidence that regenerative effects were absent.

It was found that this degree of amplification was too great for most of the measurements, causing the amplified shot disturbances to exceed the range of square detection of the detector tube. For the final measurements of e it was decreased and the selectivity band widened (which simplified the calculation of F) by inserting a 93,000 ohm resistance in multiple with each tuned plate circuit.

III. Experimental Results

14. Amplification Factor

The transmission characteristics of the amplifier are shown in Figs. 4 and 5. Fig. 4 represents the normal characteristic, with a voltage amplifi-

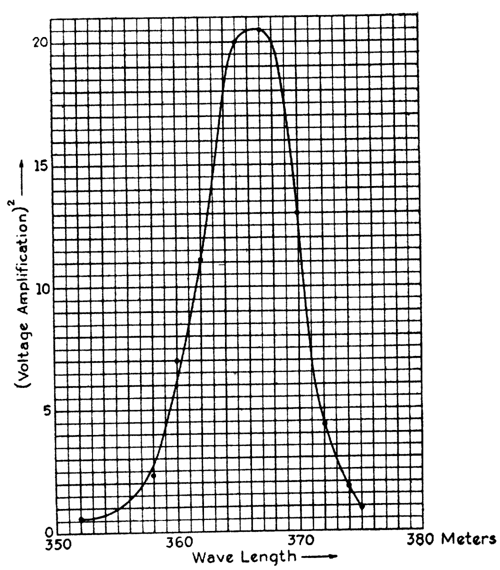

Fig. 4. Normal selectivity curve of the amplifier. The voltage amplification at the center of the band is 73000 fold.

cation of 73,000 fold at the middle of the band. This degree of amplification could be used only for those tests in which the emission was limited by space-charge, where space-charge and high effective circuit resistance both conspired to reduce the shot-effect (see section 17). When the emission was limited by temperature, the shot-effect was large, and for those measurements the amplification had to be decreased as stated above. The selectivity curve corresponding to this decreased amplification is shown in Fig. 5.

These curves represent the square of the actual voltage amplification, in the respective tests, of each component wave-length of the shot disturbance. Their ordinates represent the function from which the amplifica-

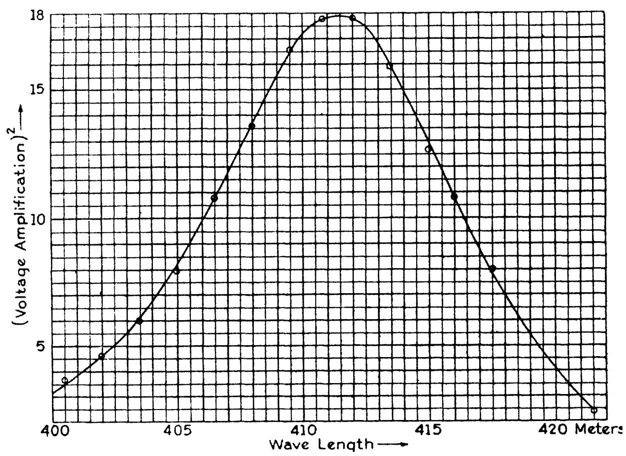

Fig. 5. Selectivity curve of amplifier with 93000 ohms in multiple with each tuned plate circuit.

tion factor F is calculated, as described in section 7. An example of graphical calculation of this factor, for an effective circuit resistance $R=3.60$ ohms, is shown in Fig. 6. Table I gives another example, for $R=2.98$ ohms, performed as a summation, in the manner described in section 7.

TABLE I

Example of calculation of amplification factor F.
$R=2.98$ ohms; $L=80\times10^{-6}$ henries; $\omega_0=4.55\times10^6$

n	δx	y_n	$\frac{\tan^{-1}y_n}{\pi/2}$	$\frac{\delta(\tan^{-1}y)}{\pi/2}$	$\varphi(y)$ (from Fig. 5)	$\frac{\varphi(y)\,\delta\tan^{-1}y)}{\pi/2}$
0	0	0	0			
1	.0024	.59	.339	.339	1.00	.339
2	.0048	1.18	.552	.213	.965	.206
3	.0072	1.77	.672	.120	.895	.108
4	.0096	2.36	.744	.072	.808	.0582
5	.0120	2.95	.792	.048	.693	.0333
6	.0145	3.54	.825	.033	.566	.0189
7	.0169	4.13	.848	.023	.439	.0101
8	.0193	4.72	.867	.019	.341	.0066
9	.0217	5.31	.880	.013	.272	.0036
10	.0241	5.90	.892	.012	.230	.0027
11	.0265	6.49	.903	.011	.190	.0021
				estimate of remainder		.005
					sum $=F=$	.788

A separate value of F is required for each test, since the value of R is different in each. Over a limited range of values of R, however, the values of F differed so little and were so nearly a linear function of R that intermediate values could be obtained with sufficient accuracy by interpolation. This may be seen from Fig. 7 in which are plotted four determinations of F covering the range of values of R used in the series of measurements of Table II.

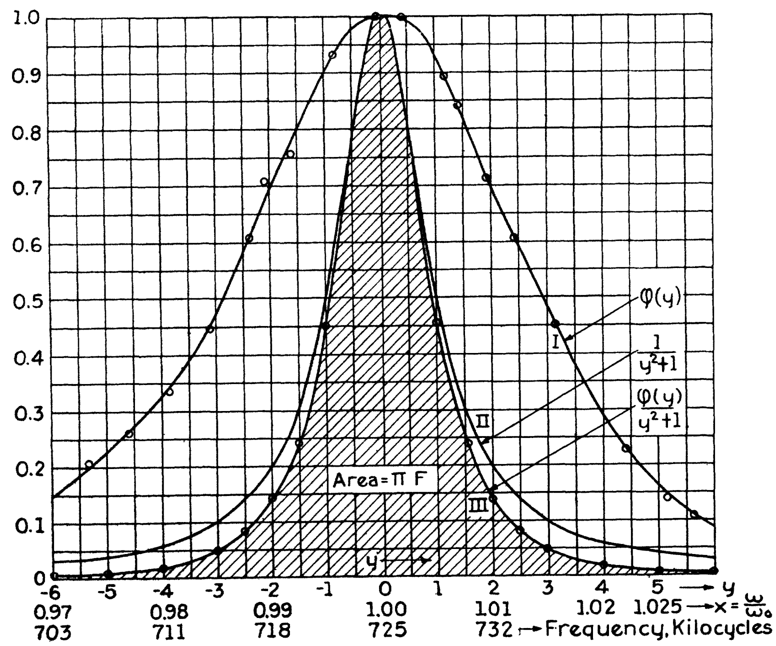

Fig. 6. Example of graphical determination of amplification factor F. Curve I is the selectivity curve shown in figure 5, plotted against $y=(1/r)(x-1/x)$, where $r=R/\omega_0 L$, and $x=\omega/\omega_0$. Curve II represents the relative intensity of the different component oscillations produced by the shot-effect. Curve III is the product of curves I and II. Its area, divided by π, is the amplification factor F.

15. Experimental Procedure

The inductance and capacity of the tuned circuit (L, C, Fig. 2) in which the shot-oscillations were to be excited, were first measured at the frequency which was to be used. The amplifier was carefully tuned to this frequency, and its relative amplification for different nearby frequencies measured (Figs. 4 and 5). The electron emission i_0 of the tube that was to produce the shot-effect ($V.T.$, Fig. 1) was then adjusted to the desired value, and the resulting amplified shot-disturbance measured by the potentiometer (Fig. 2) in the detector circuit. The amplifier was then disconnected from the shot-tube and connected to the calibrating inductor, and the current in the inductor wire quickly adjusted to give the *same reading* of the potentiometer as the shot-disturbance. The value γ of this inductor current was recorded, and also the length l of inductor wire used. From this length l the inductance of the wire was calculated, and from the inductance, the current γ and the frequency ω, the calibrating voltage v_1 impressed upon the amplifier was determined. Finally, the equivalent resistance R of the tuned circuit was measured with the shot-tube and amplifier connected exactly as in the test just made. This was done by inserting an added known resistance in this circuit and observing the ratio of impressed e.m.fs., with and without the added resistance, which gave the same current in the plate circuit of the first tube of the amplifier (acting as detector). The amplification factor F for each value of R was obtained in the manner already described (section 7). From these data the charge e of an electron can be calculated by Eq. (16).[20]

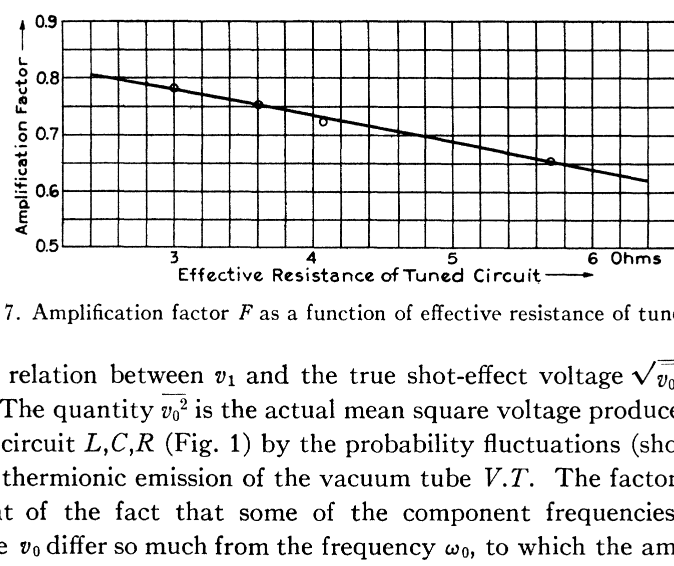

Fig. 7. Amplification factor F as a function of effective resistance of tuned circuit.

The relation between v_1 and the true shot-effect voltage $\sqrt{\overline{v_0^2}}$ is $v_1^2 = \overline{v_0^2}F$. The quantity $\overline{v_0^2}$ is the actual mean square voltage produced in the tuned circuit L,C,R (Fig. 1) by the probability fluctuations (shot-effect) of the thermionic emission of the vacuum tube $V.T.$ The factor F takes account of the fact that some of the component frequencies of this voltage v_0 differ so much from the frequency ω_0, to which the amplifier is tuned, that they are only slightly amplified. Hence, a smaller puresine voltage v_1, of frequency ω_0, produces the same amplified voltage as v_0.

The true r.m.s. voltage $\sqrt{\overline{v_0^2}}$ is given in each of the following tables under the name shot-voltage. In Table II is also given the mean shot-current $\sqrt{\overline{J^2}}$ in order to show its magnitude.

[20] This equation may be reduced to Eq. (1) section 2, by placing $v_1^2/F = \overline{v_0^2} = \overline{J^2}L^2\omega_0^2 = \overline{J^2}L/C$ whence $e = 2RC\overline{J^2}/i_0$.

16. Shot-effect of Temperature-limited Electron Emission in U.V. 199 Radiotron.

Preliminary measurements showed that space-charge caused a marked reduction of the shot disturbance.[21] It was therefore decided to make the first measurements with thermionic currents limited by temperature, with as strong an electric field at the cathode surface as possible. The radiotron U.V. 199, with plate and grid connected together (as anode) was well suited to this purpose on account of its small grid (2.5 mm in diam) and small filament (.015 mm in diam.), giving a field intensity at the cathode, with 120 volts between grid and filament, of 30,000 volts/cm. It also had the advantage of enabling us to test, first the emission from a

Table II

Shot-effect of temperature-limited electron current i_0 in a U.V. 199 radiotron.

i_0 (m-amp.)	γ (m-amp.)	l (cm)	v_1 (μ-volts)	R (ohms)	F	$\overline{v_0^2}$ (μ-volts)	$\overline{J^2}$ (μ-amp.)	e (coulombs)
1	.443	9.0	65.0	3.045	.776	73.8	.204	1.541×10^{-19}
2	.925	6.0	89.4	3.37	.763	102.2	.282	1.640
2	.602	9.0	88.3	3.37	.763	101.0	.279	1.603
3	1.05	6.0	102.7	3.65	.750	118.5	.327	1.595
3	.700	9.0	102.7	3.65	.750	118.5	.327	1.595
4	.580	12.0	113.4	3.85	.740	131.8	.364	1.570
4	.780	9.0	114.4	3.85	.740	133.1	.367	1.595
5	.835	9.0	122.5	4.06	.727	143.8	.397	1.566
5	.625	12.0	122.2	4.06	.727	143.5	.396	1.556
							mean	1.586×10^{-19}

thorium-coated tungsten filament, and then that from the same filament without thorium, by distilling off the thorium and working at a higher temperature. No difference whatever was observed in the value of shot-voltage under these two conditions so long as the emission was limited only by temperature.

Table II shows a series of measurements made in this tube, using the filament in its sensitive (thorium-coated) condition, with emission limited by temperature. The plate potential of 120 volts was furnished by dry batteries, bridged with a 2 μf condenser.

The next to last column gives the observed root-mean-square current excited in the tuned circuit by the spontaneous variations of the therm-

[21] This fact was first discovered, and called to our attention, by Mr. W. L. Carlson of the Radio Department of the General Electric Co. We at first believed that the decrease in shot noise reported by Mr. Carlson was only apparent and was accounted for, as it is to a large extent, by the high effective resistance of the tuned circuit when a space-charge limited tube (low resistance) is connected in multiple with it. The measurements reported in section 17 show, however, that there may be a real reduction of shot-effect, of several fold, due to space charge.

ionic current i_0; and the column before that gives the corresponding r.m.s. voltage at the terminals of this circuit. This voltage has the frequency spectrum shown in curve II, Fig. 6. The "equivalent input voltage" v_1 (fifth column) is a pure sine voltage, of the frequency to which the tuned circuit L, C, and the amplifier are tuned, which produces the same r.m.s. *amplified voltage* as the larger complex "shot-voltage" v_0.

Determination of Elementary Charge

The last column gives the values of e calculated from the experimental data. With the exception of the first two determinations, where the current i_0 was small, the individual determinations differ by less than 2 per cent from the mean, and the mean is within 1/3 percent of Millikan's value, 1.591×10^{-19} coulombs. The closeness of this agreement of the mean is accidental, but we believe that the errors are purely observational, provided space-charge effects are avoided, so that a reliable mean value, of much higher accuracy than this, should be obtainable from a large number of careful observations.

17. Effect of Space-Charge on Shot-Effect

If the thermionic current in a vacuum tube is limited only by the temperature of the cathode, the probability of evaporation of any one electron is, according to the present-day theories, independent of the evaporation of other electrons. This is the condition postulated in the theory of the shot-effect and assumed in the derivation of Eq. (2). This condition was complied with, as closely as possible, in the measurements reported in Table II. The same condition of independence could be realized, by avoiding space-charge limitation, in the case of photo-electric emission or of positive ions obtained by contact of an alkali vapor with a hot filament. When, on the other hand, the possibility of escape of an electron is limited, wholly or in part, by the repulsion of the electrons in the space (space-charge limitation), the theorems of unrestricted probability are no longer applicable, and Eq. (2) is not valid. The authors hope to deal with this problem in a future paper. For the present, in explanation of the measurements that follow, it is sufficient to point out that the effect of such electrostatic coupling between electrons is to reduce the amplitude of their relative motions. In the ideal case, if space-charge were to operate for a sufficiently long time, all relative motions would tend to disappear, and the electrons would move regularly spaced in a uniform stream. This condition is nearly realized in the circulating beam of electrons in the magnetron.[22]

[22] Hull, Paths of Electrons in the Magnetron (abstract in Phys. Rev. **23**, 112, 1924). The full paper will appear soon.

Preliminary measurements with two-electrode tubes showed no shot-effect when the current was limited by space-charge, so far as could be detected with the degree of amplification used for the previous (temperature-limited) measurements. Calculation showed that this was to be expected, on account of the very low a.c. resistance (5000 to 10,000 ohms) of a space-charge limited kenotron. This produced a *high* effective series resistance of the tuned circuit to which it was multiply connected, and so reduced the amplitude of oscillations produced in this circuit by the shot-effect. With increased amplification these shot-oscillations were easily detected, however, though still too small for convenient measurement.

By the use of a grid, on the other hand, it is possible to combine full space-charge limitation with any desired plate-resistance, by appropriate choice of grid mesh. Table III gives the results of measurements made in this way, using U.V. 199 radiotrons of different grid mesh. Two measurements were made with each tube, the first at low filament temperature, the second at normal temperature or higher (normal filament voltage = 3.0).

TABLE III

Effect of space-charge on shot-effect in three typical U. V. 199 radiotrons.

Constants of tube: a.c. plate resistance r_p	ampli-fication μ	Voltage across filament (volts)	Grid bias (volts)	Therm-ionic emission (m-amp.)	Effective resistance of circuit (ohms)	Shot-voltage obs. (μ-volts)	calc. (μ-volts)	obs./calc.
20000	6.0	1.9	−4.5	0.72	2.9	65.8	65.2	1.01
		3.0	−4.5	2.7	10.8	16.2	65.5	0.25
100000	29.5	2.5	0	0.97	3.84	28.0	65.7	0.43
		3.0	0	1.33	5.0	17.0	67.4	0.25
127000	31.	2.25	0	0.67	3.0	43.	61.7	0.70
		3.5	0	1.45	5.04	11.3	70.0	0.16

The first pair of measurements refer to a standard tube, operated at standard voltage (plate voltage 90, grid bias −4.5 volts). When the filament voltage is 1.9 (normal = 3.0 volts) the emission is only 1/4 its space-charge value and is limited almost entirely by temperature. Under these conditions, a normal shot-effect is produced, equal to the theoretical within the limits of experimental error. At normal filament voltage, on the other hand, with the current limited by space-charge, it is seen the shot-voltage is only 1/4 as large as that calculated by the above theory, which assumes that all the electrons are independent.

The other two tubes had much finer meshed grids, giving a higher plate resistance. With these tubes it was impossible to attain full temperature limitation, even at zero grid voltage, without reducing the plate current to too small a value for reliable measurements. The data obtained illustrate well the important fact which we wish especially to emphasize, namely, that when thermionic current is limited by space-charge, the shot-effect is only a small fraction of that to be expected from independently-moving electrons. In the transition region between full space-charge and full temperature limitation, the shot-voltage approaches more nearly to its theoretical value the more the current is depressed (in percent) below its space charge value. Thus with full space-charge, the shot-voltage is only 1/6 theoretical (last row, filament voltage 3.5); slight temperature limitation, corresponding to normal cathode temperature (filament voltage = 3.0), gives 1/4 theoretical shot-voltage; limitation by temperature to 2/3 space charge value (filament voltage = 2.5) gave 1/2 theoretical shot-voltage; limitation to 1/2 space charge (filament voltage = 2.25) gave 70 percent; and limitation to 1/4 space-charge value (filament voltage = 1.9) gave full theoretical shot-voltage within the accuracy of measurement.

TABLE IV

Variation of shot-effect with the filament temperature in a pliotron, showing the effect of space-charge. At the lower temperatures the emission current is limited by temperature, at high temperatures by space-charge.

Filament heating current (m-amp.)	Filament temperature	Thermionic emission (m-amp.)	Effective resistance of circuit (ohms)	Equivalent input voltage (μ-volts)	Amplification factor	Shot-voltage obs. (μ-volts)	Shot-voltage calc. (μ-volts)	Shot-voltage obs/calc.
140	1675°K	1.0	3.32	58.7	0.765	67.	71.7	0.93
150	1750	2.0	4.43	60.1	0.71	71.	87.7	0.82
152	1765	2.5	6.09	41.0	0.64	51.	83.8	0.61
160	1805	3.0	8.60	27.8	0.53	38.	77.2	0.49
167	1850	3.5	11.24	19.0	0.43	28.	73.0	0.39
170	1867	4.0	12.18	6.4	0.22	13.6	75.	0.18
182	1940	5.0	13.29	7.3	0.21	15.9	80.	0.20

Special U.V. 199 radiotron; plate potential = 130 volts; grid potential = −6 volts

This progressive decrease of shot-voltage in the transition region between temperature and space-charge limitation is still better exemplified in Table IV, which gives a series of measurements on the same tube at different cathode temperatures. The tube used was similar to the standard U.V. 199 radiotron in every respect except cathode diameter, which was double (normal heating current 0.200 ampere). Plate and grid voltage were kept constant at 130 and 6 volts respectively. As the cathode temperature was gradually raised, the shot-voltage fell from essentially full theoretical value to 18 percent of theoretical.

The data in Tables III and IV indicate that the measurement of shot-voltage may be a useful auxiliary weapon in attacking the problem of the part played by temperature and space-charge in the transition region. For example, it has been frequently noted that ordinary three-electrode tubes do not show "saturation" of the plate current as the cathode temperature is raised, and many explanations have been offered. The measurements in Tables III and IV indicate that space-charge is actually not complete until the cathode temperature is raised far above the value at which its emission is equal to the plate current actually used.

18. Separation of Primary and Secondary Electron Emission by Means of Shot-Effect

In Table V is given the result of a single measurement with a double-grid tube, in which the potentials were so adjusted that the current to the plate, which was connected to the tuned circuit, was zero, as indicated by any type of milliammeter. The circuit conditions were those

TABLE V

Shot-effect due to secondary electron emission in pliodynatron.

Thermionic emission from filament:	1.16	milliampere
Fraction of emitted electrons which pass through grid and strike plate:	65	percent
Current of primary electrons striking plate:	.755	milliampere
Current of secondary electrons leaving plate:	.755	milliampere
Net plate current:	0	
Effective circuit resistance:	2.22	ohms
Calculated shot-voltage		
due to primary electrons:	12.5	microvolts
due to secondary electrons:	76.6	microvolts
total (square root of sum of squares)	79.2	microvolts
Observed shot-voltage	79.0	microvolts

under which zero current is generally interpreted as the sum of two equal and opposite currents, of primary and secondary electrons. The circuit is shown in Fig. 8. It differs from that used in the other tests only in the interposition of a grid between control-grid and plate. This grid is maintained at a constant potential (130 volts) higher than that of the plate (25 volts), and its only function is to attract and carry away secondary electrons emitted by the plate.

If the zero plate current were due to deflection of the electrons by the grid wires so that none of them struck the plate, the shot-voltage would obviously be zero. The same would be true if electrons struck the plate but were reflected, as is generally assumed to take place at very low voltages. If, on the other hand, each impinging electron produces one or more secondary electrons, the escape of each of which *is a separate event*, having no exact time-connection with the impact which produced it, then the observed mean-square shot voltage should be the *sum* of the mean square voltages due to the primary and secondary electrons separately. In making the calculation allowance is to be made for the fact that the primary emission is limited by space-charge (see last section).

The calculation of shot-voltage in Table V was made on the latter hypothesis of true independent secondary emission. The agreement of the calculated and observed shot-voltage is evidence of the correctness of this hypothesis. The experiment shows that the mechanism of this secondary emission must be excitation by impact and subsequent evaporation, rather than direct splash. For a splash would give a direct time relation between impact and secondary emissions, so that the shot-effects of the two would nearly neutralize each other. The total shot-voltage would in that case be only a small fraction of the value observed.

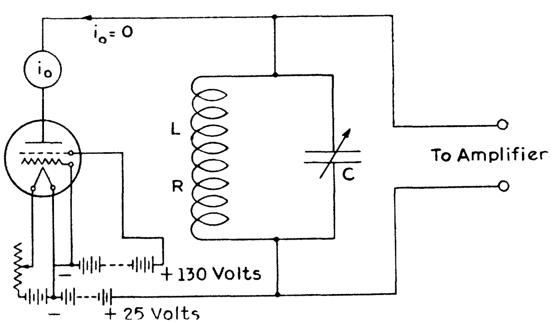

Fig. 8. Circuit for measuring shot-effect of secondary electron emission. Note that the current i_0, which produces the shot-effect, is zero. It is composed of two currents, equal and opposite in average value, but independent in phase, so that their shot-effects add together

The calculation in Table V was made in the following manner. The total current of electrons which passed through the control grid (space-charge limited) to outer grid and plate jointly was 1.16 milliamperes. Of these electrons approximately 65 percent strike the plate, according to calibration measurements recently made with positive ions by Mr. J. M. Hyatt of Union College (not yet published). Hence the primary current to the plate is .755 milliamperes. It is assumed, in accordance with the results in Tables III and IV, that this space-charge limited primary current produces only 1/6 its theoretical shot-voltage, viz., 12.5 microvolts, while the secondary current produces full shot-voltage (76.6 microvolts). The total shot-voltage to be expected is the square root of the sum of the squares of these two parts, viz., 79.2 microvolts. The observed value is 79.0 microvolts.

The closeness of the agreement of calculated and observed values is accidental, since the possible experimental error, including the fraction striking the plate and the effect of space-charge, may be as great as 5 or 6 percent. The experimental result does not exclude the possibility, therefore, of a small fraction of reflected electrons being mixed with the secondary electrons. This fraction cannot be more than a few percent, however. By greater care, this error can easily be reduced to less than one percent. Measurements of this kind offer a new and powerful means of discriminating between reflected and true secondary electrons, and of investigating the mechanism of secondary emission.

19. Discussion of Results

The foregoing measurements of shot-effect were mostly made at a single frequency (725 kilocycles). A few scattered measurements have been made over the narrow range from 600 to 1000 kilocycles, with exactly similar results. No evidence has been found of any dependence of the shot-effect upon frequency, except that involving the constants of the tuned circuit, as expressed in Eqs. (1), (13), etc. It is believed that the divergent results obtained by Hartmann are satisfactorily accounted for by the faulty method used for measuring mean-square shot-voltage, as pointed out by Fürth.

The accuracy of measurement of e by means of the shot-effect is limited only by the degree to which residual space-charge effects can be avoided. Photo-electric current (10 micro-amperes is sufficient) offers advantages in this respect, on account of the high initial velocities. All other errors in the measurements are purely instrumental and can be reduced without limit. Chief among them is deviation from a true "voltage square" law of detection, for upon this law we must rely for a faithful record of the mean square of the irregular shot-voltage. It would seem desirable on this account to substitute for the detector a thermocouple, transformer coupled to the last amplifier tube. Special care must be taken, in this case, to use for the last stage of the amplifier such a combination of tube characteristics, bias, and voltage, that the limits of linear amplification are not exceeded by any of the peaks of the shot-voltage.

A lower frequency than that used in these tests recommends itself for ease of manipulation. With too low a frequency, however, difficulty is to be anticipated in finding a detecting instrument with sufficiently long period (thermocouple with large heat capacity) to record accurately the *mean* of the varying shot-disturbance. It was observed in the present experiments that when the detecting apparatus was adjusted to the desired sensitivity, the needle of the detecting instrument (a Rawson unipivot micro-ammeter) showed continual, rapid fluctuations. These were clearly due to probability fluctuations of the shot-effect, for the needle showed a perfectly steady deflection when the shot excitation was replaced by an equal impressed voltage from a tube generator

Amplifier noises, whether due to gas, mechanical vibration, or battery fluctuation, if of constant mean value, will cause no error, since the detector adds the mean-square voltages, and the square of the shot-voltage may be reliably read as an added detected current.

An important possible source of error, which cannot be excluded a priori, is voltage fluctuation of the plate battery which supplies the shot-tube. The effect of such fluctuations, if present, would appear only when the shot-tube is conductive, hence can not be separated, like amplifier noise, by simple subtraction. Such fluctuations may be detected, however, and their effect measured quantitatively, by substituting for the shot-tube an ohmic resistance equal to the alternating current resistance of the tube. When this was done, using wire resistance, no traces of fluctuation were detected. It is worthy of mention that an India ink resistance (standard grid-leak) gave a fluctuation of the same order of magnitude as the shot-effect.

In conclusion, it seems appropriate to emphasize again the potential usefulness of this new implement of investigation, which is due to W. Schottky. We have already shown, by examples, that it provides a new means, more fundamental than any hitherto used, of differentiating between temperature and space-charge effects and between the reflection and secondary emission of electrons, and also of detecting and measuring equal and opposite currents whose resultant is zero. It is to be expected that other and still more important uses will be discovered.

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