THE CONNECTIONS BETWEEN THE FOUR TRANSVERSE GALVANOMAGNETIC AND THERMOMAGNETIC PHENOMENA

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Abstract

Three relations are deduced by reasoning of a general character between the four transverse effects, so that each of the effects may be expressed in terms of any other effect. The first relation, $Q=KP/T$, is obtained from energy considerations by assuming that the source of energy involved in the *Nernst effect* Q is provided by the *Ettingshausen effect* P, K being the thermal conductivity and T the absolute temperature. It is shown that the *Hall effect* R and the *Righi-Leduc effect* S each provides its own source of energy. The second relation, $Q=\sigma R/\rho$, where σ is the *Thomson coefficient* and ρ the specific resistance, is that of Moreau, and is obtained by assuming that the level surfaces of the Thomson e.m.f. in a metal carrying a thermal current are rotated by the Hall effect. The third relation, $P=S\sigma T/K$, involves the concept, probably new, of a temperature gradient generated by a heat current flowing down a difference of electrical potential. This new effect, which is the thermal analog of the Thomson effect, should be capable of experimental detection. It is proposed to call it the *Thomson temperature gradient*. The third relation is obtained by assuming that the isothermal lines associated with the Thomson temperature gradient are rotated by the Righi-Leduc effect. The experimental data available are not very satisfactory but they agree with the above relations within the experimental error.

In this note, I deduce by reasoning of a general character three relations connecting the four transverse effects. Of these three relations, one has already been proposed by Moreau; the other two have not been published as far as I can find, but one of them was independently deduced by Professor H. A. Lorentz at practically the same time as by me, both of us being stimulated by the approaching Solvay Conference on Conduction in Metals.

Although the reasoning by which the relations are deduced is of a general character, it is not completely general, but will impose certain restrictions on any mechanism proposed to give a detailed account of the effects. The general state of affairs is somewhat like that with regard to the application of thermodynamics to the thermo-electric circuit by Kelvin; thermodynamics strongly suggested but did not demand the relations found. The proposed relations will, therefore, need experimental verification. The experimental errors in measuring these effects are known to be very large, but I believe that within experimental error the proposed relations are satisfied.

It will be well first to review briefly the definitions of these effects.[1] All of them are transverse, that is, they take place in a plane perpendicular to a uniform magnetic field, and depend on the field. For simplicity we suppose the plane in the shape of a rectangle.

(1) *The Hall effect.* If an electric current passes lengthwise of the rectangle, then a transverse difference of electrical potential is generated, so that the current no longer flows perpendicular to the equipotential surfaces. The Hall coefficient R is defined by the equation

$$R = Ed \,/\, IH \,,$$

where E is the transverse potential difference, d the thickness of the plate parallel to H, I the total current, and H the magnetic field intensity. The coefficient is taken as positive if the electrical potential is raised on that side of the plate on which the Amperean current generating the magnetic field has the same direction as the current I.

(2) *The Ettingshausen effect.* When an electric current flows lengthwise of the rectangle, a transverse temperature difference is generated in the magnetic field. The Ettingshausen coefficient P is defined by the equation

$$P = \Delta T \times d \,/\, IH \,,$$

where ΔT is the transverse temperature difference. It is positive by convention if the plate becomes warmer on that side where I and the Amperean current have the same direction.

(3) *The Nernst effect.* If a heat current flows lengthwise of the rectangle, a transverse electrical potential difference is generated in the magnetic field. The Nernst coefficient Q is defined by the equation

$$Q = EKd \,/\, WH \,,$$

where K is the thermal conductivity, and W the total amount of heat flowing through the cross section of the plate per second. By convention Q is positive if the electrical potential is raised on that side of the plate where the Amperean current and the heat current have the same direction.

(4) *The Righi-Leduc effect.* If a heat current flows lengthwise of the plate, a transverse difference of temperature is established in the magnetic field. The Righi-Leduc coefficient S is defined by the equation

$$S = \Delta T \times Kd \,/\, WH$$

[1] The notation, definitions, and the source of the numerical data used in the following, when not explicitly given otherwise, will all be found in the recent very comprehensive book on this subject by L. L. Campbell, entitled Galvanomagnetic and Thermomagnetic Effects, published by Longmans, Green and Co., 1923.

S is positive by convention if the plate is warmer on that side on which the Amperean current and the heat current are in the same direction.

These four effects are thus seen to involve either a difference of electrical or thermal potential. Such differences may serve as sources of energy, by allowing either an electrical current to flow between the points having a difference of electrical potential or a heat current between the points having a difference of temperature. One connection which we are seeking between the effects may be found by inquiring what is the source of energy of these possible transverse currents.

We consider first the Hall effect, because it indicates most simply the character of the relations to be assumed, although it does not immediately yield any new information. In a metal, which is the source of a Hall e.m.f. under a current I, allow a transverse current i to flow. The transverse e.m.f. is RIH/d, so that the work per unit time extracted by i is $RIHi/d$. The source of this energy is to be found in the longitudinal Hall e.m.f. set up by the transverse current i. The amount of this e.m.f. is RiH/d, and the current which flows through this e.m.f. is I, making a total additional energy development by I per unit time equal to $RiHI/d$, the amount in question. An examination of the signs shows that they are as they should be. Thus we see that the Hall effect supplies its own source of energy.

Next consider the source of energy in the Nernst effect. The longitudinal heat current W generates a transverse electrical potential difference QWH/Kd, and if an electric current i flows across the plate, the energy it receives per unit time is $QWHi/Kd$. Following the hint given by the Hall effect, we seek for the source of this energy in the Ettingshausen effect of the current i. The transverse current i gives rise to a longitudinal temperature difference PiH/d, and the work which the heat current W does in flowing through this temperature difference is, by the second law of thermodynamics, $W(PiH/d)/T$, where T is the absolute temperature. Equating these two amounts of work we get

$$Q = KP / T , \tag{1}$$

which is the first of the equations of connection desired. An examination of the signs shows that they are as they should be. This was the equation also found by Professor Lorentz.

If this relation should prove to be correct it would mean that any proposed mechanism must be such that a transverse electric current, in virtue of the longitudinal difference of temperature created by it, may extract energy reversibly and in the thermodynamic amount from the primary heat current (*not* from the miscellaneous heat energy of the surroundings).

The source of energy involved in the Ettingshausen effect is, of course, by the inverse of the above, the Nernst effect, so that there is no new information here. Further, the Righi-Leduc effect provides its own source of energy, like the Hall effect, and similarly yields no information.

Of the two remaining relations, we give first that of Moreau. The guiding idea is that the essence of the Hall effect consists in a definite angular rotation of the electrical equi-potential lines in the magnetic field, irrespective of the origin of the equi-potential lines.

The angle of rotation of the lines may easily be shown to be RH/ρ, where ρ is the specific electrical resistance of the metal.

Consider the plate of the Nernst effect, carrying a longitudinal heat current. The longitudinal temperature gradient θ driving this heat flow is

$$\theta = W / Kbd ,$$

where b is the breadth of the plate. Now this longitudinal temperature gradient is associated with a longitudinal electrical gradient determined by the ordinary Thomson coefficient σ. This electrical potential gradient is equal to $\sigma\theta$. The connection is obtained as follows. If the current I flows between two points unit distance apart in a metal in which there is a temperature gradient θ, then the extra work of the current is $\sigma\theta I$, by definition of σ; hence the effective potential difference is $\sigma\theta$. The longitudinal electrical potential gradient in the Nernst plate is therefore $\sigma\theta = W\sigma/Kbd$. Now the equipotential lines associated with this gradient are rotated through the angle RH/ρ by the Hall effect, giving rise to a transverse electrical potential difference $(W\sigma/Kbd)$ $(RH/\rho)b$. But the transverse electrical potential difference generated in this way is nothing but the Nernst potential difference for which we have the expression QWH/Kd. Equating these two, we have

$$Q = \sigma R / \rho , \tag{2}$$

which is the relation of Moreau.

The third relation is to be obtained by considerations analogous to the above but involves (1) the introduction of a concept which is, so far as I know, unfamiliar, namely that of a longitudinal temperature gradient which is generated by a flow of heat down an electric potential gradient, and (2) a somewhat modified description of the Thomson effect.

Consider a metal in which there is a flow of heat under a thermal gradient and in the same direction a flow of electricity under an electrical potential gradient. There are involved here two irreversible processes, namely flow of heat against the thermal resistance of the metal, and flow of electricity against the electrical resistance; these two processes demand temperature and electrical gradients proportional to the temperature and electrical resistances. But in addition there is a reversible transfer of energy between the thermal and electrical currents. Now the only thing which can feed energy to an electrical current is an e.m.f. or difference of potential, and the only way by which a heat current can deliver work is by flowing through a difference of temperature. Hence superposed on the thermal and electrical potential differences which drive the current against resistance, there must be additional temperature and electrical potential differences corresponding to the reversible transfer. The electrical potential difference is well known, and is merely the ordinary Thomson e.m.f. The corresponding additional temperature gradient seems not to be usually considered. I propose to call it the *Thomson temperature gradient*, and denote it by the letter σ'. The definitions of the two coefficients are entirely analogous.

Electrical energy per unit time in unit length $= I\Delta E' = I\sigma\Delta T$.

Thermal energy per unit time in unit length $= W\,\Delta T'/T = W\sigma'\Delta E$.

Here ΔE and ΔT are the principal electrical potential and temperature differences that drive the currents against resistance, and $\Delta E'$ and $\Delta T'$ are the small superposed differences involved in the reversible effects. These two energies are to be put equal. Now $W = K\Delta Tbd$, and $I = \Delta Ebd/\rho$ Substituting, we get

$$\sigma' = \sigma/K\rho,$$

and

$$\Delta T' = (\sigma T/K\rho) \times \text{elec. potential diff.}$$

We digress for a moment to remark that the difference between this and the ordinary statement of the Thomson effect is in the more detailed account of the source of the energy absorbed by the electrical current in flowing through a temperature gradient. The source is usually supposed to be merely the miscellaneous heat energy of the surroundings; I have supposed it to be the primary heat current delivering energy reversibly (as in any thermodynamic engine) through a small superposed temperature gradient. This Thomson temperature gradient should be capable of experimental detection, and I propose to search for it. There are interesting questions here: Is there any surface temperature phenomenon corresponding to the supposed electrical double layers? Is there for temperature phenomena anything corresponding to the distinction between an electrical potential difference and an impressed e.m.f.? In this paper I have used these last two terms loosely, but I think without obscuring the meaning.

Returning now to the main argument, we postulate that the essence of the Righi-Leduc effect is a rotation by the magnetic field of the isothermal lines by a definite amount irrespective of their origin. This rotation may be easily shown to be SH. Consider now an electric current flowing longitudinally in a plate. In virtue of the electrical potential gradient $I\rho/bd$ which drives the current, there is set up a longitudinal Thomson temperature gradient of amount $(\sigma T/K\rho)$ $(I\rho/bd)$. The corresponding isothermal lines are rotated through the angle SH, giving a transverse temperature difference $(\sigma T/K\rho)$ $(I\rho/bd)\cdot SH\cdot b$. But this transverse temperature difference is nothing but the Ettingshausen temperature difference, for which we have the expression PIH/d. Equating these gives·

$$P = S\sigma T/K \tag{3}$$

which is the last of the relations required.

These relations may be written in various forms. Thus we may express all the coefficients in terms of the Hall coefficients

$$P = (T\sigma/K\rho)R \tag{4}$$

$$Q = (\sigma/\rho)R \tag{5}$$

$$S = (1/\rho)R. \tag{6}$$

Or we may get an important relation between all four effects not involving σ:

$$PR/QS = \rho T/K. \tag{7}$$

Equation (6) above is merely the statement that the angle of rotation of the isothermal lines is the same as that of the electrical equipotentials. This relation has been previously suggested (Campbell, page 238) but I believe on purely empirical grounds, with no theoretical basis.

When it comes to the experimental verification, we are very much embarrassed by the absence of sufficient data. The effects are very difficult to measure, not only because of their smallness, but because the great numbers of secondary effects are difficult to eliminate. Observers seldom agree and may differ even as to the sign. The effects depend greatly on the state of purity and mechanical working of the specimen. All of the quantities involved in the relations should be measured on the same specimens, and even then we have no security, because the secondary effects may not have been eliminated or the metal may be crystalline in character with different properties in different directions.

There is a very complete collection of numerical results in Campbell's book. From this I have selected the data which are adapted to comparison, consisting mostly of measurements by Unwin of the four effects on the same samples.

The results are shown in Table I, in which are given, at a temperature roughly 300° K, the four coefficients R, P, Q, and S, together with K and ρ. The electrical quantities are in absolute e.m.u. From these data KP/T is calculated, which by Eq. (1) should be the same as Q, and the combinations PR/QS and $\rho T/K$, which should be equal to each other.

TABLE I

Metal	R	P	Q	S	K
	$(\times 10^{-4})$	$(\times 10^{-9})$	$(\times 10^{-4})$	$(\times 10^{-7})$	$(\times 10^{7})$
Ag.......	−8.4	−1.65	−1.8	−2.7	4.2
Al........	−4.0	+1.06	+.42	−0.62	2.1
Cd.......	+8.8	−2.9	−1.2	+0.89	.93
Co........	+24.5	+21.6	+7.8	+1.1	.60
Cu........	−5.5	−1.6	−1.9	−2.1	3.8
Fe........	+87.	−42.6	−9.5	+5.2	.60
Ni........	−29.	+30.3	+10.	−2.5	.60
Zn........	+7.6	−2.67	−.73	+1.1	1.11
Au*.......	−6.5	−.96	−1.7	−2.5	2.9
Sb†.......	+2190.	+1940.	+176.	+20.1	.167
Bi†.......	−63300.	+35000.	+1780.	−20.5	.081

Metal	ρ	KP/T	$\rho T/K$	PR/QS
	$(\times 10^{3})$	$(\times 10^{-4})$	$(\times 10^{-3})$	$(\times 10^{-3})$
Ag	1.66	−2.2	12	28
Al.........	2.9	+.74	41	163
Cd	7.5	−.90	240	240
Co	9.7	+4.3	480	620
Cu	1.8	−2.0	14	22
Fe.........	12.0	−8.5	600	750
Ni.........	11.8	+6.1	590	350
Zn.........	6.1	−.99	165	250
Au*........	2.42	−.93	25	15
Sb†........	40.5	+110.	7,250	12,000
Bi†	160.	+950.	59,400	500,000

* From unpublished data of Professor E. H. Hall. See the forthcoming Solvay Conference Report.

† From data by Zahn, given in Campbell's book.[1]

The agreement must, I believe, be considered within experimental error in all cases, except possibly that of bismuth, which we are probably justified in neglecting because of the strongly crystalline character of the metal and the fact that the coefficients depend in a great degree on the strength of the magnetic field.

The two relations checked in the Table have been so chosen that the Thomson coefficient σ does not appear. In checking any third relation, into which σ must enter, we are badly off, because σ varies from specimen to specimen and was not determined for the specimens above.

The best data for this are probably to be found in Moreau's original papers (page 228 of Campbell). The following table is reproduced from Campbell, using Moreau's redetermined Q for cobalt.

TABLE II

Metal	Q(obs.)	Q(calc.)
Bi	+0.196	+0.149
Sb	+ .0094	+ .0090
Ni	+ .0073	+ .0026
Co	− .00146	− .00175
Fe	− .00156	− .00156
Steel	− .00060	− .00062
Cu	− .000073	− .000084
Zn	− .000054	− .000046

The agreement is certainly all that could be asked. The extreme variability of the coefficients in different samples may be seen by comparing Moreau's values for Q with those given in the table above.

Of the metals in Table I it is probable that Ag, Au, and Cu are those on which different observers might be expected to agree most closely. I have determined σ for these metals.[2] If we use my values for σ and calculate $\sigma R/\rho$ which is equal to Q by Moreau's formula, we shall find -1.3×10^{-4}, $-.75\times10^{-4}$, and $-.90\times10^{-4}$ against the respectively observed values of Q -1.8×10^{-4}, -1.7×10^{-4}, and -1.9×10^{-4}. The agreement is probably all that can be expected.

In view of the extreme uncertainty of the experimental evidence and the plausibility of the theoretical argument, I believe that we must consider the theoretical relations as verified, and in fact we can probably now calculate the more difficult of these effects from the easier ones with a result probably more correct than can be obtained by direct experiment.

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[2] P. W. Bridgman, Proc. Amer. Acad. **53**, 269-386, (1918)