# HALL EFFECT IN A CONDUCTOR DUE TO ITS OWN MAGNETIC FIELD

By W. van B. Roberts


## Abstract

If there is at each point in a conductor an electric intensity proportional to the vector product of the magnetic field and current density, there should be a difference of potential between points at different distances from the axis of a cylindrical conductor carrying a current. Rather than attempt to measure this, a qualitative confirmation of the existence of the effect was sought by use of a cylindrical specimen of bismuth having a long section of small diameter (3 to 4.8 mm) between two larger end sections (12.5 mm). When 60 cycle alternating current was sent through the specimen, the mean potential differences between the surface of the small section at the point $M$ and two points $A$ and $B$ on the surface of the larger sections at each end, was measured by means of a tuned electro-dynamometer connected from $M$ to the balance point on a semi-circular conductor connecting $A$ and $B$, at which the 60 cycle potential differences were balanced out. This Hall effect has a double frequency because it does not reverse with the current. The resulting deflections were from .1 to .3 times the theoretical upper limit, therefore in good qualitative agreement. That they were not due to thermo-electric effects was proved by the absence of the effect when an alloy with zero Hall coefficient was substituted, as well as by other tests. It is concluded that the Hall effect is always present in wires carrying currents and should cause a slight increase of resistance at high current densities.


## Theory

Both the Hall effect and the Corbino effect are consistent with the following hypothesis: At any point in a conductor there exists an electric force $e_H$ which is proportional to the resistivity $\rho$ of the material multiplied by the vector product of the current density $I$ and magnetic field $H$ at that point. Thus the Hall field is at right angles both to the lines of flow of current and to the magnetic field. Expressed as an equation,

$$e_H = c\rho[IH] \ . \qquad (1)^*$$

The question naturally arises, is there not some Hall effect in every wire carrying a current, due to the magnetic field of the current itself, or is it only an additional magnetic field that produces the effect? A very simple imaginary experiment indicates the former. Imagine a long cylindrical conductor divided up into a great number of long thin wedges so that the cross-section would be as shown in Fig. 1. If each wedge be insulated from every other by a negligibly thin space, the magnetic

* The constant of proportionality $c$ is not always absolutely constant. In bismuth, for example, the value of $c$ is less in very strong magnetic fields than in weak ones.

field produced when the ends of the wedges are connected to the same battery, will be a family of circles. Now, pick out any one wedge. Because of the magnetic field produced by the rest of the wedges there will be a potential difference between the edge of the wedge and the part on the circumference. By symmetry, this must be true of all the wedges. Hence the circumference of the conductor is an equipotential, but its potential is different from the potential of the point where the axis cuts its plane. As there should be no reason for the current to travel in a spiral along the conductor, it should make no difference whether the wedges are insulated from each other or only divided in the imagination. However, it was thought worth while checking this conclusion experimentally.

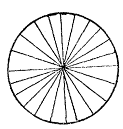
Fig. 1

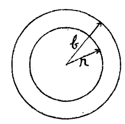
Fig. 2

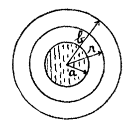
Fig. 3

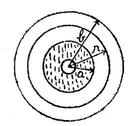
Fig. 4

The potential differences to be expected in a number of different cases, are as follows: (practical units, amperes, volts, gauss, etc., are used throughout)

Case I. Solid cylindrical conductor. Fig. 2. At a distance $r$ from the axis,

$$H=(2C/10r)\ (r^2/b^2)$$

where $C$ is the total current and $b$ is the radius of the cylinder. Substituting in Eq. (1),

$$e_H=c\rho I(C/10)\ (2r/b^2) \tag{2}$$

The potential difference between axis and circumference will then be the integral of $e_H dr$ from 0 to $b$, or $V_H=c\rho IC/10$. Now $\rho I$ is the potential gradient along the conductor, so $\rho IC$ is the number of watts dissipated in heat per cm length of the conductor, which we will call $W$. Thus finally, $V_H=cW/10$.

Case II. Hollow cylindrical conductor, inner radius $a$ (Fig. 3). In this case $H=(2C/10r)\ (r^2-a^2)/(b^2-a^2)$, so

$$V_H=(cW/10)\left\{1-2\left[a^2/(b^2-a^2)\right]\log_e(b/a)\right\}\ .$$

The function in brackets depends only upon the ratio $a/b$ and to a rough approximation falls linearly from the value 1 that it assumes when $a/b=0$ to zero when $a/b=1$. Fig. 5A shows a graph of this function.

Case III. Hollow cylindrical conductor, with return current inside and coaxial, to avoid incalculable effect of return current. (Fig. 5) In this case $H=(2C/10r)-(2C/10r)(r^2-a^2)/(b^2-a^2)$ and the potential difference between inside and outside of the pipe will be

$$(cW/10)\{2\ [b^2/(b^2-a^2)]\log_e(b/a)-1\}\ .$$

The bracketed function is plotted in Fig. 5B. It increases rapidly as $a/b$ becomes small, but $a/b$ is limited by the necessity for a finite size for the return conductor.

Case IV. Conductor of varying cross-section. (Fig. 6) By taking the line integral of $e_H$ around the small element formed by adjacent stream lines and the orthogonals, we find curl $e_H=\partial/\partial s(c\rho IH)+\psi c\rho IH$ where $\psi$ is the curvature of the orthogonal to the stream lines. Hence there will be small circulating currents superposed upon the normal current. If the

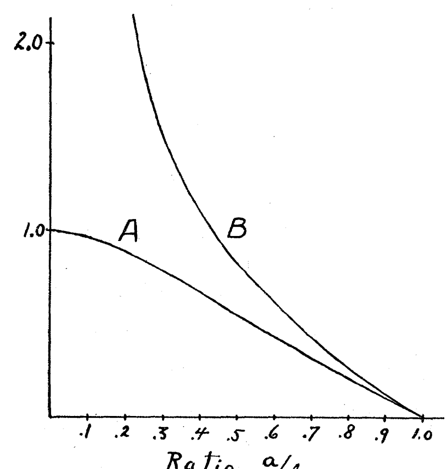


Fig. 5

intensity of the superposed current is $i$, then we have curl $(\rho i)=$curl $e_H$. Integrating curl $e_H$ from the axis to the surface, between sections of the conductor which have stream lines approximately parallel, we obtain $c_2W_2/10-c_1W_1/10$. Referring to Fig. 6, $c_1$ and $W_1$ are the values in the narrower section, and $c_2$ and $W_2$ the values in the thicker section. The integration is done in two steps. First integrating along a stream line,

$$\int \text{curl } e_H du\, ds=\int[(\partial/\partial s)c\rho IH+\psi c\rho IH]duds=\int[\delta(c\rho IH)]du$$
$$+\int(c\rho IH)\delta du=\int\delta(c\rho IHdu)=c_2\rho_2I_2H_2du_2-c_1\rho_1I_1H_1du_1\ ,$$

since $\psi\, duds=\delta\, du$, the change in $du$ in the distance $ds$. The values are those obtaining on the stream line at the limits of integration. Then making use of the fact that $u$ may be replaced by $r$ where the stream lines are parallel we have $H=(2C/10r)$ $(r^2/b^2)$, and integrating with respect to $r$ ($c$ and $\rho$ being supposed constant across the cylindrical sections) we get $c_2\rho_2 I_2 C/10$ or $c_2 W_2/10$. Likewise, at the other section we get $-c_1 W_1/10$. This result is the same as the line integral taken by the use of the formula of Case I, going along the axis (which contributes nothing, being a stream line), then out radially through the thicker section (which contributes $c_2 W_2/10$ by Case I), then along the surface of the conductor (which contributes nothing, being a stream line), then inward radially to the axis through the narrow section (which contributes $-c_1 W_1/10$ by Case I).

Thus, although it might appear that the Hall effect should not cause any alteration of the relative potentials of points $A$ and $B$ in Fig. 6 because the line integral of $e_H$ along the surface from $A$ to $B$ is zero, yet there will be an alteration due indirectly to the Hall effect, because of the resistance drop along that path of the circulating current that must flow in response to the existence of the closed line integral of $e_H$. The exact proportion of the total voltage (developed by Hall effect) that is available by tapping at $A$ and $B$ is not easy to calculate.

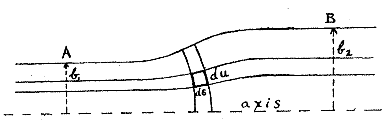


Fig. 6

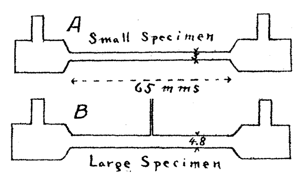


Fig. 7

## Outline of Experiment

After a careful consideration of the dimensions of the cylinders that would have to be used if a quantitative measurement using the arrangements of Cases I, II or III were to be tried, and of the chances of the cooling system being adequate to prevent thermo-electric currents that would be difficult to separate from the Hall effect, it was decided that it would be more feasible to seek only the qualitative confirmation of the theory that could comparatively easily be obtained from a modification of the arrangement of Fig. 6.

Bismuth was chosen to work with on account of its large Hall coefficient.[1] A specimen was cast in a graphite mould, 3 mm in diameter and 65 mm long, with sections 12.5 mm in diameter at each end, provided with projections so that contact could be made where the junction would not be heated. Fig. 7 shows the specimen to scale. The current was led in and out of the bismuth by copper wires 3 mm in diameter. A

[1] Albert K. Chapman, Phil. Mag., **32** (Sept. 1916)

hole was drilled in each end of the specimen, and the end of the copper was tinned and heated and pushed into the hole, the hot solder readily alloying with the bismuth. Connection was made to the projecting portions in a similar manner. Current was supplied by a 30 to 1 ratio step-down transformer. Alternating current was used to discourage thermo-electric effects as much as possible. As they both depend upon the instantaneous value of $W$, the Hall effect and thermo-electric currents both have double frequency components, but the latter should show mostly the constant component, while the former should show just as great a double frequency as the constant component.

The schematic arrangement is shown in Fig. 8. The field due to the semicircle (of radius 60 cm) is fairly well compensated for by proper choice of the resistance of the smaller semicircular shunt (about 3.5 cm radius). The instrument indicated by the arrow is capable of measuring alternating voltages of either 60 cycles or 120 cycles exclusively. The sliding contact $S$ is adjusted to a position where no 60-cycle current flows. This is also the proper position to balance out any other frequencies

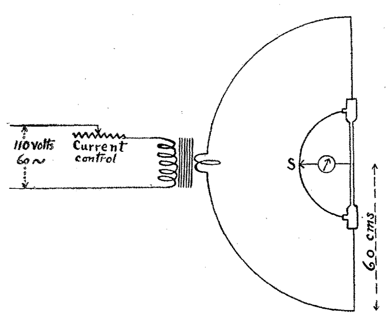


Fig. 8

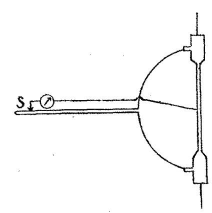


Fig. 9

that may be supplied by the transformer. If, however, Hall effect is present, currents of both zero and 120 cycles will flow through the instrument, being independent of the direction of the exciting current. A sheet of water from a fan-shaped nozzle was kept on the bismuth to keep down thermo-electric currents and prevent unbalancing at $S$ due to unequal heating of the two halves of the bismuth. The adjustment at $S$ is so critical that instead of a small semicircle, the arrangement of Fig. 9 was used. Thus, there was about 6 feet of wire for the slider to work on, and even then an adjustment to better than 1 mm was necessary.

Another difficulty lay in currents induced in the circuit of the detecting instrument by the magnetic fields due to the iron of the transformer and to the main current. These were eliminated by connecting two coils of wire (one of five turns, the other two turns, both of 3″ diameter) in series with the detecting instrument. The five-turn coil was placed about a foot from the transformer and was adjusted so as to balance out its field when the secondary was open-circuited. The other was placed further from the transformer but near the main current, and could be adjusted so as to eliminate induction due to the current. The procedure was as follows: First the detecting arrangement was fixed to detect a 60-cycle frequency only, and the deflection reduced to zero by adjustment of the slider $S$ and the two compensating coils. Then the detector was made sensitive only to 120 cycles and the deflections observed and plotted against watts per cm. The deflections were of the predicted order of magnitude. To be sure that these deflections were really due to Hall effect, three checks were employed. (1) Runs were made with a rapidly flowing stream of water and then with a much slower stream. The results were so nearly the same that thermo-electric currents could not be responsible for any very large proportion of the deflections. (2) The leads to the amplifier were reversed right at the amplifier input. This reversed the sign of the deflection, which would not have been the case if the 120-cycle current had been generated inside the amplifier as an harmonic of a 60-cycle input. (3) A duplicate specimen was cast of bismuth alloyed with 2 percent tin. This alloy has approximately zero Hall coefficient.[2] Replacing the pure bismuth by this specimen, everything else being unchanged, observations were taken as before, but this time no deflections of the same order of magnitude were observed. Hence, it seems certain that the previously observed deflections were really those predicted by the hypothesis $e_H = c\rho[IH]$.

To make doubly sure, the experiment was repeated with slight modifications. Again duplicate specimens were used, one of c.p. bismuth and the other of 2 percent tin alloy, but this time the contact to the center of the narrow part was not the phosphor-bronze knife edge previously used, but a thin rod of bismuth less than 1 mm in diameter cast integral, as shown in Fig. 7B. The diameter of the specimen being increased to 4.8 mm, this projection should disturb the stream lines very little, and since all junctions of dissimilar metals are well away from the main current, they can be kept at the same temperature. The other difference is that the detecting instrument was left adjusted to detect only 120 cycles throughout, the balance for 60 cycles being more quickly and easily got by means of a telephone receiver. Probably due to unequal cooling effect on the two halves of the bismuth rod, this balance was

[2] Albert K. Chapman, loc. cit.[1]

not always quite independent of current. So a balance was made by ear (without looking at the deflection) after each change in current, and then the deflection was read. Again the pure bismuth sample gave deflections of the expected magnitude, while the alloy gave none to speak of.

Sample observations with both small and large specimens are given in Tables I and II and in Fig. 10.

TABLE I

*Sample observations on small size specimens*[2]

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Current in amperes | | | | | | | |
| bismuth specimen: | 22 | 32 | 40 | 48 | 56 | 64 | 72 |
| alloy specimen: | not much less than for bismuth | | | | | | |
| Heat developed in watts/cm[3] | | | | | | | |
| bismuth specimen: | .035 | .09 | .15 | .23 | .32 | .43 | .65 |
| alloy specimen: | greater than for bismuth | | | | | | |
| Deflections in cm | | | | | | | |
| bismuth, water running slowly: | .04 | .10 | .17 | .24 | .34 | .45 | .61 |
| alloy specimen: | .00 | .10 | .07 | .03 | .1 | .1 | .1 |

TABLE II

*Sample observations with large size specimens*[4]

| | current (amp) | heat (watts/cm) | deflection (cm) | reversed deflection[5] | upper limit predicted |
|---|---|---|---|---|---|
| bismuth specimen (resistance .0008 ohms/cm) | 20 | .32 | .09 | .0 cm | .16 |
| | 34 | .92 | .15 | −.06 | .46 |
| | 43 | 1.46 | .21 | −.12 | .73 |
| | 53 | 2.24 | .29 | −.20 | 1.12 |
| | 61 | 2.97 | .35 | −.24 | 1.48 |
| | 64 | 3.27 | .48 | −.28 | 1.63 |
| | 80 | 5.22 | .69 | −.52 | 2.61 |
| | 96 | 7.4 | .95 | −.75 | 3.7 |
| | 117 | 10.5 | 1.40 | −1.05 | 5.2 |
| alloy specimen (resistance .0016 ohm/cm) | 58 | 5.5 | .01 | | |
| | 61 | 6.0 | .01 | | |
| | 64 | 6.5 | .02 | | |
| | 69 | 7.6 | .02 | | |

## DETECTING ARRANGEMENT

The deflection instrument used was a Leeds and Northrup electrodynamometer which gave a deflection of 2.5 mm when 1 milampere was put through both coils in series. To measure 120-cycle currents, 100 milamperes of 120-cycle current were supplied to the fixed coil, and the

[2] These readings were made with a single balancing out of the 60 cycle current at the beginning of each run.
[3] The resistance of bismuth specimen was about .002 ohm/cm, while that of alloy specimen was about twice as much.
[4] Each reading was taken after a 60 cycle balance had been made by ear.
[5] Deflection when input to amplifier is reversed.

current to be measured was put through the moving coil. As a steady torque would not result unless the two currents were absolutely synchronous, it was necessary somehow to use the second harmonic of the 60-cycle source that supplied the main current through the bismuth. This was obtained by reversing the function of the "push-pull" vacuum-tube amplifier. This is shown in the schematic circuit of Fig. 11.

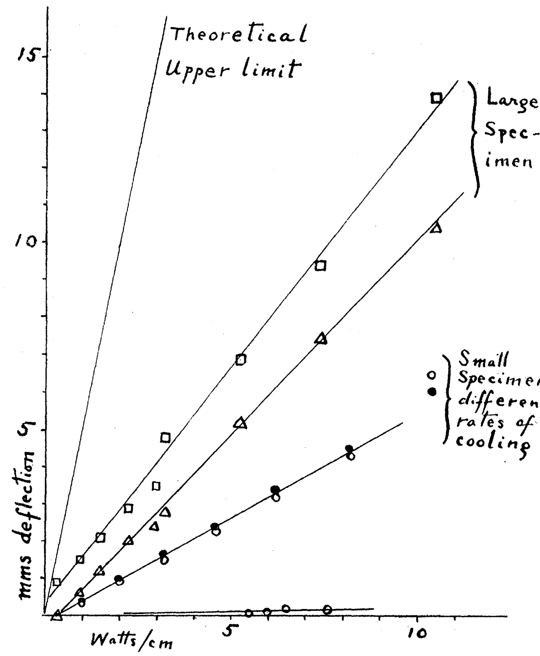


Fig. 10

Now in addition to being synchronous, the two currents in the electrodynamometer coils should be in phase. Considering the phase of the current in the moving coil as fixed, the other current could be brought into phase by shifting the phase of the input voltage to the circuit of

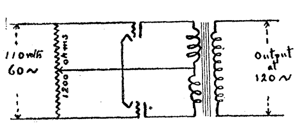


Fig. 11

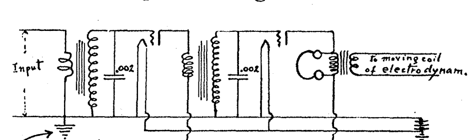


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Fig. 12. A simpler process is to have two phases 45° apart available for this input. A 45° shift of the 60-cycle input causes a 90° shift in the 120-cycle output as will be proved below. By measuring the two deflec-

tions obtained with the two stationary coil currents 90° different from each other, we deduce the deflection that would be observed if the currents in the two coils were in phase, by taking the square root of the sum of the squares of the two deflections.

Returning to analyze the action of the 120-cycle generator, assume that the plate circuit current of a vacuum tube is represented by a power series of the grid potential variations. On one grid we impress sin $pt$ and on the other $-\sin pt$, so the plate circuit currents are $\sin pt+\sin^2 pt+\ldots$ etc. and $-\sin pt+\sin^2 pt- \ . \ . \ .$ etc. and the flux in the iron core of the output transformer will contain only the even harmonics, the odd ones being cancelled out by the similarity of the primary windings. The sliding contact on the high resistance across the input allows this balance to be effected. When balanced, 100 milamperes at 120 cycles can be put through the stationary coil and considerable current at 60 cycles through the moving coil, without causing any deflection. Suppose now that the phase of the input is shifted an angle $\varphi$. The input is then $\sin(pt+\varphi)$ and $\sin^2(pt+\varphi)$ becomes a constant$+\cos(2pt+2\varphi)$, showing that the phase of the second harmonic advances twice as fast as the phase of the input. The complete circuit of the 120-cycle generator is shown in Fig. 13, which

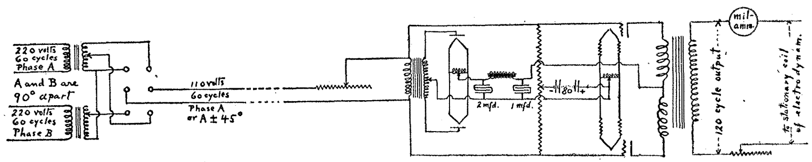


Fig. 13

includes the arrangement for phase shifting. The windings that light the filaments are low tension windings on the same core as the step-up winding that supplies the high tension. This transformer is the Radio Corporation of America model UP-1368 and the choke is their model UP-1626, and the rectifier tubes are their UV-216. The amplifier tubes are Western Electric VT-2 or E tubes, with suitable resistances in the filament leads. The output transformer is a Western Electric W-230, 6,000:50.

The sensitivity of the electro-dynamometer not being sufficient, an amplifier was used ahead of it. Two cheap audio-frequency interstage transformers were used, one as input and one as output. Only the secondary, which is designed to work into a vacuum tube, was used for the input transformer, a special primary of 45 turns with midtap being used to come nearer to matching the impedance of the circuit attached to it. The output transformer, on the other hand, has a special secondary wound with the intention of matching the impedance of the moving coil. However, there was not room to get enough turns on to do this, and no attempt was made to overcome this imperfection, as the amplifier was sufficiently sensitive anyway. The circuit is shown in Fig. 12. The middle transformer is an "Amertran." The tubes are Cunningham 301-A's. The small $.002\mu f$ condensers helped stabilize the amplifier and increased the amplification of low frequencies.

The calibration of the detecting system was made by passing 50 milamperes of 120-cycle current through a standard .01 ohm resistance. The input of the amplifier was connected across this, so that a known voltage of $5\times 10^{-4}$ volt was applied. The electro-dynamometer coils were connected, in *parallel*, to the output of the amplifier, and a 4 mm deflection resulted. As the impedance of the moving coil was 17 ohms and that of the stationary coil 32 ohms (both nearly pure resistance), and as 1 milampere in each coil gives 2.5 mm deflection, we can calculate the currents that give 4 mm. Call the stator current $i_s$ and the rotor current $i_r$. Then $32\ i_s = 17\ i_r$ and $i_s \times i_r = 4/2.5$. These equations give $i_s = .92$ and $i_r = 1.73$, approximately.

Now if only the moving coil had been connected to the output transformer, it would have received not less than 1.73 milamperes and not more than 2.65, according to the impedance of the transformer. Taking 2 as a tentative value, we summarize by saying that $5\times 10^{-4}$ volt input gives 2 milamperes output. Now we know that we are actually using 100 milamperes in the stationary coil instead of .92, so the deflection would be $(100/.92)\times 4$ or 430 mm. Hence, a deflection of 1 mm corresponds to $5\times 10^{-4}/430$ or $1.16\times 10^{-6}$ volt input under actual operating conditions.

That this is enough sensitivity to detect the Hall effect in bismuth, is seen by putting in the value of $c$ which is in the neighborhood of $7\times 10^{-5}$ for weak fields. The total generated Hall voltage is thus about $.7\times 7\times 10^{-5}\times (W_2 - W_1)/10$ or $5\times (W_2 - W_1)\times 10^{-6}$ volt (effective value), which might give a barely observable deflection for $W_2 - W_1 = 1/10$ and could give 25 mm for $W_2 - W_1 = 5$. These are, of course, upper limits, as only a fraction of the generated voltage can be picked off the outside of the specimen.

Before leaving the subject of the electro-dynamometer, it should be mentioned that the coils had to be connected together to prevent deflections due to static electrical charges, and had to be adjusted exactly at right angles, or current in the stator induced current in the rotor, tending to pull it to right angles. This same effect introduces a restoring force that adds to the restoring force due to the twist of the suspension. Since the sensitivity of the amplifier was measured in the absence of this effect, the currents in both coils being small, some correction must be made. Experiment shows that the restoring torque due to 100 milamperes in the stator is only one-tenth that of the suspension. The correction is most easily made by changing our statement that 1 mm deflection $=1.16\times10^{-6}$ volt input, to 1 mm $=1.05\times10^{-6}$ volt input.

## Discussion of Results

It will be noticed that the small specimen gave less deflection per watt than the large one. This may possibly be because connection to the amplifier had less effect on reducing the potential differences in the case of the larger lower resistance specimen, or it may be because the supply of bismuth gave out and the larger specimen was cast of a new c.p. lot.

Another peculiarity is that the deflections run uniformly larger when the connections to the amplifier are such that the harmonic (of any 60-cycle unbalance) generated by the amplifier adds to the Hall effect. This might be due to the unidirectional torque produced by the interaction of higher harmonics in the two dynamometer coils, or it may be explained as follows. In balancing out the 60-cycle current there is a small range where the 60-cycle hum cannot be heard on account of the much louder 120-cycle noise. If the connections to the amplifier are such that the Hall effect 120-cycle current and the harmonic of any 60-cycle unbalance are in phase, then the slider is set at a minimum for both frequencies. But if the amplifier connections are reversed, then the slider is apt to be set a little to one side of the true balance point, not enough to make the 60-cycle note audible through the 120-cycle noise, but just enough to let the amplifier produce a small amount of 120-cycle harmonic which in this case is 180° out of phase with the Hall effect and hence gives the illusion that a better balance is obtained a trifle off the proper point. If this explanation is correct, the larger deflections should be the best values. If some unidirectional torque is the cause of the difference then a mean of the two sets of values would be best. It makes no difference which alternative is chosen as far as the general conclusions that may be drawn go.

The fact that the deflections plotted against watts/cm give fairly straight lines does not prove anything, for thermo-electric currents or harmonics generated in the amplifier would give deflections behaving the same way; but the independence of rate of cooling, the change in sign of the deflections with reversal of connections to the amplifier, the vanishing of the deflections with the addition of 2 percent tin, and the satisfactory quantitative agreement with the predicted order of magnitude of the deflection, seem to establish the existence of the effect quite decisively. The comparison of observed deflections with the predicted upper limits indicates that the arrangement used makes available between the external contacts a few tenths of the total generated Hall voltage.

If the Hall effect causes circulating currents wherever the expression for curl $e_H$ (derived in Case IV) is not equal to zero, and the energy loss due to these currents varies as the fourth power of the main current, then the measured resistance of a conductor would be of the form $R+aI^2$ and give the possibility of detecting an apparent increase in resistance where the current density is very high. This may have something to do with the increase observed by Bridgman[6] at densities of about a million amperes per square centimeter.

I wish to thank Prof. E. P. Adams, who suggested this experiment, for his interest throughout.

Palmer Physical Laboratory
Princeton University.
May 7, 1924

[6] P. W. Bridgman, Proc. Amer. Acad. **57**, (April, 1922)