CURRENTS LIMITED BY SPACE CHARGE BETWEEN CONCENTRIC SPHERES

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Abstract

Limiting current between concentric spheres; calculation of the function $\alpha = f(r/r_0)$ in the space charge equation $i = (4\sqrt{2}/9)\sqrt{(e/m)}\, V^{3/2}/\alpha^2$.—The coefficients of the first six terms of a series for α were determined, and α^2 calculated from this series. The results were checked by an integration method which was also used to calculate values in the region where the series failed. For an emitter of radius r_0 inside a collector of radius r, values of α^2 when $\log (r/r_0) > 6.4$ are given by the equation

$$\tfrac{1}{2}\alpha^2 = 0.112 \log (\log r/r_0) + \tfrac{1}{3} \log (r/r_0) + 0.152.$$

Where the collector is the inside sphere, values of α^2 for $r_0/r > 9$ are given by the equation $(\frac{1}{2}\alpha^2)^{2/3} = 1.11\ (r_0/r) - 1.64$. It is shown that when the collector is the inside sphere the potential distribution near the collector is unaltered if the emitter is replaced by a non-emitting sphere with a diameter .677 times the original diameter.

Limiting current between coaxial cylinders and between concentric spheres.—Equations are derived for the current in terms of the radius of curvature of the emitter. It is shown that at a surface in space four-fifths of the distance from the emitter to the collector the current density is independent of the radius of curvature when r/r_0 or $r_0/r < 2$; and in the case of coaxial cylinders with the emitter inside this holds true even when $r/r_0 = 20$.

THE problem of the calculation of thermionic currents limited by space charge has to deal with three simple cases, those of parallel planes, coaxial cylinders, and concentric spheres. These three cases have the characteristic that when an electron current is flowing, the lines of force and the paths of the electrons coincide. Parallel planes[1] and coaxial cylinders[2] have been considered in previous papers, and the equations have been given for calculating the currents for these surfaces. Although it is easy to construct apparatus in which the conditions approximate closely those of the cylindrical case, it is difficult to build devices in which the heated cathode takes the form of a sphere.

Recently space charge equations have been used in a new line of experimental work[3] to measure the intensity of ionization in gaseous discharges in which a positive ion sheath forms around the negatively charged electrode and the current is limited by space charge. Surfaces of different shapes have been used for electrodes, and since the sphere has proved

[1] Langmuir, Phys. Rev. **2**, 450 (1913); Phys. Zeits. **15**, 348, (1914)

[2] Langmuir and Blodgett, Phys. Rev. **22**, 347 (1923)

[3] Langmuir, Science, **58**, 290 (1923); Gen. Elec. Rev. **26**, 731 (1923); Journ. Franklin Inst. **196**, 751 (1923)

to be particularly suitable, the space charge equations for spherical electrodes have assumed practical importance. The development of these equations and the calculation of the numerical values of the function α which they involve will be given in this paper. The derivation has been made with the intention of using the equations to calculate the flow of either electrons or positive ions, and for this reason the general case has been developed in which the current flows from an *emitter* to a *collector* without regard to the direction of the voltage between the electrodes. Thus for calculations on ionization in gaseous discharges the outer edge of the positive ion sheath itself is the emitter of ions and the negative electrode within the sheath is the collector.

For the case of space charge between concentric spheres Poisson's equation becomes

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right)=4\pi\rho \tag{1}$$

where V is the potential at a point distant r from the center, and ρ is the electron space charge. If i is the total thermionic current, v the velocity of electrons or ions at the distance r, and e and m the charge and mass of an electron or ion,

$$i=4\pi r^2\rho V. \tag{2}$$

If we neglect the initial velocities of the particles we have

$$\tfrac{1}{2}mv^2=Ve \tag{3}$$

where V is measured from the surface of the emitter. Combining Eqs. (1), (2), and (3) gives

$$\frac{d}{dr}\left(r^2\frac{dV}{dr}\right)=i\sqrt{\frac{m}{2eV}}. \tag{4}$$

This equation, like the similar equation for coaxial cylinders, probably cannot be directly integrated, but a result can be obtained in terms of a series. The final solution takes the form

$$i=\frac{4\sqrt{2}}{9}\sqrt{\frac{e}{m}}\;\frac{V^{3/2}}{\alpha^2} \tag{5}$$

where α is a function of the ratio of the radii r and r_0 of the spheres, r_0 being the radius of the emitter. It is seen from this equation that the current is independent of the actual sizes of the spheres since the radii appear only in a ratio.

Calculation of α

An equation in α can be obtained by substituting Eq. (5) in Eq. (4) and placing

$$\gamma=\log(r/r_0) \tag{6}$$

This gives

$$3\alpha\frac{d^2\alpha}{d\gamma^2}+\left(\frac{d\alpha}{d\gamma}\right)^2+3\alpha\frac{d\alpha}{d\gamma}-1=0 \tag{7}$$

From this equation the values of α can be obtained in terms of a series. The coefficients of the first six terms of the series were obtained by a method described in a recent paper,[2] which makes use of Maclaurin's series in the neighborhood of $\gamma=0$. The series for α thus calculated is

$$\alpha=\gamma-0.3\gamma^2+0.075\gamma^3-0.0143182\gamma^4+0.0021609\gamma^5-0.00026791\gamma^6+\ldots \tag{8}$$

The first three coefficients are exact and the last three are each rounded off in the last figure.

Case where $r/r_0>1$ (Emitter inside of collector). For values of r/r_0 up to 5, α could be calculated accurately to three places of decimals from the series in Eq. (8). For the larger values, however, the six terms were entirely insufficient, and an integration process was used to complete the calculations. This process was applied to the equation obtained from Eq. (7) by making the substitution

$$y=\tfrac{1}{2}\alpha^2 \tag{9}$$

which gives

$$3\frac{d^2y}{d\gamma^2}-\frac{1}{y}\left(\frac{dy}{d\gamma}\right)^2+3\frac{dy}{d\gamma}-1=0. \tag{10}$$

The method employed was described in the recent paper referred to above, and consists essentially in substituting approximate values of y and $dy/d\gamma$ in Eq. (10), solving the equation for $d^2y/d\gamma^2$, and integrating the values thus obtained for new values of y and $dy/d\gamma$ which will be more nearly correct than the approximate values substituted at the beginning of the process. These calculations were made with y in the region $\gamma=1.0$ to $\gamma=2.2$, and the values of y obtained as the result of integration agreed exactly with those obtained from the series. The process was then used to calculate y in the range $\gamma=2.2$ to $\gamma=26.8$ where the series was insufficient. The results of these calculations are given in Table I.

The following empirical equation was found to fit the calculated values of y, $dy/d\gamma$ and $d^2y/d\gamma^2$ with a high degree of accuracy for values of γ greater than 11.8,

$$y=0.112\log\gamma+\tfrac{1}{3}\gamma+0.152. \tag{11}$$

The values of y given by this equation have an error of .48 per cent at $\gamma=6.4$ and .025 per cent at $\gamma=11.8$.

Table II gives the complete range of values of α^2 up to $r/r_0=100{,}000$. The values corresponding to $r/r_0>5$ were obtained from interpolations for α from Table I, using Newton's interpolation formula.

Case where $r/r_0<1$· *(Emitter outside of collector)*. The series in Eq. (8) that was used to calculate α in the preceding case is also applicable in the inverted case where the cathode is the outside cylinder, but since γ is now negative the signs of the alternate terms are changed. In this case, also, the integration process was used to calculate α for the larger values of γ.

TABLE I

γ	$dy/d\gamma$	$y=\frac{1}{2}\alpha^2$	α
2.2	.4512	.8424	1.2980
2.4	.4435	.9319	1.3652
2.6	.4354	1.0198	1.4281
2.8	.4275	1.1061	1.4873
3.0	.4198	1.1908	1.5432
3.2	.4126	1.2741	1.5963
3.4	.4060	1.3559	1.6468
3.7	.3970	1.4763	1.7183
4.0	.3891	1.5942	1.7856
4.3	.3825	1.7099	1.8493
4.6	.3767	1.8238	1.9099
4.9	.3720	1.9361	1.9678
5.2	.3678	2.0471	2.0234
5.8	.3615	2.2658	2.1288
6.4	.3569	2.4812	2.2276
7.0	.3536	2.6944	2.3214
7.6	.3510	2.9057	2.4107
8.2	.3493	3.1157	2.4963
8.8	.3476	3.3249	2.5787
11.8	.3433	4.3607	2.9532
14.8	.3410	5.3867	3.2823
17.8	.3397	6.4077	3.5799
20.8	.3387	7.4252	3.8536
23.8	.3380	8.4402	4.1086
26.8	.3375	9.4534	4.3482

The equation which was used in this process was obtained by combining Eqs. (6) and (10) and placing $r_0/r=\sigma$, which gives

$$3\frac{d^2y}{d\sigma^2}-\frac{1}{y}\left(\frac{dy}{d\sigma}\right)^2-\frac{1}{\sigma^2}=0 \tag{12}$$

The results of calculations using the integration process in the range $\sigma=7$ to $\sigma=22$ are given in Table III.

In the neighborhood of $\sigma=22$, the term $1/\sigma^2$ in Eq. (12) is decreasing rapidly and has already become only 0.7 per cent of the second term, so that a solution of the equation neglecting the term $1/\sigma^2$ gives a good approximation formula for y which may be used to obtain values of α in the region beyond $\sigma=22$. The complete solution of

$$3\frac{d^2y}{d\sigma^2}-\frac{1}{y}\left(\frac{dy}{d\sigma}\right)^2=0 \tag{13}$$

is

$$y^{2/3}=A\sigma+B \tag{14}$$

Table II

α^2 as function of radius.
r_0 = radius of emitter; r = radius at any point P;
α^2 applies to case where P is outside emitter, $r>r_0$;
$(-\alpha)^2$ applies to case where P is inside emitter, $r_0>r$.

$\frac{r}{r_0}$ or $\frac{r_0}{r}$	α^2	$(-\alpha)^2$	$\frac{r}{r_0}$ or $\frac{r_0}{r}$	α^2	$(-\alpha)^2$
1.0	0.0000	0.0000	6.5	1.385	13.35
1.05	.0023	.0024	7.0	1.453	15.35
1.1	.0086	.0096	7.5	1.516	17.44
1.15	.0180	.0213	8.0	1.575	19.62
1.2	.0299	.0372	8.5	1.630	21.89
1.25	.0437	.0571	9.0	1.682	24.25
1.3	.0591	.0809	9.5	1.731	26.68
1.35	.0756	.1084	10	1.777	29.19
1.4	.0931	.1396	12	1.938	39.98
1.45	.1114	.1740	14	2.073	51.86
1.5	.1302	.2118	16	2.189	64.74
1.6	.1688	.2968	18	2.289	78.56
1.7	.208	.394	20	2.378	93.24
1.8	.248	.502	30	2.713	178.2
1.9	.287	.621	40	2.944	279.6
2.0	.326	.750	50	3.120	395.3
2.1	.364	.888	60	3.261	523.6
2.2	.402	1.036	70	3.380	663.3
2.3	.438	1.193	80	3.482	813.7
2.4	.474	1.358	90	3.572	974.1
2.5	.509	1.531	100	3.652	1144
2.6	.543	1.712	120	3.788	1509
2.7	.576	1.901	140	3.903	1907
2.8	.608	2.098	160	4.002	2333
2.9	.639	2.302	180	4.089	2790
3.0	.669	2.512	200	4.166	3270
3.2	.727	2.954	250	4.329	4582
3.4	.783	3.421	300	4.462	6031
3.6	.836	3.913	350	4.573	7610
3.8	.886	4.429	400	4.669	9303
4.0	.934	4.968	500	4.829	13015
4.2	.979	5.528	600	4.960	
4.4	1.022	6.109	800	5.165	
4.6	1.063	6.712	1000	5.324	
4.8	1.103	7.334	1500	5.610	
5.0	1.141	7.976	2000	5.812	
5.2	1.178	8.636	5000	6.453	
5.4	1.213	9.315	10000	6.933	
5.6	1.247	10.01	30000	7.693	
5.8	1.280	10.73	100000	8.523	
6.0	1.311	11.46			

The values of A and B were derived from the data in Table III, with the result

$$y^{2/3}=1.11\sigma-1.64. \tag{15}$$

Values of y obtained from this equation have an error of 0.5 per cent at $\sigma=9$ and 0.025 per cent at $\sigma=18$. Table II gives the complete range of values of α^2 for this case up to $r_0/r=500$. Eq. (15) was used for the calculations in the range $r_0/r=22$ to 500.

TABLE III

$\sigma=r_0/r$	$dy/d\sigma$	$y=\frac{1}{2}\alpha^2$	α
7.0	2.0454	7.675	3.918
7.5	2.1376	8.721	4.176
8.0	2.2261	9.812	4.430
8.5	2.3113	10.947	4.679
9.0	2.3935	12.123	4.924
9.5	2.4730	13.340	5.165
10.0	2.5501	14.596	5.403
12.	2.8379	19.988	6.323
14.	3.0990	25.930	7.201
16.	3.3400	32.371	8.046
18.	3.5646	39.280	8.863
20.	3.7760	46.622	9.656
22.	3.9761	54.378	10.429

Low values of α^2 for either the direct or inverted case where r/r_0 or $r_0/r<1.4$, may be conveniently calculated from Eq. (16) which is derived in the following manner. Put $\epsilon=(r_0/r)-1$ and expand $\gamma=-\log(r_0/r)$ into a series in terms of ϵ. Substitute this value of γ in the equation obtained by squaring Eq. (8) and as a result

$$\alpha^2=\epsilon^2-\frac{2}{5}\epsilon^3+\frac{77}{300}\epsilon^4-\frac{313}{1100}\epsilon^5+\ldots \tag{16}$$

Logarithmic differentiation of Eq. (5), considering i as constant, gives

$$2\frac{d\alpha}{\alpha}=\frac{3}{2}\frac{dV}{V}. \tag{17}$$

Logarithmic differentiation of Eq. (15) combined with Eq. (9) gives

$$\frac{4}{3}\frac{d\alpha}{\alpha}=\frac{1.11\,d\sigma}{1.11\sigma-1.64}. \tag{18}$$

Combining this with Eq. (17) and substituting $\sigma=r_0/r$,

$$\frac{dV}{V}=-\frac{dr}{r^2}\bigg/\left(\frac{1}{r}-\frac{1.477}{r_0}\right) \tag{19}$$

In the case of two concentric spheres, neither of which is emitting current, the voltage at any point of radius r is

$$V=A\left(\frac{1}{r}-\frac{1}{r_1}\right) \tag{20}$$

where V is measured with respect to the sphere of radius r_1. Logarithmic differentiation of Eq. (20) gives

$$\frac{dV}{V} = -\frac{dr}{r^2}\Big/\left(\frac{1}{r}-\frac{1}{r_1}\right). \tag{21}$$

Comparing Eqs. (19) and (21) we see that they become identical for all values of r if we place

$$r_1 = 0.677\ r_0. \tag{22}$$

Thus when current emitted by a spherical electrode is being collected by an inner concentric sphere of small diameter, the effect of space charge is to make the potential distribution near the collector the same as it would be if the outer sphere were not emitting current and were given a diameter 67.7 per cent of the original diameter. A similar ratio of diameters for the case of cylinders was given in the previous paper and is 0.707.

Comparison of Equations for Planes, Cylinders and Spheres

The equations given in this paper and two preceding papers for calculating thermionic currents limited by space charge are as follows:

Parallel planes (i_a = current per unit area)

$$i_a = \frac{\sqrt{2}}{9\pi}\sqrt{\frac{e}{m}}\,\frac{V^{3/2}}{x^2} \tag{23}$$

Coaxial cylinders (i_l = current per unit length)

$$i_l = \frac{2\sqrt{2}}{9}\sqrt{\frac{e}{m}}\,\frac{V^{3/2}}{r\beta^2} \tag{24}$$

Concentric spheres (i = total current)

$$i = \frac{4\sqrt{2}}{9}\sqrt{\frac{e}{m}}\,\frac{V^{3/2}}{\alpha^2} \tag{25}$$

If these three equations are written in the same form by substituting $i_l/(2\pi r_0) = i_a$ and $i/(4\pi r_0^2) = i_a$, they become for parallel planes,

$$i_a = D/x^2; \tag{26}$$

for coaxial cylinders,

$$i_a = D/(r_0 r\beta^2); \tag{27}$$

for concentric spheres,

$$i_a = D/(r_0^2\alpha^2); \tag{28}$$

where

$$D = \frac{\sqrt{2}}{9\pi}\sqrt{\frac{e}{m}}\,V^{3/2}. \tag{29}$$

If i is expressed in amperes per cm^2, V in volts, x and r in cm, and e and m are the charge and mass respectively of an electron, D has the value

$$D=2.336\times 10^{-6}V^{3/2}$$

If e and m are the charge and mass of an ion,

$$D=5.455\times 10^{-8}V^{3/2}/\sqrt{M}$$

where M is the molecular weight of the ions (oxygen atom $=16$).

Consider the case of two concentric spheres that are very close together so that their surfaces are almost parallel. If we put $r=(r_0+x)$ and $b=x/r_0=(r/r_0-1)$ we can write a series for α^2 similar to the series in Eq. (16) as follows

$$\alpha^2=b^2-\frac{8}{5}b^3+\frac{617}{300}b^4-\ \dots \tag{30}$$

This value of α^2 may be substituted in Eq. (28) and it is then seen that when b becomes so small that all the terms in the series except the first are negligible, the equation becomes identical with Eq. (26) for parallel planes.

In the same way a series for β^2 can be derived, which is

$$\beta^2=b^2-\frac{9}{5}b^3+\frac{123}{50}b^4-\dots \tag{31}$$

and this can be substituted in Eq. (27) which will also become identical with the equation for parallel planes when b is very small.

Where the surfaces of cylinders and spheres are farther apart so that the second and third terms of the series in Eqs. (30) and (31) become important and the conditions no longer closely approximate those of parallel planes, these series enable us to obtain equations for the current in terms of the total curvature of the *emitting* surfaces. This total curvature ρ_0 in the case of the emitting cylinder is $\rho_0=1/r_0$, and for the sphere is $\rho_0=2/r_0$. Combining $r=(r_0+x)$ with Eq. (31) and substituting the value of ρ_0 for a cylinder, we have

$$r_0r\beta^2=x^2\left(1-\frac{4}{5}x\rho_0+\frac{33}{50}x^2\rho_0^2-\dots\right) \tag{32}$$

Similarly from Eq. (30) we have for spheres

$$r_0^2\alpha^2=x^2\left(1-\frac{4}{5}x\rho_0+\frac{617}{1200}x^2\rho_0^2-\dots\right) \tag{33}$$

We now see that the equations for cylinders and spheres in terms of the total curvature of the emitting surface have the first two terms identical. By expanding the reciprocal of the series in Eq. (32) we obtain an expression for $1/(r_0r\beta^2)$, in which the second term is $+4/5\ (x\rho_0)$ but the third term is only $-0.02\ x^2\rho^2$ which is 3 per cent of the third term in Eq. (32). Carrying through a similar process for Eq. (33) the third term becomes $0.126\ x^2\rho^2$. Substituting these values for $1/(r_0 r\beta^2)$ and $1/(r_0^2\alpha^2)$ in Eqs. (27) and (28) and neglecting the third terms of the expansions, we have

$$i_a = \frac{\sqrt{2}}{9\pi}\sqrt{\frac{e}{m}}\,\frac{V^{3/2}}{x^2}\left(1+\frac{4}{5}x\rho_0\right) \tag{34}$$

which is the equation for parallel planes with a first order correction term for curved surfaces in terms of the curvature of the surface.

But it was arbitrary when defining the current per unit area and the total curvature to choose the area and curvature of the surface of the *emitter*. We may just as well choose the surface of the collector or any surface between them. We shall consider the general case where the current is measured per unit area of a surface of radius $r_1=(r_0+kx)$ for which the total curvature is $\rho_1=\rho_0/(1+\rho_0 kx)$ for a cylinder or $\rho_1=\rho_0/(1+\frac{1}{2}\rho_0 kx)$ for a sphere.

In this case from Eqs. (27) and (28) the current per unit area of the new surface is $i_a=D/[(r_0+kx)r\beta^2]$ for cylinders and $i_a=D/[(r_0+kx)^2\alpha^2]$ for spheres. Eqs. (32) and (33) will then become

$$(r_0+kx)r\beta^2 = x^2\left[1+\left(k-\frac{4}{5}\right)x\rho_1+\left(k^2-\frac{8}{5}\ k+\frac{33}{50}\right)x^2\rho_1^2+---\right] \tag{35}$$

and

$$(r_0+kx)^2\alpha^2 = x^2\left[1+\left(k-\frac{4}{5}\right)x\rho_1+\left(\frac{3}{4}k^2-\frac{6}{5}k+\frac{617}{1200}\right)x^2\rho_1^2+---\right]. \tag{36}$$

It now becomes apparent that for $k=4/5$ the second term in both series will vanish for all values of $x\rho_1$, and the coefficients of $x^2\rho_1^2$ will be small, having the value .02 for cylinders and .0342 for spheres. This means that for all values of $x\rho_1$ that make the third terms negligible, the current *per unit area of a surface four-fifths of the distance from the emitter to the collector* is the same as for parallel plane electrodes spaced the same distance apart as the two curved electrodes.

We have thus determined a surface in space at which the current per unit area is the same whether the electrodes are plane or curved, provided that $.03x^2\rho_1^2$ is small compared to unity. This suggests that it may be possible to locate such a surface in every instance by properly choosing the value of k. At this surface, the equations for parallel planes, coaxial cylinders, and concentric spheres must become identical, and therefore, substituting (r_0+kx) for r_0, we have from Eqs. (26) and (27)

$$x^2=(r_0+kx)r\beta^2 \tag{37}$$

from which

$$k = \frac{x}{r\beta^2} - \frac{r_0}{x}. \tag{38}$$

Similarly in the case of spheres

$$x^2 = (r_0 + kx)^2\alpha^2 \tag{39}$$

from which

$$k = \frac{1}{\alpha} - \frac{r_0}{x} \tag{40}$$

Since $x = (r - r_0)$, Eqs. (38) and (40) may be written with the radii of the curved surfaces appearing in each case only as a ratio. Values of k calculated from these equations are given in Table IV.

TABLE IV

Values of k as a function of the radii

$\frac{r}{r_0}$ or $\frac{r_0}{r}$	Cylinder $r/r_0>1$	Cylinder $r/r_0<1$	Sphere $r/r_0>1$	Sphere $r/r_0<1$
1.0	0.8000	0.8000	0.8000	0.8000
1.2	.7970	.8039	.7877	.8122
1.5	.7932	.8093	.7718	.8268
2.0	.7904	.8171	.7513	.8450
5.0	.7935	.8455	.6862	.8959
10	.8090	.8677	.6393	.9260
20	.8340	.8883	.5959	.9490
100	.9081	.9258	.5132	.9805
∞	1.0000	1.0000	.0000	1.0000

Consider the case of two coaxial cylinders where the emitter is the inner one and has a radius 1 while the collector has a radius 20. From Table IV $k=.834$, and a surface 83.4 per cent of the distance from the emitter has a radius $r_1=16.846$. The current per unit area of this surface by Eq. (27) is $i_a=D/(16.846r\beta^2)$. Substituting the value $\beta^2=1.0715$ when $r/r_0=20$ we have $i_a=D/361$ for the correct value of the current, which is the same value as $i_a=D/(19)^2$ for the current between two parallel planes.

If we had given k the approximate value 0.8 we should have had $i_a=D/347.2$ for the current per unit area of a surface with radius $r_1=16.2$. This differs from the value of current between parallel planes by 4 per cent, and since this error is $(.8x-.834x)/(r_0+.834x)$ it is clear that it is less than the error in k. Thus in this case for a range of ratio of radii from 1/1 to 1/20 the current per unit area of a surface four-fifths of the distance from the emitter is the same as the current between parallel planes within an error of 4 per cent, and over half the range the error is less than 1 per cent.

On the other hand in the case of coaxial cylinders with the emitter outside, Table IV shows that k departs much more rapidly from the value 0.8. The error introduced into calculations of current by choosing $k=0.8$ instead of the exact value k_a from the Table is $(.8x-k_a x)/(r_0+k_a x)$ where $x=(r-r_0)$ is now negative. This error in current is proportional to the error in k as k_a is to $(r_0/x+k_a)$, which shows that calculations of current using $k=0.8$ may be made with an error less than that of k only up to $r_0/r=2.55$, and above that point the error is greater than that of k, becoming infinitely great as r_0/r approaches infinity.

In the case of spheres the value $k=0.8$ yields an error of 5.6 per cent for $r/r_0=2.0$ where the emitter is inside, and 7.9 per cent for $r_0/r=2.0$ where the emitter is outside. Thus when current is flowing in either direction between any curved surfaces where the radius of the outer surface is not more than twice the radius of the inner, the amount of current flowing per unit area of a surface four-fifths of the distance from the emitter to the collector is very nearly the same as the current between parallel planes. In the case of coaxial cylinders with the emitter inside this same rule applies with similar accuracy even when the radius of one is twenty times the radius of the other.

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