ADVANCED POTENTIALS AND THEIR APPLICATION TO ATOMIC MODELS

By Leigh Page

Abstract

Generalized emission theory of electromagnetic fields assuming advanced as well as retarded potentials.—In order to reconcile electromagnetic theory with radiationless orbits, Nordstrom suggested the possibility of the fields of electrons being half retarded and half advanced, instead of being wholly retarded as usually assumed. On the emission theory this would mean that each electron is the focus of converging streams of moving elements which pass through it undeflected. Simple one-way series for the simultaneous potentials and intensities due to such a point charge are developed, and it is shown that all damping terms disappear and that the electron is capable of describing a radiationless orbit of the Bohr type, while the mass reaction is the same as for the customary retarded field. The radiation emitted during a full period vanishes, since the converging waves which constitute the advanced portion of the field bring the electron as much energy as it loses by the emission of diverging retarded waves. Nevertheless fluctuations of energy would take place during each period, and the converging and diverging waves should combine to form standing waves. The absence of any evidence for such waves constitutes a serious objection to the theory.

Introduction

THE first postulate of the Bohr theory assumes that in the stationary states of an atomic or molecular structure the laws of electrodynamics hold good with the exception of the emission of radiation and the attendant radiation reaction which the classical theory, as generally interpreted, requires whenever an electron or other charged particle describes an orbit. As second order effects deduced from the electromagnetic equations are retained in describing the motions of the electrons inside the atom, it seems not altogether consistent to discard third order effects. A path out of the dilemma has been suggested by Nordstrom,[1] who shows that if the field of a charged particle is half retarded and half advanced, instead of being wholly retarded as usually assumed, Maxwell's equations remain valid and the net energy radiated by an electron in describing a periodic orbit vanishes. The same suggestion was proposed by Lunn at the Wisconsin colloquium in 1922.

The object of this paper is to make a more detailed examination of fields of the type specified above. First, advanced fields and fields which are half retarded and half advanced will be interpreted from the point

[1] Proc. Acad. Amsterdam, **22**, 145, (1920)

of view of the emission theory. Second, simple one-way series for the simultaneous retarded and advanced potentials and field strengths of a point charge will be deduced, and it will be shown that if the field of the electron is half retarded and half advanced all terms in its dynamical equation involving odd time derivatives of the acceleration vanish, and hence there are no damping reactions, while even terms such as the mass reaction retain their accustomed value. Third, the radiation field will be investigated.

While the retarded potentials ordinarily employed in describing the field of a point charge at P specify the field at a point O and time t in terms of the relative position and velocity of the charge at the point R which it occupied at the previous time $t-[r]_r/c$, where $[r]_r$ is the distance OR, the advanced potentials specify the field in terms of the position and velocity of the charge at the point A which it will occupy at the future time $t+[r]_a/c$, where $[r]_a$ is the distance OA. The point R may be termed the *effective retarded position* of the charge for determination of the field at O at the time t, A the *effective advanced position.* While the advanced potentials, as well as the retarded potentials, satisfy the electromagnetic equations, the former have generally been discarded for the reason that it has been more in accord with the trend of scientific intuition to consider that the present is determined by the past course of events than by the future. However, if it is once admitted that the present state is uniquely determined by any past state, it follows that the future is also so determined, and hence the employment of a future as well as a past state in specifying the present marks no inherent departure from our accustomed methods of description, while it may find much justification in the simplicity which it introduces into the formulation of physical laws. In fact, in the second postulate or frequency condition of the Bohr theory, this type of formulation is already current. For Bohr's hypothesis makes the frequency of the radiation emitted by an electron during a transition from one stable orbit to another depend equally upon the energy of the orbit which it has left and that of the orbit toward which it is progressing.[2] Furthermore, from the point of view of the representation of the life history of a particle by its world line as used in the relativity theory, it would seem no less logical to describe the field due to an electron in terms of the portion of its world line extending forward in time than to describe it in terms of the portion extending backward.

[2] If it is admitted that absorption is the inverse of emission the necessity of making use of the energy of an orbit to be occupied in the future in computing the frequency cannot be avoided by assuming that in the process of emission the electron does not actually radiate energy until it is in its final orbit. For then absorption would have to occur in the initial orbit.

1. Generalization of Emission Theory to Include Advanced Fields

The emission theory[3] assumes that a charge is the source of discontinuities or *moving elements* projected uniformly outward in all directions with the velocity of light. A line of electric force is defined as the locus of all moving elements emitted in a single direction. It has been shown that this representation leads unambiguously to the electromagnetic equations and to the retarded potentials of Lienard for a point charge.[4] Now suppose that is is assumed that a point charge, instead of being a source, is a sink, toward which streams of moving elements are converging uniformly with the velocity of light. Defining a line of electric force as the locus of all moving elements which have the same direction of motion when incident upon the charge—the direction being determined in the inertial system in which the charge is momentarily at rest—the field strengths and potentials at a point O and time t can be deduced from purely kinematical considerations. As they are determined by the position and motion of the charge at a time $t+[r]_a/c$ they are advanced.

The method of deducing the advanced field strengths and potentials is so analogous to that for the corresponding retarded quantities which exist when the charge is assumed to be a source instead of a sink, that it is unnecessary to give the derivation in detail. In fact the method is precisely that given in Chapter II of the book already referred to[4] provided the direction of the velocity $\mathbf{c}$ of the moving elements is reversed so as to make them converge to the point charge instead of diverging from it. The results of the analysis, given in Heaviside-Lorentz rational units, are:

$$\mathbf{E}_a=\left[\frac{e}{4\pi r^2k^2c(1-\mathbf{c}\cdot\mathbf{v}/c^2)^3}\left\{-\mathbf{c}+\mathbf{v}+\frac{rk^2}{c^3}(\mathbf{f}\times\{\mathbf{c}-\mathbf{v}\})\times\mathbf{c}\right\}\right]_a; \tag{1}$$

$$\mathbf{H}_a\equiv\mathbf{c}\times\mathbf{E}/c$$
$$=\left[\frac{e}{4\pi r^2k^2c^2(1-\mathbf{c}\cdot\mathbf{v}/c^2)^3}\left\{\mathbf{c}\times\mathbf{v}+\frac{rk^2}{c^3}\mathbf{c}\times\{(\mathbf{f}\times\{\mathbf{c}-\mathbf{v}\})\times\mathbf{c}\}\right\}\right]_a, \tag{2}$$

where the appended subscript a denotes that the quantities within the brackets refer to the effective advanced position of the point charge. The letter $\mathbf{v}$ stands for the velocity of the charge e, $\mathbf{f}$ for its acceleration, and $k=1/(1-v^2/c^2)$.

If advanced scalar and vector potentials are defined by

$$\phi_a\equiv\left[\frac{e}{4\pi r(1-\mathbf{c}\cdot\mathbf{v}/c^2)}\right]_a, \tag{3}$$

[3] Leigh Page, Amer. Jour. Sci. **38**, 169 (1914);
Trans. Conn. Acad. Arts and Sci. **26**, 213 (1924)

[4] Leigh Page, Introduction to Electrodynamics, Ginn and Co., 1922; Chapter references are to this book.

$$\mathbf{a}_a \equiv \left[\frac{e\,\mathbf{v}}{4\pi c r(1-\mathbf{c}\cdot\mathbf{v}/c^2)}\right]_a, \tag{4}$$

then

$$\mathbf{E}_a = -\nabla\,\phi_a - \dot{\mathbf{a}}_a/c \tag{5}$$

$$\mathbf{H}_a = \nabla\times\mathbf{a}_a\,, \tag{6}$$

as may easily be verified by direct differentiation.

That $\mathbf{E}_a$ and $\mathbf{H}_a$ satisfy the field equations may be shown by precisely the same method as is used in Chapter V[4] for the retarded field strengths.

Now suppose that half the charge e gives rise to a retarded field and half to an advanced field. Then the charge emits moving elements at the same rate as it absorbs them. In other words, *the charge is merely the focus of uniformly converging streams of moving elements which pass through it undeflected and then diverge from it.* The kinematical representation of the field has many points of resemblance to Langmuir's unpublished quantel theory.

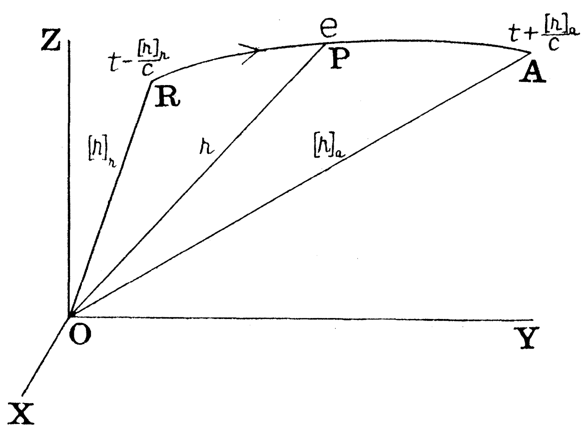

Fig. 1

2. Simultaneous Potentials and Intensities

We wish to find the simultaneous potentials at the origin O (Fig. 1) at the time t due to a point charge e which occupies the point P at this time, the coördinates of P being x, y, z. Let R be the effective retarded position of the charge and A the effective advanced position. Then if e_r is the portion of the charge giving rise to a retarded field, and e_a the portion giving rise to an advanced field, the scalar potentials ϕ and vector potentials $\mathbf{a}$ are

$$\phi_r = \frac{e_r}{4\pi}\left[\frac{1}{r(1+\dot{r}/c)}\right]_r, \tag{7}$$

$$\phi_a = \frac{e_a}{4\pi}\left[\frac{1}{r(1-\dot{r}/c)}\right]_a, \tag{8}$$

$$\mathbf{a}_r = \frac{e_r}{4\pi c}\left[\frac{\dot{\mathbf{r}}}{r(1+\dot{r}/c)}\right]_r, \tag{9}$$

$$\mathbf{a}_a = \frac{e_a}{4\pi c}\left[\frac{\dot{\mathbf{r}}}{r(1-\dot{r}/c)}\right]_a, \tag{10}$$

where the quantities in brackets are retarded or advanced depending upon whether the subscript r or a is appended.

Put $\tau \equiv r/c$, $\tau_r \equiv [r]_r/c$, $\tau_a \equiv [r]_a/c$. Then by Taylor's theorem,

$$\tau_r{}^m = \tau^m - \dot{\overline{\tau^m}}\,\tau_r + \frac{1}{2}\ddot{\overline{\tau^m}}\,\tau_r{}^2 \;.\;.\;. + \frac{(-1)^{n-1}}{\overline{n-1}\,!}\,\overset{(n-1)}{\tau^m}\,\tau_r{}^{n-1} \;.\;.\;. \tag{11}$$

The solution of the set of equations obtained by giving m all positive integral values, published previously,[5] are

$$\tau_r{}^m = \tau^m - \frac{m}{m+1}D(\tau^{m+1}) + \frac{m}{2(m+2)}D^2\,(\tau^{m+2}) \;.\;.\;.$$
$$+(-1)^{n-1}\frac{m}{\overline{n-1}!\,\overline{m+n-1}}D^{n-1}\,(\tau^{m+n-1}) \;.\;.\;.$$

where D indicates differentiation with respect to the time, or in equivalent form

$$\tau_r{}^m = \tau^m - \dot{\overline{\tau^m}}\,\tau + \tfrac{1}{2}D(\dot{\overline{\tau^m}}\,\tau^2) \;.\;.\;. + \frac{(-1)^{n-1}}{\overline{n-1}\,!}D^{n-2}\,(\dot{\overline{\tau^m}}\,\tau^{n-1}) \;.\;.\;. \tag{12}$$

Moreover, if f is any function of the time,

$$[f]_r = f - \dot{f}\tau + \tfrac{1}{2}D(\dot{f}\tau^2) \;.\;.\;. + \frac{(-1)^{n-1}}{\overline{n-1}\,!}D^{n-2}\,(\dot{f}\tau^{n-1}) \;.\;.\;. \tag{13}$$

the method of proof being that used for $\mathbf{r}_e$ in the paper just referred to.[6]

The series expansions of the advanced quantities τ_a^m and $[f]_a$ differ from (12) and (13) only in that all the terms are positive instead of alternately positive and negative.

Now consider the expression

$$\left[\frac{1}{r(1+\dot{r}/c)}\right]_r = \left[\frac{1}{r} - \frac{1}{r}\frac{\dot{r}}{c} + \frac{1}{r}\left(\frac{\dot{r}}{c}\right)^2 \;.\;.\;. + (-1)^{n-1}\frac{1}{r}\left(\frac{\dot{r}}{c}\right)^{n-1} \;.\;.\;.\right]_r$$

which appears as the retarded factor in the scalar potential ϕ_r.

[5] Leigh Page, Phys. Rev. **20**, 18 (1922)

[6] Leigh Page, loc. cit.[5] p. 21, eq. (8). Due to a typographical error τ_e appears on the left hand side of this equation instead of $\mathbf{r}_e$.

Putting $y\equiv 1/r$ for the moment, and making use of (13),

$$\left[\frac{1}{r(1+\dot{r}/c)}\right]_r = y-\tau Dy+\tfrac{1}{2}D(\tau^2 Dy)\cdots+\frac{(-1)^{m-1}}{m-1\,!}D^{m-2}\{\tau^{m-1}Dy\}\cdots$$
$$-yD\tau+\tau D(yD\tau)\cdots-\frac{(-1)^{m-2}}{m-2\,!}D^{m-3}\{\tau^{m-2}D(yD\tau)\}\cdots$$
$$+y\overline{D\tau}^2\cdots+\frac{(-1)^{m-3}}{m-3\,!}D^{m-4}\left\{\tau^{m-3}D\left(y\overline{D\tau}^2\right)\right\}\cdots$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$=y-D(\tau y)+\tfrac{1}{2}D^2(\tau^2 y)\cdots+\frac{(-1)^{m-1}}{m-1\,!}D^{m-1}(\tau^{m-1}y)\cdots$$
$$=\frac{1}{r}+0+\tfrac{1}{2}D^2\left\{\left(\frac{r}{c}\right)^2\frac{1}{r}\right\}\cdots+\frac{(-1)^{m-1}}{m-1\,!}D^{m-1}\left\{\left(\frac{r}{c}\right)^{m-1}\frac{1}{r}\right\}\cdots \quad (14)$$

Similarly the retarded factor in the x component of the vector potential a_r is

$$\left[\frac{\dot{x}}{r(1+\dot{r}/c)}\right]_r=\frac{\dot{x}}{r}-D\left\{\left(\frac{r}{c}\right)\frac{\dot{x}}{r}\right\}+\tfrac{1}{2}D^2\left\{\left(\frac{r}{c}\right)^2\frac{\dot{x}}{r}\right\}\cdots$$
$$+\frac{(-1)^{m-1}}{m-1\,!}D^{m-1}\left\{\left(\frac{r}{c}\right)^{m-1}\frac{\dot{x}}{r}\right\}\cdots, \quad (15)$$

and expanding the advanced potentials by exactly the same method

$$\left[\frac{1}{r(1-\dot{r}/c)}\right]_a=\frac{1}{r}+O+\tfrac{1}{2}D^2\left\{\left(\frac{r}{c}\right)^2\frac{1}{r}\right\}\cdots$$
$$+\frac{1}{m-1\,!}D^{m-1}\left\{\left(\frac{r}{c}\right)^{m-1}\frac{1}{r}\right\}\cdots, \quad (16)$$

$$\left[\frac{\dot{x}}{r(1-\dot{r}/c)}\right]_a=\frac{\dot{x}}{r}+D\left\{\left(\frac{r}{c}\right)\frac{\dot{x}}{r}\right\}+\tfrac{1}{2}D^2\left\{\left(\frac{r}{c}\right)^2\frac{\dot{x}}{r}\right\}\cdots$$
$$+\frac{1}{m-1\,!}D^{m-1}\left\{\left(\frac{r}{c}\right)^{m-1}\frac{\dot{x}}{r}\right\}\cdots. \quad (17)$$

Now if half the charge gives rise to a retarded field and half to an advanced field, the even numbered terms in (14) and (15) being of opposite sign cancel, and

$$\phi=\frac{e}{4\pi}\left(\frac{1}{r}+\frac{1}{2}D^2\left\{\left(\frac{r}{c}\right)^2\frac{1}{r}\right\}\cdots+\frac{1}{2(m-1)!}D^{2(m-1)}\left\{\left(\frac{r}{c}\right)^{2(m-1)}\frac{1}{r}\right\}\cdots\right). \quad (18)$$

In like manner (15) and (17) combine to give

$$a_x=\frac{e}{4\pi c}\left(\frac{\dot{x}}{r}+\frac{1}{2}D^2\left\{\left(\frac{r}{c}\right)^2\frac{\dot{x}}{r}\right\}\cdots+\frac{1}{2(m-1)!}D^{2(m-1)}\left\{\left(\frac{r}{c}\right)^{2(m-1)}\frac{\dot{x}}{r}\right\}\cdots\right). \quad (19)$$

Now
$$\mathbf{E} = -\nabla_O \phi - \dot{\mathbf{a}}/c = \nabla_P \phi - \dot{\mathbf{a}}/c$$
$$\mathbf{H} = \nabla_O \times \mathbf{a} = -\nabla_P \times \mathbf{a},$$
where the subscripts O and P denote differentiation at the points O and P respectively.

Therefore
$$E_x = \frac{\partial}{\partial x}\phi - \frac{1}{c}\frac{\partial}{\partial t} a_x \cdot$$

Carrying out the differentiation with respect to x it is found that

$$E_x = \frac{e}{4\pi}\left\{ -\left(\frac{x}{r^3}\right) + \frac{1}{2c^2} D\left(x^2 D\left\{\frac{1}{r^2}\frac{r}{x}\right\}\right) + \frac{3}{24c^4} D^3\left(x^4 D\left\{\frac{1}{r^2}\left(\frac{r}{x}\right)^3\right\}\right) \cdots \right.$$
$$\left. + \frac{2m-3}{2(m-1)!\, c^{2(m-1)}} D^{2m-3}\left(x^{2(m-1)} D\left\{\frac{1}{r^2}\left(\frac{r}{x}\right)^{2m-3}\right\} \cdots \right\}, \qquad (20)$$

$$H_x = \frac{e}{4\pi c}\left\{ -\frac{z\dot{y} - y\dot{z}}{r^3} + \frac{1}{2c^2} D^2\left(\frac{z\dot{y} - y\dot{z}}{r}\right) + \frac{3}{24c^4} D^4\left(r\left\{ z\dot{y} - y\dot{z}\right\}\right) \cdots \right.$$
$$\left. + \frac{2m-3}{2(m-1)!\, c^{2(m-1)}} D^{2m-2}\left(r^{2m-1}\left\{ z\dot{y} - y\dot{z}\right\}\right) \cdots \right\}. \qquad (21)$$

These simultaneous expansions of the electric and magnetic intensities have the advantage of being in a form such that each term is of a single order of magnitude specified by the exponent of the factor $1/c$. Thus the first term in (20) is the zeroth order term specifying the electrostatic field, the second term is of the second order and is responsible for the mass reaction, etc. As here written the series apply to the case where the field is half retarded and half advanced. If the field is taken to be wholly retarded, odd order terms of the same form and with negative signs must be added, whereas if the field is wholly advanced, the same odd order terms with positive signs must be introduced.

The reaction on the electron of its own field may be most easily calculated relative to the inertial system in which the electron is momentarily at rest. Thus the mass reaction is obtained from the second order term in (20) and is the same whether the field is retarded, advanced, or partly retarded and partly advanced. Performing the indicated differentiation with respect to the time, this second order term is seen to be

$$E_{x2} = \frac{e}{8\pi c^2}\left\{ -\frac{\ddot{x}}{r} - \frac{x}{r^2}\left(\ddot{r} - 2\frac{\dot{r}^2}{r}\right)\right\}$$
$$= -\frac{e}{8\pi c^2}\left\{\frac{\mathbf{f}_x}{r} + \frac{x}{r}\frac{\mathbf{f}_r}{r}\right\},$$

where $\mathbf{f}$ is the acceleration and $\dot{r}$ has been put equal to zero since the electron is at rest. If the electron is assumed to be a uniformly charged spherical shell of radius a relative to the system in which it is momentarily at rest (Lorentz electron), this gives for the mass reaction the familiar expression $-(e^2/6\pi ac^2)\mathbf{f}$.

Now the damping terms in the expression for the reaction on the electron of its own field are those involving the odd derivatives of the acceleration $\mathbf{f}$. From a consideration of dimensions alone it is clear that these terms must contain odd powers of $1/c$, that is, they must have their origin in the odd order terms in the expansion of the electric intensity. But all the odd order terms are missing from (20). *Therefore, if the field of each element of charge of which the electron is composed is half retarded and half advanced, there are no damping terms of any order whatsoever in the dynamical equation. Therefore an electron of this type can describe a periodic orbit such as is postulated in the Bohr theory.* Furthermore these statements are true even if the distribution of charge in the electron is not the simple uniform surface distribution assumed by Lorentz.

It has been noted that equations (20) and (21) may be applied to the case of wholly retarded or wholly advanced fields provided the missing odd order terms are reinstated. For instance, the so-called radiation reaction is due to the third order term

$$E_{x3}=\mp\frac{e}{12\pi c^3}D^2\left(x^3D\left\{\frac{1}{r^2}\left(\frac{r}{x}\right)^2\right\}\right)=\pm\frac{e}{6\pi c^3}\dddot{x}=\pm\frac{e}{6\pi c^3}\dot{\mathbf{f}}_x$$

where the upper sign is to be taken in the case of retarded fields and the lower signs in the case of advanced fields. When integrated over the electron this term gives rise to the reaction $\pm(e^2/6\pi c^3)\dot{\mathbf{f}}$, which as is well known, is independent of the shape or distribution of charge of the particle. If the field is retarded, this reaction constitutes a damping term in the equation of motion, resulting in irreversible dissipation of energy, whereas if the field is advanced, the charge in sign makes it an accelerating term leading to an increase in energy of the moving particle.

By selecting the appropriate term in (20) the reaction on the electron of any order may be calculated by a simple integration.[7]

3. The Radiation Field

We wish to investigate the field at a point P at a distance from the orbit of the electron very great compared to the dimensions of the orbit itself. Choose some fixed point O in the neighborhood of the orbit (the nucleus of the atom, for instance) as origin, and denote by $\mathbf{M}$ a unit vector in the

[7] Integration formulas for the Lorentz electron have been published in Phys. Rev. **11**, 394 (1918)

direction OP. The terms in the expression (1) for the advanced electric intensity and in the corresponding expression for the retarded electric intensity which vary with the inverse square of the distance will be insignificant compared to those which vary with the inverse first power. Therefore both $\mathbf{E}_r$ and $\mathbf{E}_a$ at P are perpendicular to $\mathbf{M}$ and

$$\mathbf{H}_r=\mathbf{M}\times\mathbf{E}_r, \quad \mathbf{H}_a=-\mathbf{M}\times\mathbf{E}_a.$$

The Poynting flux is

$$\mathbf{s}=c\mathbf{E}\times\mathbf{H}=c(\mathbf{E}_r+\mathbf{E}_a)\times\{\mathbf{M}\times(\mathbf{E}_r-\mathbf{E}_a)\}=c(\mathbf{E}_r{}^2-\mathbf{E}_a{}^2)\mathbf{M}.$$

Now, if half the charge gives rise to a retarded field and half to an advanced field, expression (1) for the advanced electric intensity differs from the corresponding expression for the retarded electric intensity only in the sign of the vectors $\mathbf{c}$ and $\mathbf{v}$. That is, $\mathbf{E}_a$ is obtained from $\mathbf{E}_r$ by reversing the direction of the flow of time. Therefore during a complete period of the motion of the electron, $E_a{}^2$ goes through the same cycle of values as $E_r{}^2$ and the total energy passing through a unit cross section during this time vanishes. At any instant, however, $E_a{}^2$ and $E_r{}^2$ are not in general equal, and therefore there are back and forth fluctuations of energy. For this reason it cannot be concluded from the fact that the net radiation during a full period vanishes that the damping and accelerating terms in the equation of motion are at all times zero. It is necessary rather to carry out the analysis in section (2).

While simple series for the advanced field strength at a great distance from the charge may be deduced by the same method as has been previously published for the retarded field, it will be sufficient in the subsequent discussion to make use of the first approximation obtained by neglecting $\mathbf{v}$ as compared with $\mathbf{c}$ in (1). Hence

$$\mathbf{E}_r=(e/8\pi rc^2)\{[\mathbf{f}]_r\times\mathbf{M}\}\times\mathbf{M}, \tag{22}$$

$$\mathbf{E}_a=(e/8\pi rc^2)\{[\mathbf{f}]_a\times\mathbf{M}\}\times\mathbf{M}. \tag{23}$$

and, to a first approximation, the flux of energy is

$$\mathbf{s}=(e^2/64\pi^2r^2c^3)\{[f_s{}^2]_r-[f_s{}^2]_a\}\mathbf{M}, \tag{24}$$

where f_s is the normal component.

This expression consists of a positive and a negative term. The positive term represents the irreversible radiation of energy from the half of the charge which gives rise to a retarded field. The negative term represents an absorption of energy converging on the half of the charge which is responsible for the advanced field. In fact a net radiation of energy during a full period of the motion has been obviated by superimposing on the diverging wave system radiated out from the electron on the classical theory a system of converging waves bringing energy in to the electron from the outermost confines of space. These two sets of progressive waves traveling in opposite directions give rise to standing waves, and therein lies the chief difficulty with the theory. For these standing waves would be expected to cause a second electron over which they might pass to oscillate, and such oscillations would be expected to lead to detectable effects.

The standing waves can be illustrated very simply in the case of a charge executing linear simple harmonic vibrations. Let P be on the perpendicular bisector of the line of vibration. Then

$$[f]_r = A \cos\{\omega(t - r/c) + \delta\},$$
$$[f]_a = A \cos\{\omega(t + r/c) + \delta\},$$
$$E = (eA/4\pi rc^2)\cos(r/c)\cos(\omega t + \delta),$$
$$H = (eA/4\pi rc^2)\sin(r/c)\sin(\omega t + \delta).$$

Having obtained a set of potentials and intensities which are consistent with the classical differential equations of the electromagnetic field and which explain the possibility of motion in a periodic orbit without radiation, the question arises as to how radiation or absorption may occur during transitions from one stationary orbit to another. An answer which the preceding investigation suggests at once is that the retarded and advanced portions of the charge of the electron may not remain coincident during a transition. It is clear from (24) that if these two portions of the charge describe similar orbits of different sizes about the nucleus with the same frequency, emission exceeds or falls below absorption according as the retarded or the advanced portion of the charge describes the orbit of greater size.

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