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THE ABSORPTION COEFFICIENTS OF HOMOGENEOUS X-RAYS BETWEEN WAVE-LENGTHS 0.1 A AND 0.71 A

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Abstract

Using the general radiation from a tungsten target, dispersed by reflection from a crystal, the total mass absorption in *fourteen elements of atomic number 6 to 83* (C, Mg, Al, S, Fe, Ni, Zn, Ag, Sn, W, Pt, Au, Pb and Bi) was determined for various wave-lengths from .10 A to .51 A by the ordinary ionization measurements. The Kα and Kβ radiation from a Mo target was used to give values for wave-lengths .63 A and .71 A. Very good agreement was found with the results of Richtmyer and Duane wherever these are comparable. For the elements from C(6) to W(74) the results are represented within a few per cent by the formula $\mu/\rho=\tau/\rho+\sigma/\rho=C\lambda^{2\ 92}N^{2.92}+\sigma/\rho$, where $C=7.82(10)^{-3}$ for the K absorption and $1.00(10)^{-3}$ for the L absorption and the scattering coefficient σ/ρ is assumed to increase with N from .144 for Al to .50 for W at .12 A, and also with the wave-length. The jump in the value of the fluorescent absorption coefficient τ/ρ from L to K absorption is the ratio of C_K to C_L or 7.8. The atomic absorption $\tau_a=2.18(10)^{-26}\lambda^{2.92}N^4$ (Bragg-Owens law). For the heavier elements Pt, Au, Pb and Bi the values of C_K get progressively lower to $4.7(10)^{-3}$ and those of C_L increasingly higher to $1.12(10)^{-3}$, the ratio or jump decreasing to 4.1 for Bi.

Introduction

An excellent summary of our knowledge of the absorption of x-rays when they pass through matter is given by A. H. Compton in the October, 1922, Bulletin of the National Research Council. On page 32 there is a table giving the mass absorption coefficients of a number of elements from Li to Pb for wave-lengths from .1A to 1.0A. For the most part these results are taken from the works of Hewlett,[1] Richtmyer,[2] and Duane[3] and are for homogeneous rays from a tungsten target reflected from a calcite crystal.

A careful examination of these results shows a considerable divergence of the mass absorption given by the different observers for the same

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[1] C. W. Hewlett, Phys. Rev. **17**, 284 (1921)

[2] Richtmyer, Phys. Rev. **18**, 13, (1921)

[3] W. Duane, Proc. Nat. Acad., March 1922

substance and same wave-lengths. In the case of aluminum those of Hewlett are uniformly lower than those of Richtmyer. Those of Duane, over the short range investigated, agree well with those of Richtmyer. Only two results for the K absorption of an element of higher atomic number than that of silver are given, those of Hull and Rice[4] for lead, obtained eight years ago. It therefore seems desirable, considering the importance of the subject, to have a more complete set of absorption coefficients, comprising a greater range of wave-lengths and atomic numbers, especially the K absorption coefficients of the high atomic number elements. Experiments for this purpose have been carried out by the author during the last two years, and a summary of the results was given at the meeting of the American Physical Society at Cincinnati, December 1923. This paper gives the results and conclusions in a more detailed manner.

If a narrow beam of homogeneous x-rays passes through a layer of any substance whose thickness is d and density ρ the loss of energy is given by, $I_d = I_0 e^{-\mu d}$ where I_0 represents the energy on entrance, and I_d the energy on exit.

The quantity $\mu/\rho = (\log I_0 - \log I_d)/\rho d$, is known as the mass absorption coefficient. From μ/ρ can be calculated the atomic absorption coefficient μ_a by use of Avogadro's constant and the atomic weight.

The loss of energy represented by μ/ρ is due to the energy absorbed to produce the various fluorescent radiations represented by τ/ρ, and the energy lost by scattering etc. represented by σ/ρ.

Hence, $$\mu/\rho = \tau/\rho + \sigma/\rho$$

and, $$\mu_a = \tau_a + \sigma_a.$$

It has generally been accepted that these absorption coefficients can be represented empirically by the following expressions:

$$\tau/\rho = C\lambda^n N^p$$

$$\tau_a = C_a \lambda^n N^q$$

where λ is the wave-length in angström units; N the atomic number; C and C_a constants for each fluorescent series; n, p, q, empirical exponents.

Since μ/ρ alone can be measured by experiment it is necessary in some manner to assume values of σ/ρ, before τ/ρ can be estimated. In the range of λ from .1 A to 1.0 A, σ/ρ in general is a quite large part of μ/ρ, and for short wave-lengths and low values of N it is indeed the greater part, hence the uncertainty of σ/ρ renders confirmation of the formulas difficult. The most usual values accepted for the exponents are, $n=3$, $p=$ nearly 3, and $q=4$.

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[4] Hull and Rice, Phys. Rev. **8**, 836 (1916)

When for any substance a wave-length is reached which is very close to its emission wave-length Kγ, the absorption coefficient suffers a very abrupt change, becoming 8 times as great in the case of passing from L to K. If the value of $(\tau/\rho)_K$ for the K absorption just after this change takes place is known, and the value of $(\tau/\rho)_L$ for the same wave-length which would have been observed if this change had not taken place is also known, we can divide $(\tau/\rho)_K$ by $(\tau/\rho)_L$ and obtain what we may call the "jump" J from L to K series. If the above formulas are sufficient to represent the values of $(\tau/\rho)_K$ and $(\tau/\rho)_L$ over the complete possible range of N, this jump should always be the same, about 8, and consequently C_K and C_L should be constants.

EXPERIMENTAL ARRANGEMENTS

The modern method of measuring the absorption coefficients of homogeneous rays is well standardized, and it is not necessary to go into details here. The present work follows closely the methods used by Richtmyer and Duane.

A fine beam of the general radiation from a Coolidge Universal Tungsten tube was reflected from the (100) planes of a selected crystal mounted on a Bragg x-ray spectrometer. Both calcite and rock-salt crystals were used.

The resulting narrow spectrum was intercepted by a fine slit in the ionization chamber which was filled with methyl iodide vapor. Readings were carried out as follows. The zero position of the spectrometer was carefully determined by setting on the Kα lines of Tungsten and also on the Kα and Kβ lines of a Mo water-cooled tube. The tube was then excited to the desired voltage and the ionization chamber set to receive the definite wave-length to be tested. From the chamber angle 2θ, and the crystal spacing d, the value of λ was calculated from Bragg's classical equation $N\lambda = 2\,d \sin\theta$.

Ionization readings were then taken both before and after an absorbing layer was placed in position between the target and the crystal. For any wave-length chosen, a complete absorption test to extinction was always made on some substance, such as aluminum or copper. By plotting the log I against the thickness of layer, a resulting straight line would ensure that all corrections had been made, and one would be sure that he was dealing with a pure homogeneous radiation. Then, keeping this wave fixed in position and the voltage constant, measurements were made on all the other elements.

This method was used with all the wave-lengths observed from $\lambda = .1$ A to $\lambda = .5$ A. The measurements at $\lambda = .63$ A and $\lambda = .71$ A were made with the Mo tube the α line being observed in the first, second, and third orders, and the β line in the first and second orders.

Instrumental errors should not be greater on the average than 1 per cent. The chief difficulty, as has also been mentioned by Richtmyer, is to obtain very thin layers of the high atomic weight elements which are homogeneous. In all cases the mass of matter traversed was obtained by weighing a layer of known area.

RESULTS

The values of μ/ρ obtained in these experiments are shown in Table I. In Fig. 1 are plotted the results for the light elements C to S, in Fig. 2 those for the heavier elements Fe to Ag.

TABLE I

Mass absorption coefficients μ/ρ

Wave-length	C	Mg	Al	S	Fe	Ni	Cu	Zn	Ag	Sn	W	Pt	Au	Pb	Bi
.100A									1.04		3.80	3.40	3.60	3.40	2.90
.110			.165		.32	.37	.39	.46	1.23	1.50	4.36	4.10	4.20	3.92	3.50
.120	.148	.160	.175	.20	.38	.44	.47	.54	1.52	1.80	5.00	5.10	4.85	4.64	4.40
.130		.165	.185	.22	.43	.51	.56	.64	1.82	2.20	5.85	6.30	5.90	4.00	2.52
.138	.150	.170	.190	.24	.49	.57	.63	.74	2.15	2.55	6.70	7.30	6.00	2.90	2.20
.150		.175	.205	.27	.58	.67	.78	.89	2.63	3.00	8.10	4.30	3.60	2.37	2.44
.162	.152	.190	.218	.31	.68	.80	.94	1.06	3.18	3.60	5.00	2.60	2.63	2.78	2.90
.170	.155	.198	.228	.32	.70	.95	1.05	1.20	3.58	4.00	3.32	2.84	2.88	3.10	3.32
.180		.207	.243	.35	.82	1.10	1.22	1.36	4.10	4.70	2.53	3.24	3.30	3.52	3.86
.190	.160	.220	.256	.38	.93	1.27	1.40	1.57	4.70	5.40	2.76	3.70	3.76	4.00	4.42
.200		.232	.275	.42	1.07	1.45	1.55	1.77	5.50	6.30	3.20	4.16	4.28	4.64	5.10
.220	.165	.260	.310	.50	1.40	1.87	2.00	2.32	7.40	8.30	4.20	5.25	5.50	5.90	6.48
.240	.169	.293	.356	.58	1.75	2.28	2.50	2.80	9.60	10.8	5.10	6.65	6.95	7.40	8.30
.260		.330	.410	.68	2.22	2.83	3.05	3.50	12.2		6.10	8.00		9.50	10.4
.280	.182	.375	.475	.80	2.75	3.38	3.70	4.30	14.8		7.20	9.60		11.5	12.5
.300	.190	.430	.545	.93	3.30	4.10	4.50	5.10	17.9		8.60	11.5		13.6	14.8
.320	.200	.480	.630	1.08	3.95	4.87	5.25	6.20	21.1		10.1	13.5		16.2	17.7
.340	.210	.570	.730	1.23	4.65	5.75	6.38	7.30	24.5		12.0	15.8		19.7	21.0
.360	.216	.660	.850	1.42	5.40	6.70	7.55	8.50	29.2		14.5	18.2		23.0	
.380	.230	.760	.970	1.58	6.25		8.80	10.0	34.0		17.0	21.2		27.2	
.400	.245	.875	1.11	1.78	7.25		10.2	11.6	38.2		19.8	24.5		31.8	
.420	.260	.995	1.25	1.95	8.30		11.7	13.5	43.5		22.8	28.0		36.5	
.440	.275	1.13	1.41				13.2	15.4	49.5		26.5	31.8		41.2	
.450	.282	1.20	1.48				14.0	16.4	53.0		28.0	33.5		43.5	
.460	.291	1.27	1.56				14.9	17.5	37.0		30.0	36.0		46.5	
.484	.312	1.44	1.75				17.2	20.2	8.8		34.3	41.5		53.0	
.500			1.94				19.0	22.0	10.5		38.0	45.5		58.0	
.630	.55		3.78	6.9			37.8	43.0	20.5		75.0	87.0		101.	
.710	.68		5.35	9.9			53.7	60.0	28.5			119.		140.	

These results cover a range of λ and N which is about the same as investigated by Richtmyer, Hewlett, and Duane. There is extremely good agreement between the results of the author and those of Richtmyer, and also those of Duane in the case of Al, Cu, and Ag. The results for C agree well with those of Hewlett at short wave-lengths, but are generally higher than his at longer wave-lengths. All elements show a steady but slower decrease as λ approaches .1 A. There is no indication as yet of any abrupt change. In Figs. 3 and 4 are plotted those portions of the results for the high atomic number elements W, Pt, Au, Pb, and Bi between .10 A and .24 A, which include the critical absorption wave-lengths Kγ. It is to be noticed that the observations are carried right through the abrupt changes of μ/ρ.

These results for high weight elements show very clearly a distinct difference from similar results for lower weight elements. For example, if we compare the results for W and Bi, we see that in the L absorption

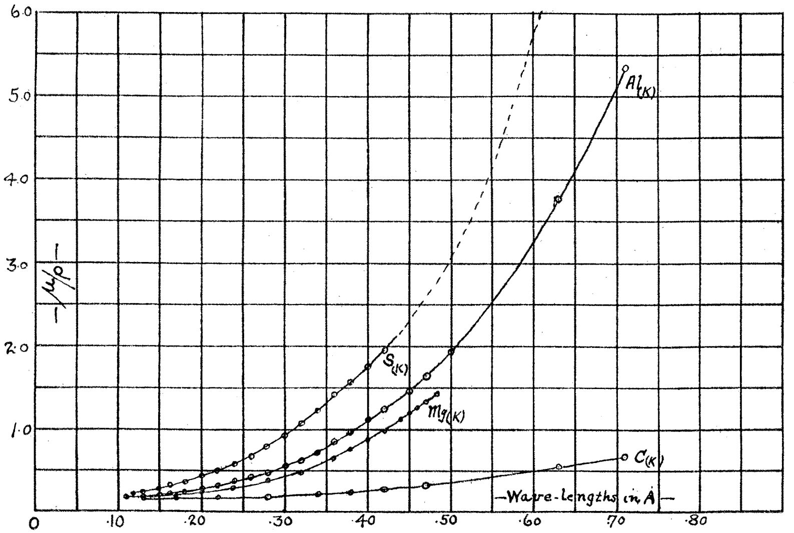

Fig. 1. μ/ρ as a function of wave-length for C, Mg, Al and S.

the graphs for Bi lie above that for W in accordance with the order of the atomic numbers, but in the K absorption this is reversed. The same phenomenon is true for all the heavy elements. In short, the sudden rise, or "jump" from L to K absorption is much less than would be

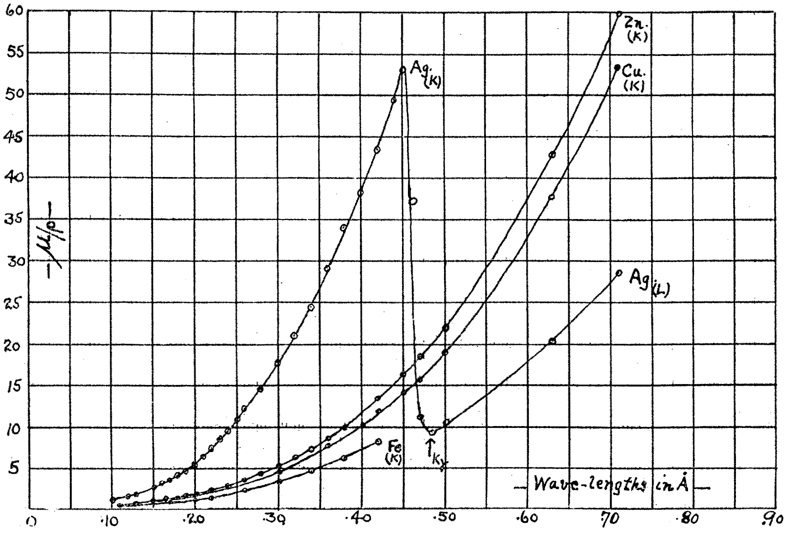

Fig. 2. μ/ρ as a function of wave-length for Fe, Cu, Zn and Ag.

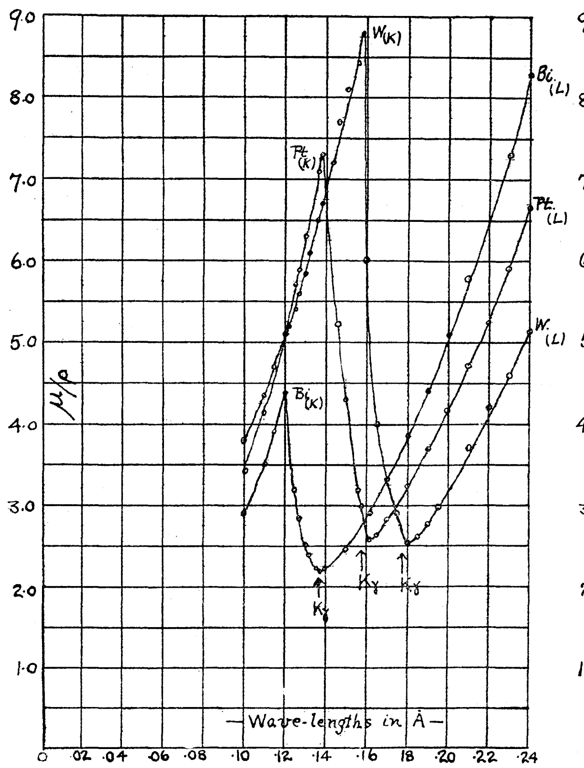

Fig. 3. μ/ρ as a function of wave-length, for Bi, Ptand W.

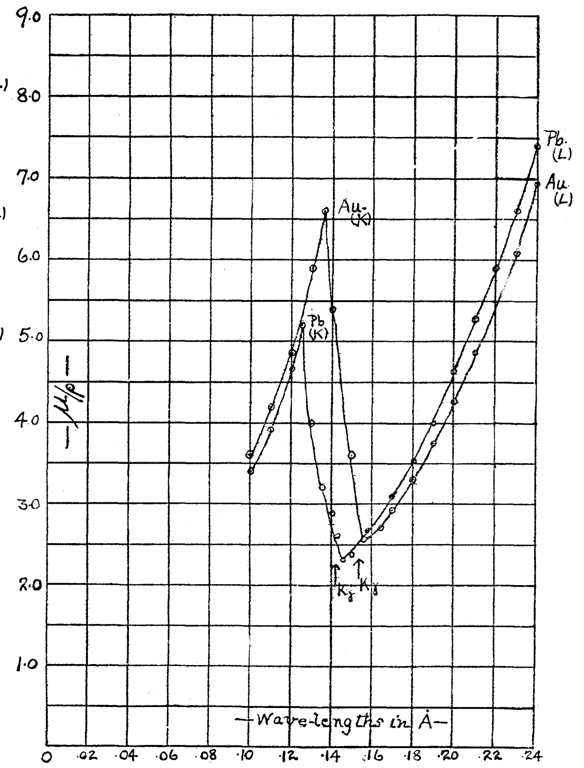

Fig. 4. μ/ρ as a function of wave-length, for Au and Pb.

predicted by extrapolation of the formulas given above, and moreover this jump becomes progressively less as N increases.

In Table II are shown these "jumps" J calculated from the graphs and corrected for scattering from assumed values of σ/ρ.

The values of $(\tau/\rho)_K$ are the highest K values experimentally obtained and λ is the wave-length at which they occurred. The values of $(\tau/\rho)_L$ are those at the same wave-length of the L absorption which would have been obtained if the "jump" had not occurred. With the high weight atoms are included also Ag, Mo (from Richtmyer) and Cu (from Barkla).

TABLE II

Relations between the K and L absorption coefficients at the absorption limit $K\gamma$

Element	N	λ	$(\mu/\rho)_K$	σ/ρ	$(\tau/\rho)_K$	$(\mu/\rho)_L$	$(\tau/\rho)_L$	J
Cu	29	1.46 A	440	—	440	55	55	8.0
Mo	42	.60	81	.40	80.6	11.0	10.6	7.6
Ag	47	.45	53.5	.45	53.0	7.5	7.0	7.5
W	74	.159	9.00	.55	8.45	1.95	1.40	6.0
Pt	78	.138	7.35	.63	6.72	1.90	1.27	5.3
Au	79	.136	6.65	.64	6.01	1.85	1.21	5.0
Pb	82	.125	5.25	.69	4.56	1.70	1.01	4.5
Bi	83	.120	4.40	.70	3.70	1.60	.90	4.1

It is readily seen that, while this "jump" in the absorption coefficients in passing from the L to the K levels is approximately 8 times for light elements, it becomes progressively less as N increases, showing a quite rapid decline from $N=74$ to $N=83$.

The values of J here given depend upon the values of σ/ρ assumed and, since we have no very exact knowledge of σ/ρ for high weight atoms, they can be considered at this stage as only approximate. There seems, however, no doubt that they progressively decrease with the increase of N.

Whether the absorption coefficients change so abruptly as to constitute a discontinuity is open to question. The results here indicated with high atomic number elements would seem to show that the abrupt rise slants backward towards shorter wave-lengths and that actual measurements can be made in this changing region, even with slit widths of .1 to .2 mm.

At the same meeting mentioned above, Richtmyer read a paper on the relations of the absorption coefficients in the immediate neighborhood of the K critical absorption, which showed results almost identical with those of the author, confirming the idea that the value of J decreases steadily with the increase of N.

A very careful analysis of all the values of μ/ρ in this work was made to test the accuracy of the empirical formulas discussed above. This was done by actual calculation by trial and error instead of the method usually employed of plotting μ/ρ against N^3. It was found that, if suitable values of σ/ρ were chosen, over a wide range of λ and N up to 47, the fluorescent absorption coefficient τ/ρ could be expressed by a formula of the form,

$$\tau/\rho = C\lambda^n N^p,$$

and the atomic coefficient by

$$\tau_a = C_a\lambda^n N^q.$$

σ/ρ was taken for Al at $\lambda = .12$ A as .144 and at $\lambda = 1.0$ A as .20, with increasing values as N increased.

It was found that on the average $n = 2.92$ (instead of the usual value 3), $p = 2.92$, and $q = 4$ best fitted the results.

TABLE III

Empirical data with reference to $\tau/\rho = C\lambda^{2.92}N^{2.92}$ *and* $\tau_a = C_a\lambda^{2.92}N^{4}$

Element	N	σ/ρ at $\lambda = .12$A	C_K ($\times 10^{-3}$)	C_L ($\times 10^{-3}$)	$(C_a)_K$ ($\times 10^{-26}$)	$(C_a)_L$ ($\times 10^{-26}$)
C	6	.145	7.76		2.21	
Mg	12	.144	7.48		2.05	
Al	13	.144	7.82		2.19	
S	16	.15	7.92		2.10	
Fe	26	.16	7.67		2.10	
Ni	28	.17	7.85		2.08	
Cu	29	.18	7.73	.97	2.14	.267
Zn	30	.20	8.02	1.02	2.20	.279
Mo	42	.28	7.47	.97	2.09	.269
Pd	46	.32	7.81	1.02	2.20	.287
Ag	47	.35	7.73	1.00	2.15	.275
Sn	50	.40	7.43	1.00	2.13	.288
W	74	.50	7.66	1.00	2.23	.290
Pt	78	.58	6.71	1.04	1.96	.304
Au	79	.60	6.33	1.03	1.84	.301
Pb	82	.67	5.42	1.08	1.59	.302
Bi	83	.70	4.71	1.12	1.37	.324

Extrapolation of these formulas to high atomic weight elements for the K series gave predicted values much higher than those actually measured. The accord was better in the case of the L absorption. The complete data are shown in Table III. The results for Mo are taken from Richtmyer and those for Sn from Bragg and Pierce. The universal constant $(C_a)_K$ is $= 2.18 \times 10^{-26}$ for $N = 2.92$ (K absorption) and $(C_a)_L = .29 \times 10^{-26}$ (L absorption). $C_K = 7.82 \times 10^{-3}$, and $C_L = 1.0 \times 10^{-3}$.

It would seem that these formulas do not hold throughout the whole range of λ and N, and certainly not near a critical absorption region.

Very notable attempts to deduce a theoretical formula for τ_a have been made recently by De Broglie[5] and by H. A. Kramers[6] on the quantum theory. Both of these formulas indicate a decrease to be expected in J as N increases, which would be in accord with the results of this paper.

UNIVERSITY OF CINCINNATI,
January 25, 1924.

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[5] L. de Broglie, Jour. de Phys. et Rad. 1922

[6] H. A. Kramers, Phil. Mag. Nov. 1923