THE CURRENT-VOLTAGE RELATION IN THE CORONA

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Abstract

Current-voltage relation for the corona discharge between a cylinder and a coaxial wire.—*An equation* is derived on the basis of a small region (of radius a) of intense ionization around the wire, outside of which the current is carried only by ions of one sign, whose space density ρ is constant. At the boundary between the two regions the field is assumed to be the minimum field required to start the corona, and the field beyond is taken to be the sum of the electrostatic field and that due to the space charge. Putting in the experimental result that a is a linear function of the applied voltage V, the equation for the current i is put in the form $i=cV(V-V_0)/(V_1-V)$, when c is a constant proportional to the mobility. *Measurements* with a brass tube 17.8 cm long and 4.75 cm inside diameter, through which a slow stream of dried or moist air was passed, show good agreement with the above equation as far as variation with V is concerned, for voltages from 4.7 kv to 8.8 kv, for temperatures from 290°K to 417°K and for moisture content up to 44 percent. The theory is evidently imperfect, however, as the absolute values of the mobilities come out from 2 to 4 times the values obtained in small fields although the variation with absolute temperature is correct.

THE relation between current and voltage in the corona discharge between a hollow cylinder and a wire placed at its axis has been studied by Almy,[1] who gave an empirical equation, and by Schaffers[2] and Townsend,[3] who have derived theoretical equations. Both Schaffers and Townsend make use of the fact that ionization takes place principally within a small region surrounding the wire, and obtain relations involving the mobilities of the ions. Schaffers' equation, however, yields mobilities as high as one hundred times as great as those obtained by direct measurement, and Townsend's equation is not in a convenient form, because it expresses the current as an implicit function of the voltage. These difficulties may be removed, as will be shown below, by a treatment of the problem somewhat similar to that of Schaffers, but taking account of the fact that the region of intense ionization expands as the voltage is increased.

From the fact that the ionization takes place almost wholly within a certain short distance of the wire, it follows that beyond this distance,

[1] Almy, J. C., Am. Jour. of Sci. (4), **12,** 175, 1901
[2] Schaffers, V., Phys. Zeits. **15,** 405, 1914
[3] Townsend, J. S., Phil. Mag. (6), **28,** 83, 1914

the current is carried by ions all of the same sign, constituting a space charge. For example, if the wire is positive, both positive and negative ions are formed in the region immediately surrounding the wire, but since the negative ions move in toward the wire, and the positive ions move outward, the space outside of the ionizing region contains positive ions only. Our attention will be directed principally to this region of space charge, i.e., outside of the region of intense ionization. If the field intensity is not too great, nor the gas pressure too low, the motion of the ions that carry the current is somewhat similar to the fall of very small particles through a viscous fluid, and their velocity is proportional to the intensity of the field.

The current density at any point is given by

$$j=\rho u=\rho kX$$

where ρ = volume density of charge; u = velocity of ions; k = mobility of ions; X = electric field. Hence the total current per unit length of wire is

$$i=2\pi r\rho kX. \tag{1}$$

If the current is small, the field X will be nearly that of the electrostatic case, namely,

$$X=-\frac{V}{r\log(R/r_0)}$$

where R = radius of cylinder; r_0 = radius of wire.

This expression, however, will not be used as we are confining our attention to the space outside of the region of intense ionization. The visible glow gives an indication of the extent of the ionization, and photographs[4] of the discharge show fairly definite limits to the luminous region. Let a = radius of the cylindrical surface which we shall regard as the inner boundary of the region in which Eq. (1) is applicable. Then the potential difference between the cylinder and the wire will be nearly equal to that between the cylinder and the surface $r=a$, since a is only a little greater than the radius r_0 and the ionizing potential of the gas is only a few volts; and to a fair approximation we may write

$$X=-\frac{V}{r\log(R/a)} \tag{2}$$

Combining this relation with Eq. (1),

$$i=-2\pi\rho k\frac{V}{\log(R/a)}. \tag{3}$$

This equation is not in convenient form for use, because the density of charge ρ depends upon the potential difference; but by means of Poisson's equation another relation may be obtained, which makes possible the elimination of ρ.

[4] Peek, F. W., Jr., Dielectric Phenomena in High Voltage Engineering, p. 76.

Assuming the dielectric constant of the gas to be unity, Poisson's equation for this case may be put in the form

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi}{dr}\right)=-4\pi\rho$$

where ϕ = potential. This equation is easily integrated, for it will be seen from Eq. (3), in which r does not appear, that ρ cannot be a function of the radius. Indeed, for a particular potential difference V, with the corresponding current i, all the other quantities in Eq. (3) are constants, and therefore ρ must be constant, or the charge density is uniform throughout the region considered. The first integration of Poisson's equation yields

$$r\, d\phi/dr=-2\pi\rho r^2+C_1.$$

Now a certain definite voltage V_0 is required to start the corona current, and corresponding to this starting voltage is a definite field intensity X_0 at the wire, which causes the air to break down, and may be called the "starting" field. When the voltage is raised above the starting value it seems reasonable to believe that the surface at which the "starting" field is maintained is no longer the wire but will be located on a cylinder of radius a in the gas. But this surface is the inner boundary of the region to which we are applying Poisson's equation, and therefore the constant of integration C_1 may be determined from the condition that $d\phi/dr=-X_0$ when $r=a$. But by comparison with Eq. (2),

$$X_0=-\frac{V_0}{a\log(R/a)}.$$

This condition gives

$$C_1=2\pi\rho\, a^2+\frac{V_0}{\log(R/a)}.$$

But both ρ and a are small, so that the first term of the right-hand member of the equation may be neglected,[5] and we may write

$$r\frac{d\phi}{dr}=-2\pi\rho\, r^2+\frac{V_0}{\log(R/a)}.$$

Integrating this equation between the limits a and R,

$$V=-\pi\rho(R^2-a^2)+V_0.$$

Now a^2 may be neglected in comparison with R^2, giving

$$V=-\pi\rho\, R^2+V_0. \tag{4}$$

Substituting in this equation the value of ρ from Equation (3),

$$i=\frac{2kV\,(V-V_0)}{R^2\log(R/a)}. \tag{5}$$

[5] In an extreme case, where $i=25$ (10^{-6}) amp. per cm, and $V=8.5$ kv, ρ is about 2.2 e.s., and a is always a small fraction of a centimeter, while X_0 (which is less than the last term of this equation) is about 39 kv per cm, or 130 e.s.u.

It remains to express the distance a in terms of other quantities. The surface $r=a$ must be closely related with the region from which light is emitted in the corona discharge. From the photographs mentioned above it is seen that the apparent size of the wire increases as a linear function of the applied voltage. It thus seems possible that a may be expressed as such a function:

$$a=\alpha+\beta V.$$

This will introduce an empirical feature into the resulting equation but it will be possible to obtain the values of α and β from the experimental curves. When the applied potential is just sufficient to cause the corona to start, the ionization begins at the surface of the wire, so that

$$a=r_0 \qquad \text{when} \qquad V=V_0,$$

and the equation may be put in the form

$$a=r_0+\beta(V-V_0).$$

Then

$$\log\frac{R}{a}=\log\frac{R}{r_0}-\log\left[1+\frac{\beta}{r_0}(V-V_0)\right].$$

The quantity $(\beta/r_0)(V-V_0)$ has usually the magnitude of a few tenths, so that the log $[1+(\beta)r_0)(V-V_0)=\beta/r_0(V-V_0)$, approximately. Eq. (5) may now be written

$$i=\frac{2kV(V-V_0)}{R^2\left[\log\frac{R}{r_0}-\frac{\beta}{r_0}(V-V_0)\right]}=\frac{2kV\ (V-V_0)}{R^2\frac{\beta}{r_0}\left[\frac{r_0}{\beta}\log\frac{R}{r_0}+V_0-V\right]}$$

Collecting constant quantities, and letting $2kr_0/R^2\beta=C$ and

$$\frac{r_0}{\beta}\log\frac{R}{r_0}+V_0=V_1$$

the equation may be put in the final form

$$i=\frac{CV\ (V-V_0)}{V_1-V}. \tag{6}$$

The accuracy of Eq. (6) was tested by comparison with available curves of the current-voltage relation,[6] and found to be very satisfactory. As a further check, a number of experimental determinations of this relation were made, and the range of conditions was extended by using different temperatures and different humidities of the air. Alternating current was used, and the starting potential was computed in each case from the curve of the current-voltage relation.

Experimental Method

The corona tube was of brass, 17.8 cm long and 4.75 cm in diameter inside, with a central copper wire, No. 29 B. & S. gauge, held in place by

[6] Univ. of Ill., Eng. Exp. Station, Bulletin No. 114, p. 37, 38.

glass plates attached to the ends of the tube by bakelite cement. The voltage applied to the tube was read by means of a potential transformer and an ordinary voltmeter, and the corona current was measured by a thermal cross in an evacuated tube, connected to a sensitive galvanometer. In order to remove the ozone, which forms rapidly, and to insure that the current was measured in air rather than in a mixture of air and ozone, a steady slow stream of air was maintained through the apparatus, passing through sulphuric acid and over phosphorus pentoxide before entering the corona tube, and escaping through a tube whose mouth was just below the surface of water, in order that the rate of bubbling might be used as a rough indication of the speed of flow. Possible diffusion of water vapor from the water back into the apparatus was prevented by the insertion of a second tube of pentoxide.

For a study of the effects of humidity, the air, instead of passing through the drying tubes, was forced in succession through two bottles of water, each provided with an electric heater, and was led around the drying tube at the outlet.

For the control of temperature, the corona tube was placed in a large wooden box, lined with asbestos, and provided with electric heaters. This box also contained brass cylinders about 20 cm long and 7.5 cm in diameter, through which the air passed. In order to insure complete drying, or complete saturation, the flow must evidently be quite slow, and at such speeds it is not practicable to heat the air by passing it through *small* heated tubes. The speeds were far below the critical velocity, and in the viscous flow thus obtained, the transfer of heat from the tube to the air is very slight. The cylinders gave satisfactory heating by providing reservoirs in which the air remained for some time, since they were of sufficient size to allow free convection currents. The controlling temperature measurements were made with copper-constantan thermocouples, two on the corona tube itself, and one in the air stream near the entrance to the tube. Other junctions were placed in the reservoirs of both dry and moist air streams, and in the second water bottle, which was outside of the wooden box, in a smaller box of its own, heavily lagged with hair felt.

It was found by trial that irregularities in readings could be avoided by keeping the rate of flow of air below a certain limit, and allowing about two minutes with the current off, between successive readings. To obtain definite humidities, the second water bottle was heated to some temperature below that of the corona tube, and it was assumed that the air was saturated at the temperature of the water. Since the rate of flow was always small, and the top of the bottle contained a considerable air space, this assumption seems to be justified. Before the air reached the corona tube, its temperature was raised, and consequently its relative humidity lowered, although the absolute humidity was unchanged. To insure this condition, the moist air was driven through the system for perhaps half an hour, or until the corona current had fallen to a steady value, before the first readings were taken. After each run with moist air, hot dry air was driven backwards through the corona tube and through the line as far as the water bottle, in order that when the temperature was subsequently lowered, there should be no condensation in the lines.

All observations were made at atmospheric pressure, which was about 74 cm of mercury.

Results

The results of the experiments are indicated in the following table, which shows the constants obtained for a number of current-voltage

Table I

Constant of equation: $i=\frac{C\,V(V-V_0)}{V_1-V}$

Group	No.	Temp	C (10^{-6} amp.)	V_0 (kv)	V_1 (kv)	k	moisture (per cent)
I	1	290°K	6.69	3.80	24.65	5.5	
	2	321	13.5	3.78	31.0	6.5	
	3	360	11.6	3.65	25.4	7.0	
II	4	298	4.98	2.94	22.9	3.3	
	5	417	8.99	2.55	29.6	4.4	
III	6	374	21.9	3.44	48.5	6.4	
	7	376	13.9	3.37	32.8	6.2	
	8	347	14.55	3.07	35.1	6.0	
IV	6	374	21.9	3.44	48.5	6.4	0
	9	376	7.51	2.63	30.6	3.5	44
V	7	376	13.9	3.37	32.8	6.2	0
	10	378	14.4	3.63	39.4	5.2	4.5
VI	8	347	14.55	3.07	35.1	6.0	0
	11	346	7.84	3.36	24.2	4.9	2.1
	12	346	8.20	3.42	27.4	4.0	8.3

The coefficient β, which has to do with the increase of the ionizing region, ranges from 0.00046 to 0.00105 cm. per e.s. unit of potential.

relations, some of which are represented on the curves. The mobilities were computed by means of the defining equations for C and V_1, which give, using electrostatic units,

$$k=\frac{CR^2}{2(V_1-V_0)}\log_e\frac{R}{r_0}.$$

For tabulation, the mobilities have been reduced to cm per sec. per volt per cm. Since the starting potential and the constants C and V_1 are very greatly affected by conditions of surface, only those runs should be compared that were taken within a short time of each other. Related runs are indicated in the table by the various groups, and comparisons should be made only *within* groups.

The effects of temperature and humidity are best indicated by the mobilities, and comparisons within groups are shown in Tables II and III.

TABLE II.

Effect of temperature on mobility

Run numbers	Ratio of temperatures	Ratio of mobilities
1,2	1.19	1.18
1,3	1.24	1.28
2,3	1.12	1.085
4,5	1.40	1.34
6,8	1.08	1.06
7,8	1.08	1.04

TABLE III

Effect of humidity on mobility

Run number	Temperature	Per cent moisture	Ratio of mobilities*
9	376°K	44	0.55
10	378	4.5	0.84
11	346	2.1	0.82
12	346	8.3	0.67

*Ratio of mobility in moist air to that in dry air.

The accuracy with which the equation represents experimental data is indicated on the plots, where in each case the curve is drawn by the equation, and the circled points show the direct result of experiment. It is evident that the equation fits the observations well within the limits of experimental error, over a considerable range of temperatures and humidities of the air. A further test is furnished by the computed mobilities. It has been found[7] that over a wide range of temperatures the mobilities of ions in air are proportional to the absolute temperature; and Table II shows that within groups of related runs, the mobilities computed by means of this equation exhibit that proportionality. The values obtained for the mobilities, however, are from two to four times as great as those usually found by direct measurement, which are of the

[7] Phillips, P., Roy. Soc. Proc. **78**, 167 (1906)
Kovarik, A. F., Phys. Rev. **30**, 415 (1910)
Erikson, H. A., Phys. Rev. **6**, 345 (1915)

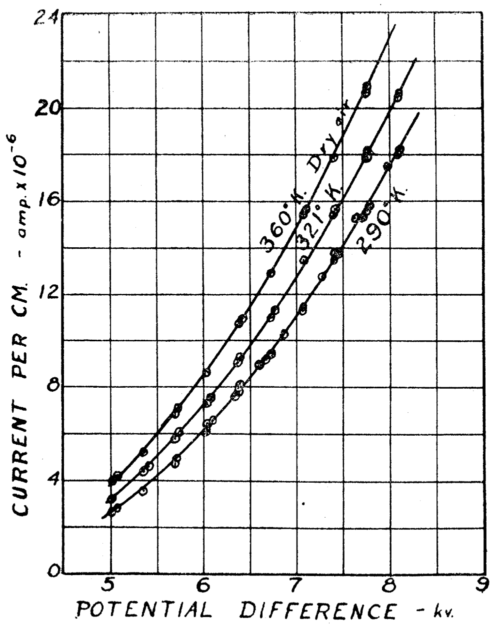

Fig. 1. Corona current curves for various temperatures, in dry air.

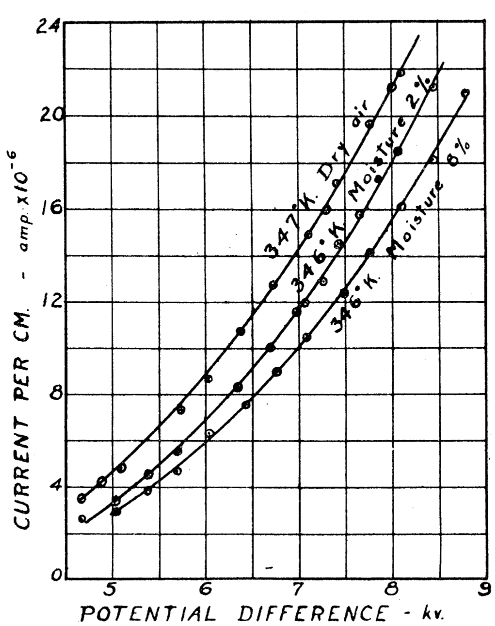

Fig. 2. Corona current curves for various humidities.

order of 1.6 to 2.0 (in terms of the volt) for the negative ion, and 1.3 to 1.6 for the positive ion.[8]

These high values of the mobility may be due to a combination of two causes. In the first place the effect of the space charge has not been completely taken into account. Its effect would be to diminish the field just outside the glow. The neglect of this effect would be to give values of the mobility too high as calculated from the equation. In the second place the presence of electrons would give rise to a higher value of k. Franck[9] has measured mobilities as high as 12.26 in cylindrical fields but his further investigation showed that this high value was due to the presence of electrons.

Table III shows the well known effect of water vapor in decreasing the mobilities.

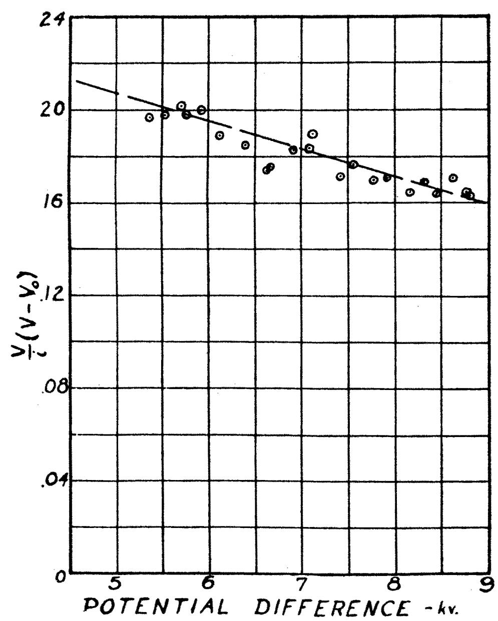

Fig. 3. $V(V-V_0)/i$ as a function of voltage.

Comparison with Previous Equations

Comparison of Eq. (6) with those of Almy, Schaffers, and Townsend, shows these points:

(1) Almy's equation fits experimental data fairly well over a limited range of voltages, but cannot fit over any wide range. According to his

[8] Kovarik, A. F., Roy. Soc. Proc., **86**, p. 154, (1912)
Lafay, A., Comptes Rendus, 173, p. 75, July, (1921)

[9] Franck, J. Ann. der Phys. **21**, 972 (1906)

equation, the quantity $V(V-V_0)/i$ should be a constant; while by Eq. (6) it is a linear function of the voltage. This relation is shown for a typical run in Fig. 3, in which, although the method of plotting brings experimental errors into prominence, it is clear that the value is not a constant, and a linear relation is at least a possible interpretation of the plot.

(2) Schaffers' equation fits experimental data satisfactorily over the range used, 5 to 9 kv, but yields mobilities as much as 100 times as great as those found by direct measurement.

(3) Townsend, by the use of his equation and Watson's[10] data, has computed mobilities in agreement with those found by direct measurement, but his equation is not very useful, because of its form.

(4) The equation developed in this article fits experimental data over the range of voltages used, and over a considerable range of temperatures and humidities. It possesses the following advantages over the other equations: it is accurate over a wider range of voltages than Almy's; is in a much more convenient form than Townsend's; and yields mobilities in rough agreement with known values, which Schaffers' equation fails to do.

UNIVERSITY OF MICHIGAN,
June 16, 1923.

[10] Watson, E. A., Electrician, **64,** 709, Feb. 11, (1910)