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THE RECOIL OF ELECTRONS FROM SCATTERED X-RAYS

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Abstract

Quantum theory of the recoil of electrons from scattered x-rays.—This is an extension of the quantum theory of scattering suggested by Compton, which assumes that each directed x-ray quantum is scattered by a single electron. Expressions for the *distribution of recoil velocities, of energies and of ranges* are developed for each of two postulates, assuming (1) the scattered radiation consists of directed quanta, and (2) the scattered radiation proceeds as spherical waves. On the first postulate the maximum recoil energy is shown to be $E_m = h\nu_0 \times 2\alpha/(1+2\alpha)$, where $\alpha = h/mc\lambda_0$; the recoil electrons are shown to be concentrated at angles near the direction of the primary beam; and from the distribution of energy, using a relation given by C. T. R. Wilson, the distribution of ranges is found to be such that two-thirds have tracks shorter than half the maximum range. The maximum range increases rapidly with frequency. The values for the maximum ranges in the case of x-rays (.34 to .48 A) are computed to be about one-third of those observed by Wilson for his fish tracts, but the difference may be due to the lack of homogeneity of the rays used. The relative number of recoil electrons to photo-electrons increases with the frequency and is in agreement with observations by Wilson. The second postulate, however, leads to a value for E_m only one-fourth that given above, a value which is inconsistent with that derived from a consideration of radiation pressure and which leads to values for maximum ranges one-fiftieth of those observed by Wilson. Other experimental observations are cited which also lead to the conclusion that the first postulate is much more likely to be true than the second, hence, that each quantum of scattered radiation is probably emitted in a definite direction.

In recent papers one of the writers has developed a quantum theory of the scattering of x-rays,[1,2,3] designed primarily to account for the change in wave-length observed when x-rays are scattered. The postulate upon which this theory is based is that x-rays are scattered quantum by quantum, each from a single electron. The change in momentum of the x-ray quantum on being scattered results in a recoil of the scattering

[1] Compton, Bull. Nat. Res. Council, No. 20, p. 19 (1922)
[2] Compton, Phys. Rev. **21**, 207 and 483, 1923
[3] Cf. also P. Debye, Phys. Zeits. April 15, 1923

electron with a velocity which may be a considerable fraction of the speed of light. When recoiling from the x-ray quanta which they have scattered, these electrons should appear as a type of secondary β-rays. The "recoil electrons" are, however, sharply distinguished from those secondary β-rays known as "photo-electrons," which are ejected with an energy comparable with $h\nu$, in that their energy is less by a factor of the order of γ/λ, where $\gamma = h/mc = 0.0242$ A.[4] In view of their relatively small energy, it is not surprising that at the time this theory was proposed the production of such recoil electrons by x-rays had not been observed. Evidence was, however, presented[5] to show that the secondary β-rays excited by hard γ-rays in the lighter elements are of this type.

Recently a new type of track has been observed almost simultaneously by C. T. R. Wilson[6] and by W. Bothe,[7] in photographs of the passage of x-rays through moist air. These tracks are very short compared with the usual photo-electron tracks, and occur in rapidly increasing numbers as the wave-length diminishes. A tentative suggestion is made by Bothe that these tracks are due to H particles ejected from the water vapor with an energy of about $h\nu$.* This hypothesis leaves unexplained, however, the fact noticed by Wilson that the short-range tracks always proceed in the initial direction of the x-ray beam. Wilson concludes that both the direction and range of the short-range tracks are in agreement with the suggestion that a single electron scatters an x-ray quantum and in so doing receives the momentum of the quantum. Evidence is given below which strongly supports this conclusion. Wilson's discovery of these recoil tracks, following upon the other successes of the theory, makes the evidence very convincing that the postulate of the scattering of whole quanta by individual electrons is sound.

There are, however, two essentially different methods by which an electron may scatter a quantum. In the postulate as first presented it was supposed that an electron receives the radiation quantum from a definite direction and scatters it in a different but equally definite direction. On this view the velocity and direction of recoil of the scattering electron will depend upon the angle at which the quantum is scattered. It may be imagined, on the other hand, that while the energy and momen-

[4] Cf. Compton, l. c.[1] p. 27

[5] Compton, l. c.[1] p. 71

[6] C. T. R. Wilson, Proc. Roy. Soc. A, **104** (Aug. 1, 1923)

[7] W. Bothe, Zeits. f. Phys. **16**, 319 (July 19, 1923)

*Note added March 6: In a second paper, in which Bothe studies these new rays by an ionization method (Zeits. f. Phys. **20**, 237, 1923), he shows that they are electrons instead of H particles. Their range, as he measures it, is slightly less than the theoretical value, Eq. (22), instead of slightly greater, as measured by Wilson.

tum of the primary quantum are received from a definite direction, the energy thus received is scattered in spherical waves in all directions. In this case every scattering electron will recoil in the direction of the primary ray with a momentum equal to the difference between the momentum of the primary ray and the resultant momentum of the spherical scattered ray. While the first form of the postulate is perhaps a more obvious consequence of the general quantum principle, the second form is in better accord with the interpretation of the quantum suggested by C. G. Darwin,[8] and has been used by C. T. R. Wilson in accounting for the short tracks observed in his photographs.[9] By studying the motions of the recoil electrons it should be possible to choose between these two forms of the quantum hypothesis.

Theory of the Recoil Electrons

Their energy. Using the assumption that each quantum of the primary radiation is scattered in a definite direction by a single electron, it has been shown[10] that the relative velocity of the recoil electron is

$$\beta=2\alpha \sin \tfrac{1}{2}\varphi \frac{\sqrt{1+(2\alpha+\alpha^2) \sin^2 \frac{1}{2}\varphi}}{1+2(\alpha+\alpha^2) \sin^2 \frac{1}{2}\varphi}, \tag{1}$$

where $\beta=v/c$ and $\alpha=\gamma/\lambda_0$, γ being $h/mc=0.0242$ A, and λ_0 being the wave-length of the incident x-rays. φ is the angle between the primary and the scattered x-ray quanta. The expression for the kinetic energy corresponding to this velocity, as derived first by Debye,[3] is somewhat simpler, being

$$E=h\nu_0\frac{2\alpha \sin^2 \frac{1}{2}\varphi}{1+2\alpha \sin^2 \frac{1}{2}\varphi}. \tag{2}$$

Debye shows further that the angle θ between the primary ray and the path of the recoil electron is given by the expression

$$\tan \theta = -1/(1+\alpha) \tan \tfrac{1}{2}\varphi. \tag{3}$$

Combining these two expressions, it follows that the energy of the recoil electron ejected at an angle θ with the incident ray is

$$E=\frac{2\alpha\ h\nu_0}{1+2\alpha+(1+\alpha)^2 \tan^2\theta}=\frac{2\alpha\ h\nu_0 \cos^2 \theta}{(1+\alpha)^2-\alpha^2 \cos^2\theta}. \tag{4}$$

The energy of the recoil electron is thus, for small values of α, nearly proportional to $\cos^2\theta$. Its maximum value is at $\theta=0$, where

$$E_m=h\nu_0\cdot 2\alpha/(1+2\alpha). \tag{5}$$

[8] C. G. Darwin, Nat. Acad. Sci. Proc. 9, 25 (1923)
[9] Wilson, loc. cit.[6], p. 15
[10] Compton, loc. cit.[2] p. 487

We may calculate the energy of recoil on the second scattering postulate if we notice that the total energy as well as the total momentum of the system (radiation+electron) is the same before and after scattering. The energy equation thus becomes

$$h\nu_0+0=\epsilon_s+mc^2\ (1/\sqrt{1-\beta^2}-1), \tag{6}$$

where ϵ_s is the energy of the scattered radiation; and the momentum equation is

$$h\nu_0/c+0=\mu_s+m\beta c/\sqrt{1-\beta^2}, \tag{7}$$

where μ_s is the momentum of the scattered radiation. To secure a relation between ϵ_s and μ_s we may now make the assumption that the incident radiation is scattered when the electron has a relative velocity $\bar{\beta}=\alpha/(1+\alpha)$, since this is the velocity which the electron must have in order to give the observed change of wave-length according to the Doppler principle.[11] To an observer moving forward with the scattering electron at the velocity $\bar{\beta}c$, the scattered radiation would appear distributed symmetrically in the backward and forward directions, and its total momentum would therefore be zero. The effective velocity of the radiant energy ϵ_s is hence $\bar{\beta}c$, and its resultant momentum is

$$\mu_s=(\epsilon_s/c^2)\bar{\beta}c=(\epsilon_s/c)\ \alpha/(1+\alpha), \tag{8}$$

where, as before, $\alpha=h\nu_0/mc^2$.

By eliminating ϵ_s and μ_s from Eqs. (6), (7), and (8), we find for the relative velocity of the recoil electron, $\beta=\alpha/(1+\alpha)=\bar{\beta}$. Thus the final velocity of recoil of the scattering electron is just that required on the Doppler principle to give rise to the observed change in wave-length. The kinetic energy of a recoil electron with this velocity is

$$E'=h\nu_0\cdot\frac{1}{\alpha}\left\{\frac{1+\alpha}{\sqrt{1+2\alpha}}-1\right\}=h\nu_0\cdot\frac{\frac{1}{2}\alpha}{1+2\alpha}\ (1-\tfrac{1}{4}\ \alpha^2+\ \ .\ .\ .). \tag{9}$$

Since α is usually small compared with unity, the energy of recoil according to this form of the quantum postulate is almost exactly ¼ of its maximum value (5) according to the first form of the postulate. This result (9) is in accord with the approximate values of the velocity and energy calculated on similar assumptions by one of the writers[4] and by C. T. R. Wilson.[6]

Attention should be called to a difficulty connected with this view of the scattering process. Eqs. (6), (7), and (8) state that the energy and momentum principles are satisfied and that the wave-length change shall be that which is experimentally observed. The equations do not, however, result in kinetic energy of the recoiling electron identical with that

[11] Compton, loc. cit.[2]

which we should calculate from the work done upon the electron by the radiation pressure. The impulse imparted to the electron by the radiation is obviously equal to the difference in momentum of the primary ray and of the scattered ray, i.e.,

$$\int f dt = h\nu_0/c - \mu_s;$$

or substituting the value of μ_s from Eqs. (6), (7), and (8),

$$\int f dt = (h\nu_0/c)\ (1-\alpha+\ .\ .\ .).$$

It is clear, however, that this impulse is imparted while the radiation is being scattered, that is, according to our assumptions, while the electron is moving forward with a velocity $\bar{v} = \alpha c/(1+\alpha)$. The work done on the scattering electron by the radiation pressure is hence

$$W = \bar{v}\int f dt = h\nu_0\,[\alpha/(1+\alpha)]\ (1-\alpha+\ .\ .\ .). \tag{10}$$

Instead of being equal to the final kinetic energy of the recoiling electron, as given by Eq. (9), this amount of work is about twice as great. It seems

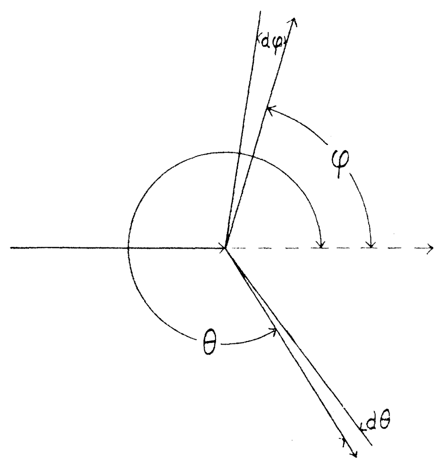

Fig. 1. Electrons which scatter the x-rays in directions between φ and $\varphi+d\varphi$ recoil in directions between θ and $\theta+d\theta$.

impossible to develop a scattering theory on the second form of the quantum postulate (that each scattered quantum proceeds in all directions) without encountering some inconsistency of this character. From the theoretical standpoint we should therefore prefer expression (4) to expression (9) as a statement of the energy of the recoil electrons.

Distribution of the recoil electrons. Let us now determine, on the view that the scattered rays proceed in definite directions, the relative number of electrons which will recoil at different angles. We may suppose, as in Fig. 1, that if the scattered ray proceeds at an angle between φ and $\varphi+d\varphi$ with the incident ray, the recoil electron moves at an angle between

θ and $\theta+d\theta$. Then if $P_\varphi d\varphi$ is the probability that a scattered quantum will lie between φ and $\varphi+d\varphi$, and if $P_\theta d\theta$ is the probability that a recoil electron will be ejected between the angles θ and $\theta+d\theta$, it is clear that

$$P_\theta d\theta = P_\varphi\, d\varphi. \tag{11}$$

In an earlier paper it was shown that[12]

$$P_\varphi\, d\varphi = \tfrac{3}{8}\sin\varphi\, d\varphi\, \frac{(1-\beta^2)\left\{(1+\beta^2)\,(1+\cos^2\varphi)-4\beta\cos\varphi\right\}}{(1-\beta\cos\varphi)^4},$$

where $\beta=\alpha/(1+\alpha)$. In terms of α this becomes,

$$P_\varphi\, d\varphi = \tfrac{3}{8}\sin\varphi\, d\varphi\, \frac{(1+2\alpha)\left\{1+\cos^2\varphi+2\alpha(1+\alpha)\,(1-\cos\varphi)^2\right\}}{(1+\alpha-\alpha\cos\varphi)^4}. \tag{12}$$

It follows from Eq. (3) that

$$\tan\tfrac{1}{2}\varphi = -\frac{1}{1+\alpha}\cdot\frac{1}{\tan\theta},$$

whence

$$\left.\begin{aligned} \sin\varphi &= -\frac{2(1+\alpha)\tan\theta}{(1+\alpha)^2\tan^2\theta+1} \\ \cos\varphi &= \frac{(1+\alpha)^2\tan^2\theta-1}{(1+\alpha)^2\tan^2\theta+1} \\ d\varphi &= \frac{2(1+\alpha)\,d\theta}{\cos^2\theta\left[(1+\alpha)^2\tan^2\theta+1\right]}. \end{aligned}\right\} \tag{13}$$

Substituting these values in Eq. (12) we obtain for Eq. (11),

$$P_\theta d\theta = -\frac{3(1+\alpha)^2\,(1+2\alpha)\left\{(1+\alpha)^4\tan^4\theta+(1+2\alpha)^2\right\}}{\left\{(1+\alpha)^2\tan^2\theta+(1+2\alpha)\right\}^4}\cdot\frac{\sin\theta}{\cos^3\theta}d\theta.$$

When we write $a=(1+\alpha)^2$ and $b=(1+2\alpha)$, this becomes

$$P_\theta d\theta = -\frac{3ab(a^2\tan^4\theta+b^2)}{(a\tan^2\theta+b)^4}\;\frac{\sin\theta}{\cos^3\theta}d\theta. \tag{14}$$

The probability that a recoil electron will strike unit area placed at a distance R and at an angle θ is

$$P_{R,\theta} = -\frac{P_\theta d\theta}{2\pi\, R^2\sin\theta d\theta} = \frac{3}{2\pi R^2}\;\frac{ab(a^2\tan^4\theta+b^2)}{(a\tan^2\theta+b)^4\cos^3\theta}. \tag{15}$$

The total number of recoil electrons is, however, equal to the total number of scattered quanta, which has been shown to be,[13]

$$n=(8\pi/3)\ I\ Ne^4/b\ h\nu_0\ m^2c^4,$$

where I is the energy per square cm of the incident ray whose frequency is ν_0, and N is the number of electrons effective in scattering the x-rays.

[12] Compton, loc. cit.[2] p. 492
[13] Compton, loc. cit.[2] p. 493

Combining this with Eq. (15), we find for the number of recoil electrons per unit area,

$$nP_{R,\theta}=4\frac{IaNe^4}{h\nu_0\ R^2m^2c^4}\cdot\frac{a^2\tan^4\theta+b^2}{(a\tan^2\theta+b)^4\cos^3\theta}. \tag{16}$$

Multiplying this by the energy of each recoil electron (Eq. 4), we find for the total energy of the recoil electrons which, if undeviated after scattering the x-ray, should traverse unit area at a distance R and an angle θ with the primary x-ray beam,

$$I_r=\frac{8Ia\alpha\ Ne^4}{R^2m^2c^4}\cdot\frac{a^2\tan^4\theta+b^2}{(a\tan^2\theta+b)^5\cos^3\theta}. \tag{17}$$

The concentration of the effect due to the recoil electrons at angles near the direction of the primary beam becomes apparent when we plot from this Eq. (17) the energy per unit solid angle of the recoil electrons ejected in different directions. This is done in curve A of Fig. 2 for such great

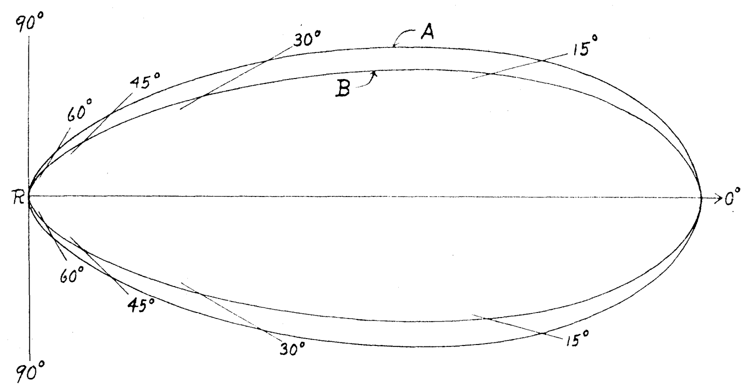

Fig. 2. Spatial intensity distribution of the recoil electrons calculated: A for long waves and B for very short waves ($\lambda=.024$ A) showing a strong concentration near the direction of the incident x-rays.

wave-lengths that a and b are sensibly equal to 1, and in curve B for $\lambda=0.024$ A, corresponding to hard γ-rays. It will be seen that the form of the distribution curve varies but slightly with the wave-length. It is easy to see from this figure, if it is these recoil electrons which constitute the secondary β-rays excited by γ-rays in light elements, how one might conclude with Rutherford[14] and Wilson[6] that the β particles are ejected nearly in the direction of the incident x-rays or γ-rays.

Energy and range of recoil electrons. It remains to determine the probability that a recoil electron will be ejected with a definite energy. If an

[14] E. Rutherford, "Radioactive Substances etc.," p. 276

electron recoiling at an angle θ has an energy E, the probability that the energy of recoil will lie between E and $E+dE$ is

$$P_E dE = P_\theta d\theta. \tag{18}$$

But according to Eq. (4),

$$\tan^2 \theta = k/aE - b/a,$$

where $k = 2\alpha h\nu_0$. Thus

$$\frac{\sin\theta}{\cos^3\theta}\, d\theta = \tfrac{1}{2}\, d(\tan^2\theta) = -\frac{kdE}{2aE^2}.$$

Substituting these values in Eq. (14), and noting according to Eq. (5) that the maximum energy of a recoil electron is $E_m = k/b$, Eq. (18) becomes

$$P_E dE = \tfrac{3}{2}\left(1 - 2\,\frac{E}{E_m} + 2\,\frac{E^2}{E_m{}^2}\right)\frac{dE}{E_m}. \tag{19}$$

This expression is of the same simple form whatever the frequency of the primary rays. In view of the fact, however, that E_m increases with the frequency, the formula can be applied strictly only in case the incident x-rays are homogeneous. Even in this case it is obvious that a correction will usually have to be made for the energy required to remove the recoil electron from the atom.

C. T. R. Wilson has found[6] that the length of his β-ray tracks is proportional to the square of the energy of the β-ray. Writing s for the length of the track and $1/p^2$ as the constant of proportionality, this fact may be expressed as $s = E^2/p^2$, or $E = ps^{\frac{1}{2}}$. When this value of E is substituted in Eq. (19), the probability that the length of the track of a given recoil electron will lie between s and $s+ds$ is found to be,

$$P_s ds = \tfrac{3}{4}\ (1 - 2\ \sqrt{(s/s_m)} + 2(s/s_m)\ ds/\sqrt{s_m s}, \tag{20}$$

whence,

$$P_s s_m = \tfrac{3}{4}\ [\sqrt{(s_m/s)} - 2 + 2\ \sqrt{(s/s_m)}\ ], \tag{21}$$

where s_m is the maximum length of the recoil electron tracks.

If we calculate the relative number of tracks of different lengths s/s_m, we find, according to Eq. (21), the values plotted in Fig. 3. It is found that more than two thirds of the tracks are of less than half of the maximum range, and more than one third are of less than one tenth the maximum range. The value of this maximum range may be calculated from the expression for the maximum energy, Eq. (5). Combining this with $s = E^2/p^2$, and writing $\alpha = h\nu_0/mc^2$, we have

$$s_m = \frac{1}{p^2}\cdot\frac{4h^4\ \nu_0{}^4}{(mc^2 + 2h\nu_0)^2}. \tag{22}$$

It is thus seen that the maximum range increases rapidly with the frequency. These results may be subjected to experimental test.

Another experimental test can be made by comparing the lengths of the photo-electron tracks with those of the recoil electrons. If homogeneous x-rays of frequency ν_0 are used, the highest speed photo-electrons will possess energy $h\nu_0$, whereas the highest speed recoil electrons will possess energy $h\nu_0 \times 2\alpha/(1+2\alpha)$ according to Eq. (4), or approximately $h\nu_0 \times \alpha/2(1+2\alpha)$ according to Eq. (10). The ratio of their energies will thus be either $2\alpha/(1+2\alpha)$ or $\alpha/2(1+2\alpha)$ respectively. The corresponding

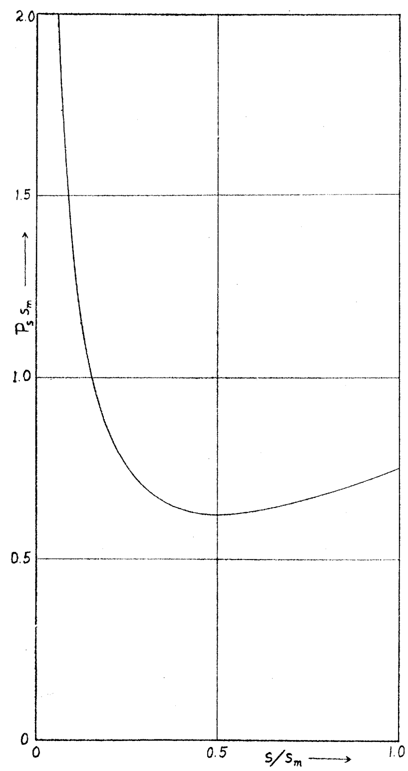

Fig. 3. Most of the recoil electrons have ranges in air less than half the maximum range s_m.

ratios of the lengths of the paths of the photo-electrons and the recoil electrons are

$$R = 4\ \alpha^2/(1+2\alpha)^2 \tag{23}$$

and

$$R' = \alpha^2/4\ (1+2\alpha)^2. \tag{23'}$$

Number of recoil electrons. Each recoil electron represents the loss of one quantum of energy from the primary beam, just as does each photo-electron. It follows that the ratio of the number of recoil electrons to the number of photo-electrons should be equal to the ratio of the x-ray energy spent in scattering to that spent in exciting photo-electrons. Thus if the total absorption coefficient of the x-rays in the medium is written as $\mu=\tau+\sigma$, where τ represents the energy spent in exciting photo-electrons and σ that dissipated by scattering, the ratio of the number of recoil electrons to the number of photo-electrons is

$$n_r/n_p=\sigma/\tau. \qquad (24)$$

This ratio may be estimated approximately from absorption measurements. Since it is found that τ is proportional to λ^3, whereas σ is nearly independent of the wave-length, it follows that for great wave-lengths the photo-electrons will predominate, whereas for small wave-lengths the recoil electrons will be greater in number.

Comparison with Wilson's Cloud Expansion Experiments

Number of tracks. Using the experimental data of Hewlett,[15] we estimate for the wave-length 0.5 A at which Wilson begins to observe the tracks which he attributes to recoil electrons, that $\tau/\rho=0.3$ and $\sigma/\rho=0.2$ per gram, where ρ is the density of the air. Thus there should be about 1.5 times as many photo-electrons as recoil electrons for this wave-length. If his shortest x-rays were about 0.3 A, they should, by similar calculation, have produced about 3.5 times as many recoil electrons as photo-electrons. These numbers are in satisfactory accord with Wilson's observation[6] that the relative number of short range tracks increases rapidly as the wave-length decreases, being greater in number than the long range tracks for wave-lengths shorter than about 0.45 A. This agreement affords a strong confirmation of his conclusion that these short tracks are due to recoil electrons.

Wilson observes in general a predominance of "sphere" tracks over the short range or "fish" tracks. As the frequency increases the sphere tracks increase rapidly in number,[6] and are accompanied by the development of fish tracks. These observations coincide in detail with what would be expected in view of Eqs. (21) and (22). The great predominance of points or sphere tracks in Wilson's photographs may be attributed to the relatively great probability of tracks of vanishingly small range, and on the other hand, the rapid development with increasing frequency of these points into tracks of measurable length is in agreement with the expression for the maximum range of a single particle as a function of the frequency, given in Eq. (22). The close general agreement between the experiments and the theory leads to the conclusion that the sphere tracks as well as those of definite range must be considered together in any study of x-ray scattering. In order that quantitative comparisons be-

[15] C. W. Hewlett, Phys. Rev. **17**, 284 (1921)

tween experiment and theory may be made, it is desirable that data be secured on the tracks produced by homogeneous x-rays.

Range of tracks. Eqs. (23) show that the recoil electrons with the longest paths should go 16 times as far on the directed quantum hypothesis as on the spherical radiation view. Wilson observes that "When the x-rays are hard enough to eject β-particles of 1.5 cm range, fish tracks of ranges up to about 0.4 mm appear; their range increases as the frequency of the incident radiation is increased, but rarely exceeds 1.5 mm, even when the long tracks have a range exceeding 3 cm."[6] The wave-length required to produce a photo-electron track of 1.5 cm length, according to Wilson's data, is about 0.48 A, whence $\alpha = .0242/.48 = .050$. According to Eq. (23) the longest recoil tracks should thus be $1.5 \times 4\alpha^2/(1+2\alpha)^2 = 0.12$ mm. While this is considerably shorter than the observed tracks of 0.4 mm, it is at least of the correct order of magnitude. Eq. (23), however, would predict a track of only 0.008 mm length, which is very much too short.

Similarly, corresponding to the long tracks of 3 cm range, for which the wave-length is 0.34 A, Eq. (23) predicts recoil tracks of 0.5 mm length and Eq. (23′) of 0.03 mm. The difference between the theoretical range of 0.5 mm and the observed range of 1.5 mm is perhaps no greater than might result from the fact that heterogeneous x-rays were used by Wilson in these experiments. For the number of photo-electrons excited in air increases rapidly with increasing wave-length whereas the prominence of the recoil electrons decreases with increasing wave-length. Thus the effective wave-length for the photo-electrons must have been greater than that for the recoil electrons. This consideration certainly accounts for a part of the difference between the theoretical and the experimental values. In order to obtain a more exact test of Eq. (23) it will be necessary to excite the recoil electrons by more nearly homogeneous x-rays.

The present experiments of Wilson suffice to show, however, that Eq. (23′), which leads to a range differing from the experimental value by a factor of about 50, is not correct. This indicates that we must abandon the assumption upon which the equation is based, that the scattered radiation is emitted in spherical waves. Both from the standpoint of the experimental evidence and from the internal consistency of the theory *we therefore seem forced to the conclusion that each quantum of scattered x-rays is emitted in a definite direction.* It would appear but a short step to the conclusion that all radiation occurs as definitely directed quanta rather than as spherical waves.

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