THE PHYSICAL CRITERION FOR DETERMINING THE PITCH OF A MUSICAL TONE

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Abstract

Effects upon pitch and quality of musical sounds of eliminating certain component frequencies.—A high quality telephone system was used to reproduce musical sounds from the voice, the piano, the violin, the clarinet and the organ without any appreciable distortion. Into this telephone system electrical filters were introduced which made it possible to eliminate any desired frequency range. Results with this system show that only the quality and not the pitch of such musical sounds changes when a group of either the low or high frequency components is eliminated. Even when the fundamental and first seven overtones were eliminated from the vowel *ah* sung at an ordinary pitch for a baritone, the pitch remained the same. These results were checked by a study of synthesized musical tones produced by ten vacuum tube oscillators, with frequencies from 100 to 1000 at intervals of 100. It was found that three consecutive component frequencies were sufficient to give a clear musical tone of definite pitch corresponding to 100, and that in general when the adjacent components had a constant difference which was a common factor to all components, a single musical tone of pitch equal to this common difference was obtained, but not otherwise. *Explanation of the results in terms of mechanism of the ear.* Recent work on hearing has shown that the transmission mechanism between the air and the inner ear has a non-linear response which accounts for the so-called subjective tones. When the components of low frequency are eliminated from the externally impressed musical tone, they are again introduced as subjective tones before the sound reaches the nerve terminals. Calculation of the magnitude of these subjective tones from the non-linear constants of the ear shows that the results on pitch are what might be expected.

Sound spectra of ten typical musical sounds, obtained with an electrical automatic harmonic analyser to be described by Wegel and Moore, are given for *ah* sung at pitch d, $\bar{a}$ sung at a, piano c_1, piano c', violin g', clarinet c, organ pipe c_1 for three pressures, and organ pipe c'.

THIS paper describes some experiments which show that the fundamental and a large number of harmonics from a compound tone may be eliminated without changing the pitch of the tone. This is contrary to the general belief that the pitch of a musical tone is determined by the vibration frequency of the fundamental. Although the idea of producing a musical tone which has a lower pitch than that corresponding to any single component frequency is not entirely new, the extent to which this method of producing low pitched tones can be carried has not been appreciated.

Composition of Musical Tones

A musical tone is usually composed of a large number of component frequencies. In the voice and in most musical instruments, it is well known that these components are harmonics, i.e. the component frequencies stand in the ratios of the integers 1, 2, 3, etc.; the characteristic quality of the tone is determined by their relative magnitudes.

Sound spectra for various types of musical sounds are shown in Fig. 1. In the charts shown in this figure the abscissas give the frequency in

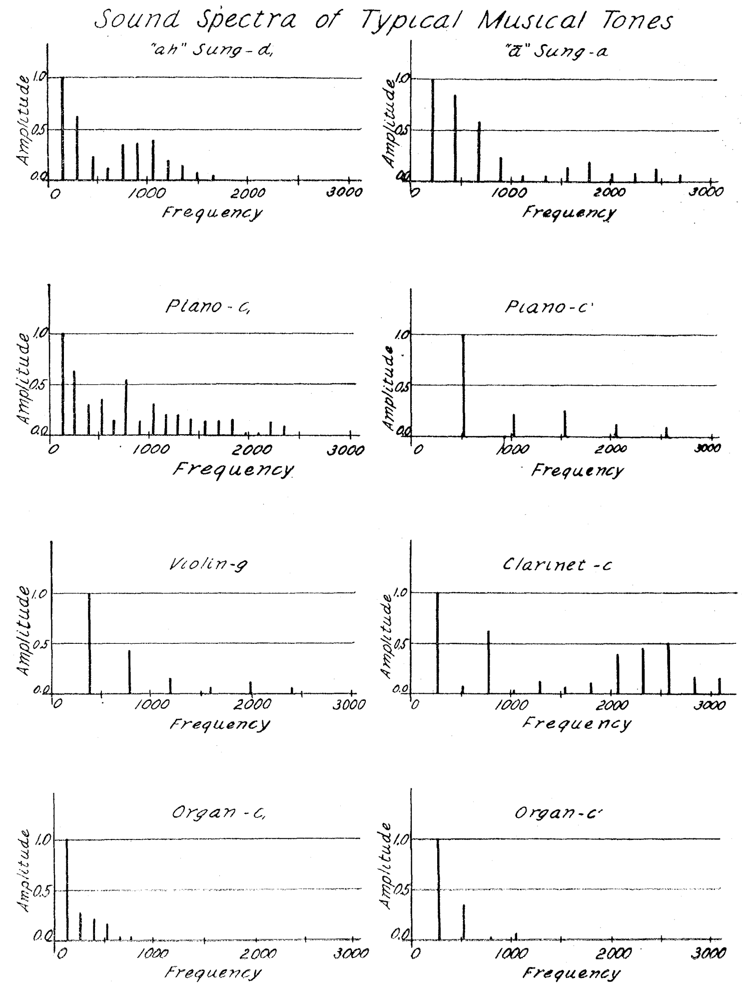

Fig. 1

double vibrations, or cycles per second. The ordinates give the relative amplitude of the harmonics, plotting the fundamental as unity. The spectra for the piano represent the average values of the components during the first few seconds after the strings are struck. The sound spectra for organ pipes change very markedly when the blowing pressure changes. Those shown represent a typical case.

The analyses of these tones were made by the electrical harmonic analyzer which has been recently developed in the Bell System Research Laboratories. A full report covering the design and operation of this analyzer will soon be published by Wegel and Moore,[1] who are responsible for its development. The sounds to be analyzed are picked up by a condenser transmitter[2] which converts the sound wave into a faithful electrical copy. This electrical copy of the sound wave is then amplified and sent into the electrical harmonic analyzer for analysis. The analyzer consists essentially of a resonant circuit for which the frequency of resonance may be varied from 80 to 6,000 in steps of only a few cycles. Consecutive readings of the current transmitted through this circuit for each value of the resonant frequency are recorded photographically. The operations are entirely automatic, so that it requires only about five minutes from the time the machine is started until a completed picture of the sound spectrum comes out of the camera-box.

Measurement of the Pitch of a Musical Tone

The pitch of a tone as usually understood is the place on the musical scale to which the tone is referred. A pure tone,—that is, one consisting of a single component or frequency—has a definite pitch which anyone experienced in music will correlate with that frequency. By changing the frequency of such a pure tone its pitch can be made the same as that of any compound musical tone. Consequently a vibration frequency number can be associated with each pitch although, as will be seen presently, a component having such a frequency may not be present in the musical tone. The pitches as used in this paper are expressed on the International Tempered Scale, $a = 435$ cycles per second.

The pitch of the musical tones used in this investigation were determined by comparison with a standard, giving a fairly pure tone. This standard tone was produced by a telephone receiver, which was connected to a vacuum tube oscillator. Its pitch was adjusted to any desired value by making the proper setting on the oscillator.

The question might be asked, What happens to a tone if some of the upper harmonics are eliminated? Its is well known that the pitch is

[1] Wegel and Moore, presented before A.I.E.E. at Philadelphia, Feb. 8, 1924
[2] Wente, Phys. Rev. (2) **10**, 39, 1917

TABLE 1

Effect of the Elimination of Various Components on the Pitch and Quality of Various Musical Sounds

Source	Pitch	Eliminated components	Eliminated frequencies	Pitch change	Quality
Voice–*ah*	d(145)	F	0–250	No change	Inappreciable change
		F & 1–2	0–500	" "	Small change
		F & 1–4	0–750	" "	Large change
		F & 1–7	0–1250	" "	Very large change
		F & 1–9	0–1500	Uncertain	Noise
		6–∞	1000–∞	No change	Small change
		3–∞	500–∞	" "	Large change
		F& 1–2 & 6–∞	0–500 & 1000–∞	" "	Very large change
Voice–*ā*	a(218)	F	0–250	No change	Slight change
		F & 1–2	0–750	" "	Sounds like *ah*
		F & 1–4	0–1250	" "	Small change
		F & 1–5	0–1500	" "	Between *ah* & *ō*
		6–∞	1500–∞	" "	"
		3–∞	750–∞	" "	Sounds like *ō*
		F & 1–2 & 8–∞	0–750 & 2000–∞	" "	Very weak *ah*
Piano	c(129)	F	0–250	No change	Small change
		F & 1–2	0–500	" "	Metallic
		F & 1–5	0–750	" "	Clanging
		For more harmonics eliminated the tone lost all musical character			
		6–∞	750–∞	No change	No brilliance
Piano	c''(517)	F′	0–750	No change	Small change
		F & 1	0–1250	" "	Metallic
		All harmonics	750–∞	" "	Pure tone Musical brilliance lacking
Violin	g'(388)	F	0–500	No change	Large change
		F & 1	0–1000	" "	Very large change
		F & 1–2	0–1500	Uncertain	Non-musical
		2–∞	1000–∞	No change	Violin quality gone
Clarinet	c'(259)	F	0–500	No change	Large change
		F & 1–2	0–1000	" "	Very large change
		F & 1–4	0–1500	" "	Non-musical
		7–∞	2000–∞	" "	Large change
		2–∞	750–∞	" "	Pure tone (no clarinet quality)
Organ pipe	c(129)	F	0–250	No change	Small change
		F & 1–2	0–500	" "	Large change
		F & 1–4	0–750	Uncertain	Noise
		15–∞	2000–∞	No change	Very small change
		6–∞	750–∞	" "	Small change
Organ pipe	c'(259)	F	0–500	No change	Large change
		F & 1–2	0–1000	Uncertain	Non-musical
			2000–∞	No change	Small change
			750–∞	" "	Sounds dull

unaltered, but the quality of the tone is changed. An equally interesting question is, What happens to a tone if the fundamental and some of the lower harmonics are eliminated? The answer to this question is the same as to the first one, namely, the pitch remains the same but the quality is altered.

Results

For the purpose of obtaining quantitative data relating to this question, use was made of the high-quality telephone system which has been described in previous papers.[3] The efficiency of this telephone system is approximately constant for the various frequencies throughout the range used in this investigation. Also, for the range of intensities which were used, the system has a linear response, so that no frequencies other than those in driving force were introduced. Consequently, the sound coming out of the receiver is a faithful copy of that which goes into the transmitter. Electrical filters[4] were introduced into this system, so that any portion of the spectrum could be eliminated. The characteristics of these filters were such that the amplitudes of the eliminated harmonics were reduced to values between .001 and .0001 of those obtained without the filter.

The results of the tests with the musical sounds indicated in Fig. 1 are shown in tabular form in Table 1. The judgments of pitch and quality were made by three persons familiar with music. In every case they agreed unanimously in the statements made in the 4th and 5th columns. The musical tones were produced at loudness[5] values between 70 and 80 units. These loudness values correspond approximately to intensities of 10^7 and 10^8 times the minimum audible intensity. In the second column of this table, the letter F refers to the fundamental and the numbers refer to the overtones, thus (F & 1–6) means that the fundamental and the first six overtones were eliminated. It is seen that the vowel *ah* sung at a pitch *d* is affected only slightly in pitch or quality when the fundamental and first two overtones are eliminated. Even with the fundamental and first six overtones eliminated, the pitch still very definitely corresponds to the pitch of a pure tone with the frequency of the fundamental, namely, 145 cycles per second. The harmonic analysis of this filtered tone shows no frequencies below 1000 cycles per second. Eliminating all of the overtones above the sixth changes the quality by about the same amount as eliminating the fundamental and first and second overtones. The data also indicate that if the fundamental and all of the upper and lower harmonics except the third, fourth and fifth

[3] R. L. Wegel, Journal A.I.E.E., October, 1921

[4] H. Fletcher, Jour. Franklin Inst. June 1922; also Phys. Rev. **15**, 513, 1920

[5] For the meaning of the term loudness as used here see article by H. Fletcher, Jour. Franklin Inst., Sept. 1923

are eliminated, the remaining compound tone has the same pitch as the fundamental, although the quality of the sound is very different from that of the sound *ah*.

As indicated in the table, similar results were obtained for the vowel *ā*, sung at the pitch *a*. Still other vowels were tried with similar results. In general, neither the quality nor the pitch of notes from a rich baritone or contralto voice is appreciably affected by eliminating the fundamental and the first two to three overtones. If, however, higher overtones are eliminated, the musical quality (in particular the richness) is noticeably affected and this is true even though the omitted overtones are all above the fifteenth. The high harmonics do not seem to be so essential for good quality in a soprano voice. Experimental tests showed the rather unexpected result that the elimination of all the harmonic frequencies above 2000 cycles affects the musical quality of a bass, a baritone or a contralto voice to a greater extent than the quality of a high soprano voice.

The table shows that the quality of the principal musical instruments is much more seriously affected by the elimination of the lower parts of their characteristic sound spectra than the quality of the sung vowels by a similar elimination. In any case such eliminations do not change the pitch, for this remains constant as long as the filtered sound can be recognized as a musical tone.

These results were confirmed in a very striking manner by using ten separate vacuum tube generators for producing the component frequencies. These generators were adjusted to give the frequencies 100 to 1000 at intervals of 100. They were all connected to a special telephone receiver and the currents regulated so that the pressure amplitude of the components of the sound emitted by the receiver were equal. By suitable switching arrangement any one of the components could be eliminated. When they were all impressed upon the receiver a full tone resulted which had a definite pitch corresponding to 100 cycles per second. The elimination of the 100-cycle component produced no noticeable effect. The elimination of any other single component had no effect upon the pitch and almost none upon the tone quality, although by careful listening, its introduction and withdrawal could be detected in most cases. Even with the first seven components eliminated, leaving only 800, 900 and 1,000, the pitch corresponded to a frequency of 100. When only two components were left, they were heard as separate tones, the fundamental subjective tone at 100 being still plainly audible but much weaker than either component. Any three consecutive components were sufficient to give the tone a pitch corresponding to 100, as, for example, 200, 300, 400 or 600, 700, 800 etc. When four consecutive components were sounded, the fundamental subjective tone was very prominent. When all of the components were sounded this fundamental seemed to be louder than the other components and dominated the tone.

The tests just described were made when the loudness[5] of the 700-cycle tone was at 90. When only three components, 700, 800, 900 were used and the loudness of the combination greatly decreased, it was found that the 100-cycle subjective tone disappeared when the loudness of the combination was approximately 45 units. At this loudness, the three tones were heard as separate tones, the 900 cycle one being the last to disappear as the loudness approached zero. When five or more consecutive components were used, the pitch seemed to remain the same for low values of the loudness even down to zero, although for these very low values, it was very difficult to judge pitch.

If the components 200, 400, 600, 800 and 1,000 were used, the pitch corresponded to 200 cycles, i.e. to the octave of the compound tone discussed above. This tone still had the same pitch when the 200 and 400 cycle components were eliminated. Any two consecutive pairs gave the subjective tone 200 but only very weakly. Combination 300, 600 and 900 gave a harmonious sound, the listener having the tendency to hear the combination as separate musical tones.

From the results which have been described, one might conclude that the pitch of a musical tone was determined by the common difference in the frequencies of the harmonics, rather than by the frequency of the lowest component. This conclusion suggested trying a combination of frequencies which are separated by a common difference, but which are not necessarily multiples of this common difference. The combination 100, 300, 500, 700 and 900 was tried and it was found to have no definite pitch, but sounded like a noise. However, one could distinctly hear the subjective tone at 200 cycles. Similarly, the combinations 100, 400, 700, 1,000 and 100, 500, 900 and 200, 500, 800 were tried and found to have no definite pitch and to be entirely lacking in musical quality.

Further evidence of the above phenomenon was made possible by means of a carrier telephone system which was available in the laboratory. The technique of carrier telephony makes it possible to displace all the frequencies constituting a compound tone by the same absolute amount upward or downward. Thus, if the compound tone of ten components which has been described were transmitted through such a system when the carrier at the transmitting end differed from the carrier at the receiving end by 30 cycles, the frequencies received would be 130, 230, etc. up to 1030. It is found that such a shift destroys the musical quality which the original tone possessed. If the fundamental is very predominant this shift raises the pitch, but the inharmonic tones produce a harshness and the tone loses its musical character.

Structure of "Harmonic" Tones from Such Wind Instruments as the Bugle

In this connection it is interesting to examine the structure of those tones produced on wind instruments by changes in the blowing intensity rather than by changes in the length of the vibrating air column. When

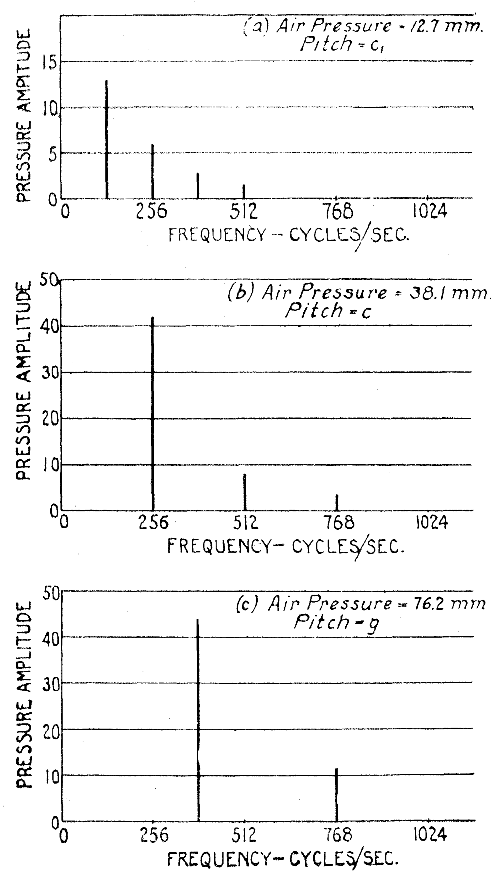

Fig. 2

the air pressure blowing an organ pipe or horn is continuously increased, the pitch of the emitted tone corresponds first to the fundamental and then it suddenly jumps to that corresponding to the first overtone and then to that corresponding to the second overtone, etc. As is well known, it is this effect that makes it possible to produce the different notes on a bugle. One might expect to find all the harmonics of the fundamental in each of these notes. However, this would be contrary to the observations we have just described. In fact, we find from a few experiments upon organ pipes that the overtones are all present in appreciable amount when and only when the pitch of the tone is that corresponding to the fundamental but when the pitch corresponds to the first overtone, only those components which are multiples of the first overtone are perceptible. This is clearly shown in the sound spectra given in Fig. 2, obtained by means of the harmonic analyzer which has been described. They represent the sound emitted by an organ pipe when it was blown with the various pressures indicated on the charts.

Explanation of the Results

It is possible to explain these experimental facts concerning pitch from our conception of the mechanism of the ear. The recent work[6] on this subject which has been carried on in the Bell System Laboratories, has shown that the ear displays a non-linear response to external applied forces. This non-linearity produces subjective tones; all the summation, the difference, and the harmonic frequencies as well as the impressed frequencies produce nerve stimulation. When the fundamental and first few overtones are eliminated from the external tone, they are again introduced by the ear mechanism as subjective tones although of course with different intensities. The changes in the relative intensities of the components impressed upon the nerve endings produce the observed changes in quality.

For example, in Fig. 3, the top chart (a) shows that sound spectrum described above, which was produced by 10 oscillators working into a single telephone receiver. The currents from the oscillators were adjusted so that the pressure amplitudes produced in the outer ear canal were all equal to approximately 100 dynes per square centimeter. When these component tones are transmitted through the ear mechanism the relative amplitudes of the elements producing nerve stimulation are quite different from those shown in Chart (a). If the component tones are impressed separately upon the ear as single tones then the relative amplitudes of these elements which stimulate the inner ear would be represented by chart (b). The ordinates represent the relative sensitivity of the ear to the various frequencies.[7] If the ear had a linear response this same chart would represent the relative amplitudes when all the components were sounding simultaneously.

[6] See Wegel and Lane, Phys. Rev. **21**, 705, 1923; also Fletcher[4]
[7] See Fletcher and Wegel, Phys. Rev. **19**, 553, June 1922

The third chart (c) shows the redistribution brought about by the non-linearity of the ear, that is, it represents the actual spectrum produced in the average inner ear that is excited by the tone represented in chart

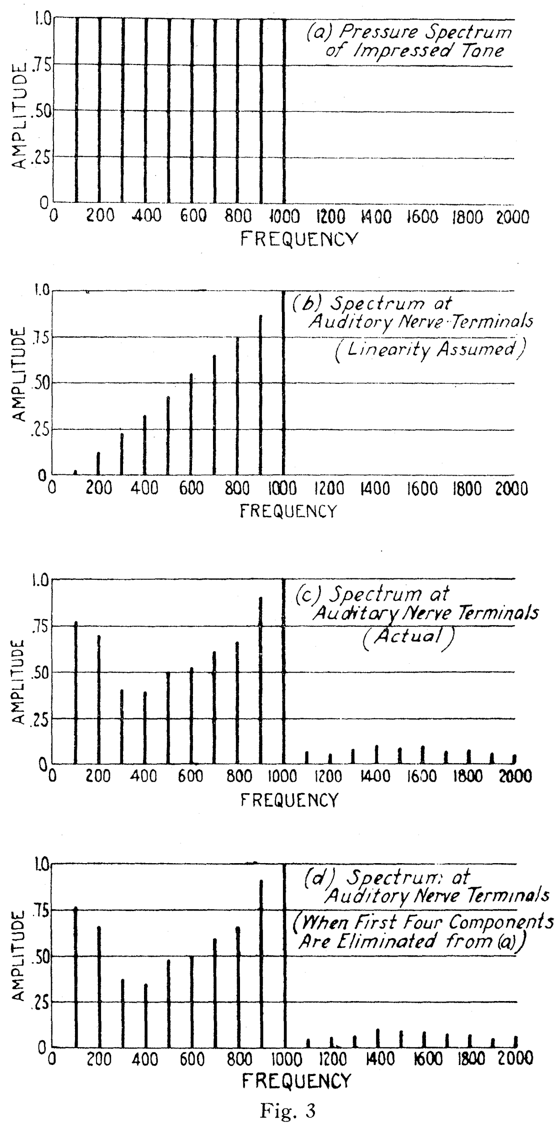

Fig. 3

(a). The fundamental is greatly increased due to the nine difference tones from adjacent components uniting to reinforce the fundamental.

The fourth chart (d) shows the spectrum in the inner ear when the first four components are eliminated from the impressed tone. That is, it represents the condition when the oscillators which are sending out 100, 200, 300 and 400 vibrations per second are switched off. Since the lower components in the inner ear spectrum are principally due to the higher frequency components of the pressure spectrum, the elimination of the first few components of the latter spectrum produces very little effect upon the former.

The constants used in computing these redistributions were obtained from experimental measurements by Wegel and Lane[6] of the magnitudes of the subjective tones. A comparison of the last two charts shows very clearly why one should expect the pitch of the two tones to be the same. If the impressed tones are all decreased without changing their relative magnitudes the resulting inner ear spectra will be different from that shown in these charts. Consequently the quality of the musical tone as well as its intensity will change as the loudness of the musical tone approaches zero. For these low intensities the inner ear spectrum approaches that shown in chart (b).

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