

THE CONDUCTIVITY OF FLAMES FOR RAPIDLY
ALTERNATING CURRENTS

BY H. A. WILSON AND A. B. BRYAN

ABSTRACT

Conductivity of salted flames for high frequency currents.—*Theory.* Expressions are developed for the capacity and series resistance of a parallel plate condenser in a flame, which enable certain constants to be computed from the measurements of Bryan given in the preceding paper. *Calculated density of positive ions* increases from .03 e.s.u. in the unsalted flame to 1.5 e.s.u. for the flame sprayed with a solution of 10 gm/liter K_2CO_3 . For the smaller concentration, it varies about as the square root of the potassium present, as was to be expected. The variation with frequency and electric field is slight. *Calculated mobility of electrons* is found to diminish from 200 to 60 m/sec. per volt/cm as r.m.s. field increases from 9 to 64 volts/cm, in good agreement with values found by Loeb for electrons in pure nitrogen at room temperatures. It decreases markedly as the concentration of potassium increases.

THE experiments described in the preceding paper by A. B. Bryan show that a pair of electrodes in a Bunsen flame containing an alkali metal vapor have a capacity for rapidly alternating currents depending on the amount of vapor present, on the frequency of the electrical oscillations and on the potential difference. There is also a resistance which depends on the same variables. The theory of the conductivity has been discussed previously by H. A. Wilson and E. Gold¹ who also made measurements of the capacity but did not determine the resistance. In this paper the results obtained by A. B. Bryan are compared with the theory of Wilson and Gold, which has been extended so as to include the resistance as well as the capacity. The theory enables the number of ions present in the flame and also the mobility of the negative ions to be calculated from the experimental results.

Consider two parallel metal plates with the space between them filled with a strongly ionized gas, the negative ions being free electrons. The mobility of free electrons is very large compared with that of positive ions so that the motion of the positive ions due to an alternating electric field between the plates is negligible compared with that of the electrons. The electrons will vibrate in the alternating field and those near the plates will strike them and so be removed from the gas. We shall therefore have a layer of electrons vibrating between the plates. Inside this layer

¹ Wilson and Gold, *Phil. Mag.* **11**, 497, 1906

there will be as much positive electricity as negative and outside it only positive ions. If D is the distance between the plates and $D-d$ the thickness of the layer of negative electrons then, following Wilson and Gold, it is easy to show that

$$V = 2\pi\rho d(2t_1 - d) - XD$$

where V is the potential difference between the plates, X the electric intensity, supposed uniform, in the layer of electrons, ρ the charge per cc on the positive ions and t_1 the distance of the layer of electrons from the negative plate. Let $x = t_1 - d/2$ so that

$$V = 4\pi\rho dx - XD \quad (1)$$

x is then the displacement of the layer of electrons from its mean position and $\frac{1}{2}d$ is the amplitude of the vibration. We shall suppose that the inertia of the electrons may be neglected so that

$$dx/dt = -kX \quad (2)$$

where k is the electron mobility or velocity for unit electric field. The current density j in the layer of electrons is given by

$$j = -\rho \frac{dx}{dt} + \frac{1}{4\pi} \frac{dX}{dt} \quad (3)$$

since it is the sum of the convection and displacement currents.

These equations may be solved in the case of an alternating potential difference, given by $V = V_0 e^{ipt}$ by assuming $x = x_0 e^{ipt}$, $X = X_0 e^{ipt}$ and $j = j_0 e^{ipt}$. Substituting these values we get

$$V_0 = 4\pi\rho dx_0 - X_0 D \quad (4)$$

$$x_0 ip = -kX_0 \quad (5)$$

$$j_0 = -\rho x_0 ip + X_0 ip / 4\pi \quad (6)$$

and we also have the condition that the amplitude of x is to be put equal to $\frac{1}{2}d$.

Eliminating x_0 and X_0 we obtain

$$j = \frac{V_0 p \epsilon^{i(pt+\gamma)}}{4\pi} \sqrt{\frac{(4\pi\rho k)^2 + p^2}{(4\pi\rho kd)^2 + p^2 D^2}} \quad (7)$$

$$\tan \gamma = \frac{(4\pi\rho k)^2 d + p^2 D}{4\pi\rho k p (D-d)} \quad (8)$$

$$d = (1/4\pi\rho k) [\sqrt{(8\pi V_0 \rho k^2)^2 + \frac{1}{4}(pD)^4} - \frac{1}{2}(pD)^2]^{\frac{1}{2}} \quad (9)$$

and

$$X = V_0 p \epsilon^{i(pt+a)} / \sqrt{p^2 D^2 + (4\pi\rho kd)^2}$$

where $\tan a = 4\pi\rho kd / pD$.

When an alternating potential difference $V_0 e^{ipt}$ acts on a condenser of capacity C_1 with a resistance R_1 in series, the current is equal to

$$V_0 \epsilon^{i(pt+\gamma)} / \sqrt{R_1^2 + (pC_1)^{-2}}$$

where $\tan \gamma = (R_1 p C_1)^{-1}$

Comparing these expressions with those previously found for the strongly ionized gas, noting that the total current $c=ja$, where a is the area of the electrodes in the gas, we get

$$C_1 = \frac{a}{4\pi} \left[\frac{(4\pi\rho k)^2 + p^2}{(4\pi\rho k)^2 d + p^2 D} \right] \quad (10)$$

$$R_1 = \frac{16\pi^2 \rho k (D-d)}{a [(4\pi\rho k)^2 + p^2]} \quad (11)$$

Eqs. (9) (10) and (11) enable ρ , k , d and X to be calculated from the experimentally determined values of C_1 and R_1 found by A. B. Bryan as described in the preceding paper.

Eqs. (10) and (11) on putting $C_0 = a/4\pi D$ give

$$(C_1 - C_0)/R_1 C_1 C_0 = 4\pi\rho k \quad (12)$$

Putting $4\pi\rho k = z$ and $d/D = F$, (10) then gives

$$F = (C_0/C_1) (1 + p^2/z^2) - p^2/z^2 \quad (13)$$

and (9) gives

$$k = (FD^2/2V_0) \sqrt{p^2 + z^2 F^2}$$

The value of ρ is then given by $\rho = z/4\pi k$. The following table gives the values of F , k , ρ and \bar{X} , the r.m.s. value of X calculated from Bryan's values of C_1 and R_1 . C_0 was equal to $8.23 \mu\mu f$ or 7.4 cm and D was 0.5 cm .

TABLE I

C_1 ($\mu\mu f$)	R_1 (ohms)	V_0 (volts)	f (per sec.)	K (g/liter)	F (d/D)	k ($\frac{\text{cm/sec}}{\text{volt/cm}}$)	ρ ($\frac{\text{e.s.u.}}{\text{cc}}$)	\bar{X} ($\frac{\text{volts}}{\text{cm}}$)
12.8	4500	82.2	6×10^5	1.0	.589	6100	0.42	64.
13.75	4580	66.7	6×10^5	1.0	.548	7300	0.39	51.
15.4	4550	47.5	6×10^5	1.0	.492	9300	0.36	35.
18.4	6060	26.9	6×10^5	1.0	.382	10000	0.29	25.
25.3	5000	11.4	6×10^5	1.0	.290	19300	0.25	9.4
20.6	2860	21.15	10×10^5	1.0	.363	24000	0.28	17.
21.4	3470	21.15	8×10^5	1.0	.352	19000	0.30	16.6
23.5	4030	21.15	6×10^5	1.0	.327	14400	0.36	15.2
29.4	4700	21.15	4×10^5	1.0	.267	8800	0.56	13.4
39.8	4150	21.15	2×10^5	1.0	.205	6000	1.04	7.6
9.7	6420	11.1	6×10^5	0	.60	28200	.028	14.1
15.7	7130	11.1	6×10^5	0.1	.423	24300	.089	11.6
25.3	5170	11.1	6×10^5	1.0	.287	19000	.221	10.0
39.3	3930	11.1	6×10^5	5.0	.191	12800	.50	10.0
75.2	4560	11.1	6×10^5	10.0	.087	4230	1.49	13.9

We should expect ρ to depend only on the amount of salt in the flame. Of the eleven values of ρ obtained while a solution containing one gram per liter of potassium carbonate was being sprayed into the flame, the majority do not differ much from about 0.35. The theory can be regarded only as a first approximation so that the variation of ρ is not greater than might have been expected. The theory is probably not

applicable unless the frequency is very large and so fails in the case of the two lowest frequencies of 2×10^5 and 4×10^5 , for which the values of ρ are considerably greater than the mean value.

The last five rows in the table show the variation of ρ due to changing the concentration of the salt in the flame. ρ increases with the amount of salt as we should expect. With the smaller concentrations ρ increases about as the square root of the concentration but with the larger concentrations is more nearly proportional to the concentration. These results agree with the earlier measurements of the conductivity of K_2CO_3 in flames using steady currents.²

The relation between the r.m.s. field strength \bar{X} and the mobility k is shown in Fig. 1, where the seven values of k obtained when spraying a K_2CO_3 solution containing one gram per liter are plotted. It appears that the mobility falls as \bar{X} increases.

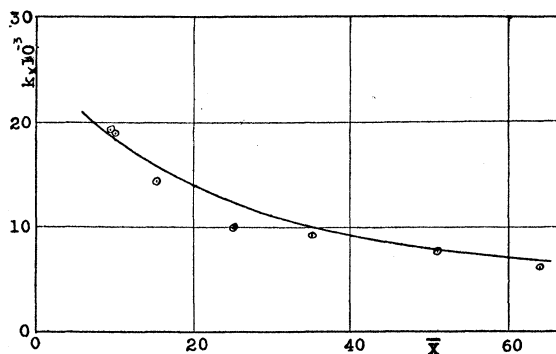


Fig. 1. Variation of mobility of flame electrons with r.m.s. field. Results of Bryan compared with values given by Loeb's formula.

Loeb³ has found that the average mobility of electrons in pure nitrogen at a 760 mm pressure is equal to $571000/(21 + \bar{X})$. The curve drawn in Fig. 1 represents the values of the mobility given by Loeb's formula. The agreement between curve and points seems to indicate that the mobility of electrons in gases is not much affected by temperature at constant pressure.

We should expect the mean free path to be proportional to the absolute temperature and the velocity of agitation to vary as the square root of the absolute temperature, so that for small fields the mobility might be expected to vary as the square root of the absolute temperature at constant pressure. The agreement between Loeb's mobilities at the ordinary temperature and those found in the flame suggests that the increase in the

² Smithells, Dawson and Wilson, Phil. Trans. A 193, 89, 1899

³ Loeb, Phys. Rev. 19, 24, 1922

velocity of agitation due to the field makes the velocity of agitation of the electrons nearly proportional to the absolute temperature so that the mobility is nearly independent of the temperature. However, since the calculation of the mobilities in the flame is based on an approximate theory of the conductivity and since flame experiments are subject to considerable errors, too much weight must not be given to this agreement.

In nitrogen at ordinary temperatures Townsend⁴ found the electron velocity to be 5.2×10^5 cms per sec. when $X/p = 0.25$, where p is the pressure in mm of mercury. Using Loeb's formula we can calculate the velocity of the electrons in the flame when $X/p = 0.25$. Since $p = 760$, we require the velocity when $X = 190$. It comes out $2710 \times 190 = 5.15 \times 10^5$ cm per second and so agrees well with that found by Townsend.

The first measurements of the mobility of the negative ions in flames were made by H. A. Wilson⁵ who got about 1000 cm per sec. for one volt per cm by a rough method only capable of giving the order of magnitude of k . Numerous other estimates have been made, among which is that of E. Gold⁶ who got $k = 12900$ for very small values of X . H. A. Wilson⁷ found $k = 2450$ from measurements of the Hall effect with X about 30 volts per cm. The high values for the mobility of the negative ions in flames indicate that they are free electrons, as was first suggested by Lenard.

The values of k found with different amounts of salt, given in the last five rows of the table, indicate that k diminishes as the amount of salt increases. This may be due to the electrons getting attached to potassium atoms during a part of their existence as negative ions, which is possible when X is small, as was true in all these cases.

The values of k found with different frequencies, given in the second five rows of the table, indicate that k diminishes with the frequency. This variation may be apparent rather than real, however, since the results for the two lowest frequencies are open to suspicion, as already mentioned and the other three do not show much change.

DEPARTMENT OF PHYSICS,
THE RICE INSTITUTE, HOUSTON, TEXAS,
September 1, 1923.

⁴ Townsend, *Phil. Mag.* **42**, 873, 1921

⁵ Wilson, *Phil. Trans. A* **192**, 499, 1899

⁶ Gold, *Proc. Roy. Soc. A* **79**, 61, 1907

⁷ Wilson, *Phys. Rev.* **3**, 375, 1914