THE MOTION OF ELECTRONS IN NITROGEN

By H. B. Wahlin

Abstract

Mobilities of electrons in nitrogen have been determined in fields ranging from 1 volt/cm to 55 volts/cm at atmospheric pressure. For low fields the mobility has a value of 18,000 cm/sec. but decreases rapidly for higher fields. The results agree for low fields with Compton's mobility equation, viz., $u=a/[1+(1+Bx^2)^{\frac{1}{2}}]^{\frac{1}{2}}$, but in order to obtain full agreement it must be assumed that the mean free path of the electron decreases as the velocity of the electron increases and also that the impacts of electrons with nitrogen molecules are not completely elastic. The *mean free path* of the electrons in nitrogen at a pressure of 760 mm and in thermal equilibrium with the molecules has been determined from the limiting mobility as the field in which the mobility is determined approaches zero. The value thus found is .000131 cm. This is about 2.7 times the kinetic theory value.

SINCE the discovery that when electrons are liberated in a gas they do not become attached immediately to the molecules but in most gases remain free for a considerable time and in some cases never unite to form negative ions, considerable interest has been aroused in the study of motions of electrons through gases under the action of an electric field. The factors involved in such a study are more numerous and somewhat more complicated than those encountered in the case where the negative carriers consist of charged atoms or molecules, for in this case or in any case where the carriers have approximately the same mass as the molecules of the medium through which they are moving, there will be equipartition of energy between the ions and the molecules of the gas, except for the comparatively small amount of energy gained in the field between impacts; therefore, since the free time between impacts is nearly a constant and independent of the voltage, the velocity of the ions in the direction of the field will be proportional to the field intensity. When the carriers consist of electrons only, this ceases to be the case, for due to the great difference in mass between the electrons and molecules of the gas, the electrons will not for the lower speeds lose all the energy gained in the field on impact with the molecules, hence the energy will continue to increase until it reaches a terminal value. This terminal energy will be greater, the greater the field intensity. Consequently, the free time between impacts decreases with the increased field and the velocity in the direction of the field will no longer be proportional to the field strength.

Electronic motion through a gas may be studied most readily by a determination of the mobility of electrons for different values of the field intensity. Townsend and Bailey[1] using a new method in which the velocity in the direction of the field is measured by means of the magnetic deflection of the electron stream, have succeeded in determining not only this velocity but also the factor by which the energy of the electrons exceeds that of the molecules. This method is not very suitable, however, for determining the mobility in low fields or at high gas pressures, that is, for small values of the field strength divided by the pressure. Loeb[2] has applied the Rutherford alternating potential method to the determination of electronic mobilities in a number of gases and has made measurements in nitrogen at pressures ranging from 75 mm to 606 mm, and in fields ranging from 3.8 volts/cm to 52 volts/cm.

Results are herewith presented extending these values from fields of 1 volt/cm to 55 volts/cm, at a pressure of 760 mm and with frequencies ranging from 5300 cycles/sec. to 88,000 cycles/sec.

The method of procedure was essentially the same as that described in an earlier paper.[3] The Rutherford alternating potential method was used and the electrons were obtained photo-electrically from one of the two parallel plates between which the mobility was measured.

As a source of alternating potential, a vacuum tube oscillator of the Hartley type was used and the voltage was determined by means of an electrostatic voltmeter having a range of 30 to 120 volts. A quadrant electrometer having a phosphor-bronze suspension was used to measure voltages below thirty. This was calibrated on direct voltage and had a sensibility such that it gave 250 scale divisions with an applied potential of 30 volts, for determining the mobility in fields down to 8 volts/cm, and a sensibility such as to give a deflection of 170 scale divisions with 15 volts for lower voltages. Since with an alternating potential, the electrometer must be used idiostatically, the deflections for the lower voltages were not very large and an error of 10 per cent could easily be introduced in reading the voltage. For this reason, a number of mobility curves were taken with the same frequency and the mobility taken as the average value found from these curves.

Mobilities were determined with plate distances of 1.55 to 2.4 cm.

A standard wave-meter having a range of 3000 to 23,500 meters was used to determine the higher frequencies. For the lower frequencies, a circuit of the type shown in Fig. 1 was adopted. Here L_1 is an induct-

[1] Townsend and Bailey, Phil. Mag. **42,** 873, 1921
[2] Loeb, L. B., Phys. Rev. **19,** 24, Jan. 1922
[3] Wahlin, H. B., Phys. Rev. **21,** 517, 1923

ance which was coupled inductively with the oscillator; L is a calibrated variable inductance and C a variable condenser of known capacity; T is a pair of phones. L and C were adjusted to give a minimum of sound in the phones, and the frequency was computed from the relation

$$n = 1/2\pi\sqrt{LC}$$

The same values for the frequency were obtained within the limits of error of measurement when L was coupled inductively to the oscillator and the phones placed across C, the circuit being adjusted for a maximum of sound in the phones. The circuit diagrammed seemed preferable,

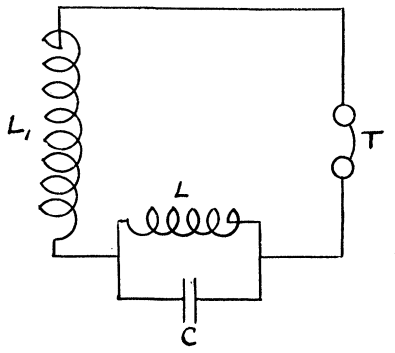

Fig. 1. Circuit used.

however, since it eliminated any possible capacity effect of the phones. From the value of the frequency determined in this way and the value of the voltage for the intercepts of the mobility curves, an approximate value of the mobility may be determined from the equation

$$u = (\pi/\sqrt{2})\ (nd^2/V) \qquad (1)$$

where d is the distance between the plates, V is the effective value of the voltage intercept and n is the frequency.

The nitrogen was obtained from a commercial tank and was purified by passing it over hot copper oxide, hot copper, potassium hydroxide, calcium chloride, phosphorus pentoxide and through two traps cooled with liquid air before it entered the measuring chamber. The chamber was washed four or five times with the purified gas before any readings were taken. This was necessary for otherwise, due to attachment of electrons to impurities, the mobility would decrease in the low fields. Readings were taken immediately after the chamber was filled, so as to eliminate any effect due to impurities diffusing from the walls. However, no change in the mobility was produced by allowing the gas to remain in the chamber for twelve hours and since no set of readings extended over a period as long as this, the effect due to impurities from the

walls was negligible. In order to minimize any effect due to space charge action at the emitting plate, the photo-electric emission was kept as small as was practicable.

The mobility curves were obtained by applying the alternating potential to the emitting plate for a given time (in most cases 20 sec.) and then connecting the electrometer to the collecting plate and measuring the deflection. The voltage was then varied and the process repeated until a sufficient number of readings had been obtained to plot a curve of electrometer deflections against voltage.

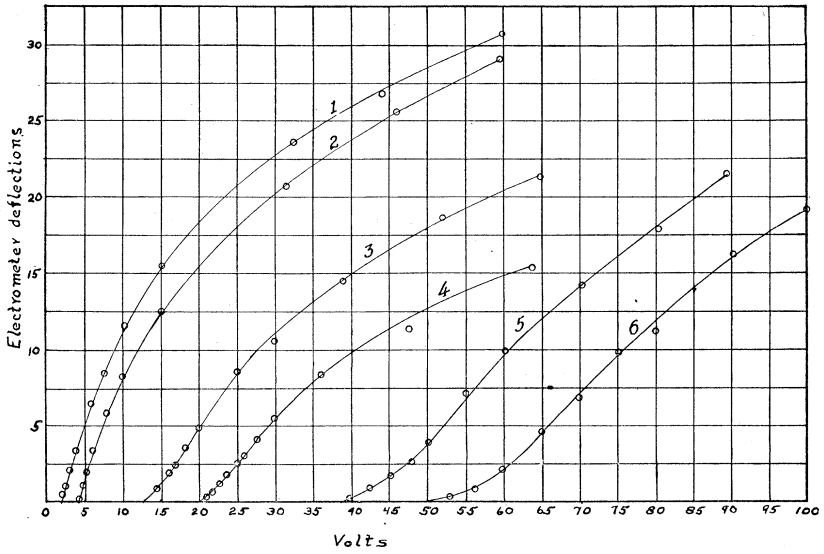

Fig. 2. Mobility curves for various frequencies.

Fig. 2 illustrates the types of curves obtained.

TABLE I

Curves shown in Fig. 2

Curve	Voltage intercept (corrected)	n	d	u
1	1.9	6,300	1.55	17,700
2	3.9	12,850	1.55	17,600
3	13.3	37,500	1.55	15,100
4	20.2	50,500	1.55	13,300
5	40.9	71,100	1.55	9,420
6	52.5	77,000	1.56	7,920

Fortunately the intercepts of the mobility curves are sharper at low voltages than at high ones. If this were not the case, anything but an approximate determination would be impossible, for a small error in determining the intercept in low fields would change the results appreciably.

In Figs. 3 and 4 the values of the mobility computed from Eq. (1) are plotted against the fields in which they were determined.

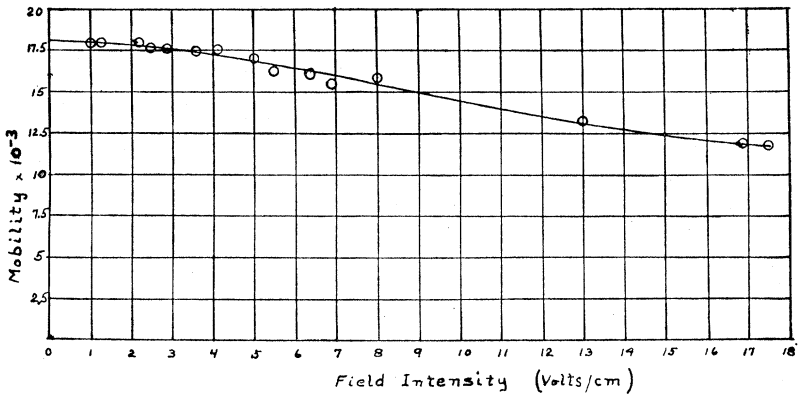

Fig. 3. Mobility for fields to 18 volts/cm.

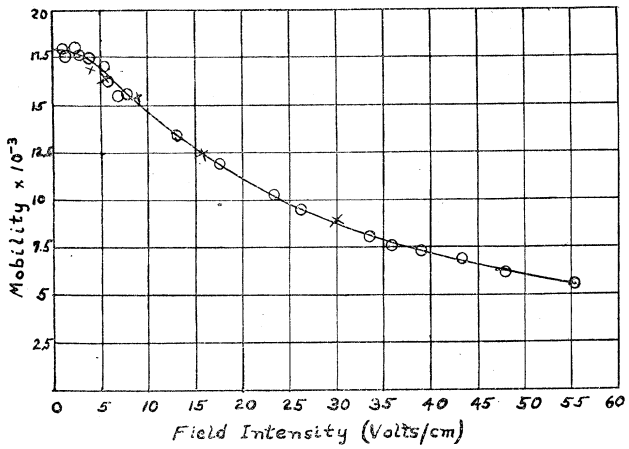

Fig. 4. Mobility for fields to 55 volts/cm.

TABLE II

Mobility as a function of field

V/d	*n*	*u*	*V/d*	*n*	*u*
1.01	5,300	17,900	8.0	37,000	15,900
1.16	6,210	18,100	13.0	50,500	13,300
1.22	6,300	17,700	17.5	60,100	11,850
2.2	11,500	18,000	23.2	68,500	10,300
2.9	15,050	17,600	26.1	71,100	9,420
3.6	18,600	17,400	33.5	77,000	7,950
4.2	21,500	17,500	35.8	79,100	7,520
5.0	25,000	17,000	39.0	81,300	7,250
6.3	29,700	16,200	43.3	84,500	6,800
5.5	16,670	16,200	48.0	85,300	6,110
6.9	30,900	15,500	55.5	87,700	5,500

As will be noticed the mobility in fields below 3 volts is nearly a constant, but decreases quite rapidly as the field is increased above this value. For this reason, there will be an error in the value of the mobility as computed from Eq. (1) for higher fields. This equation is derived from the expression

$$dy/dt = uX \sin 2\pi mt$$

and assumes that u is a constant and independent of the field strength. As may be seen from Figs. 2 and 3 this is not the case, and the equation must be written

$$dy/dt = f(x)X \sin 2\pi nt = f(X \sin 2\pi nt)X \sin 2\pi nt \tag{2}$$

where x is the instantaneous value of the field.

Unless the form of $f(x)$ is known, an exact determination of the mobility using a sine wave alternating potential is, in general, impossible. If the function is known, however, it is possible to compare the frequencies determined experimentally as a function of the voltage intercepts with those computed from theoretical considerations.

K. T. Compton[4] has recently determined the form of $f(x)$ and has found that

$$f(x) = \frac{.815el}{\sqrt{m}[aT + (a^2T^2 + 1.76l^2e^2x^2/f)^{\frac{1}{2}}]^{\frac{1}{2}}} \tag{3}$$

where e and m are the charge and the mass of the electron aT is the mean energy of agitation of the gas, f is the fraction of the electron energy lost at each impact, l is the mean free path and x is the field intensity. This equation can be written in the form

$$u = a[1 + (1 + Bx^2)^{\frac{1}{2}}]^{-\frac{1}{2}} \tag{4}$$

On substituting this in Eq. (2) above we get:

$$\frac{dy}{dt} = \frac{aX \sin 2\pi nt}{[1 + (1 + BX^2 \sin^2 2\pi nt)^{\frac{1}{2}}]^{\frac{1}{2}}}$$

For the case where the electrons just reach the collecting plate during a half cycle of the alternating potential this becomes:

$$d = \int_0^d dy = aX \int_0^{1/2n} \frac{\sin 2\pi nt\, dt}{[1 + (1 + BX^2 \sin^2 2\pi nt)^{\frac{1}{2}}]^{\frac{1}{2}}}$$

Placing $2\pi nt = \theta$ and solving for n this becomes

$$n = \frac{aX}{\pi d} \int_0^{\pi/2} \frac{\sin\theta\, d\theta}{[1 + (1 + BX^2 \sin^2\theta)^{\frac{1}{2}}]^{\frac{1}{2}}}$$

This is an elliptic integral which on reduction gives:

[4] Compton, K. T., Phys. Rev. **22**, 333, Oct. 1923

$$n=\frac{a\sqrt{2r}}{\pi dB}\left[2E(k_1\varphi)-F(k_1\varphi)\right] \tag{5}$$

where $r=\sqrt{BX^2+1}$, $k=\sqrt{(r+1)/2r}$ and $\varphi=\sqrt{(r-1)/(r+1)}$. Since $r \geqq 1$, k is $\leqq 1$ and the solution holds for any value of X.

If a and B can now be determined, n may be plotted as a function of X and the values thus obtained compared with the experimental values.

The constant a can be determined immediately from the limiting mobility as X approaches zero. Eq. (4) becomes for this case:

$$u_0=a/\sqrt{2}$$

Since the experimental value found for u_0 is 18,000 cm/sec./volt/cm, $a=25{,}400$. If it is assumed that the electrons make elastic impacts with the molecules, B may be computed also, for as Compton[4] has shown, Eq. (3) then reduces to

$$u=\frac{.815el}{\sqrt{m}[aT+(a^2T^2+l^2Me^2x^2/1.134m)^{\frac{1}{2}}]^{\frac{1}{2}}}.$$

Here M is the molecular weight of the gas.

From the determined value of a, $l=.000131$, and since $M=28$, $B=.658$. These values of a and B are not in agreement with the results of these experiments, as may be seen from Fig. 5, curve 1 where the values of n computed from Eq. (5) are plotted against the effective values of X. The circles represent the experimental values. Curve 2 is the plot of n with $a=25{,}400$ and $B=1/10$ and curve 3 with $a=25{,}400$ and $B=.027$. This last curve agrees quite well with the experimental values for low fields so that Compton's equation may be said to hold in this region. The deviation at higher fields may very well be due to a decrease in the mean free path of the electron with increasing fields. This effect would lower the mobility and consequently the frequency necessary to produce an intercept at a given field intensity. Such a change has been observed by Townsend and Bailey,[1] working with higher fields than those employed here.

If the value of B is taken as .027 it must be assumed in order to obtain agreement with Eq. (3) that f is larger than would be the case if the energy loss on impact were due to a momentum transfer only; in other words, that the electrons make partially elastic impacts with the nitrogen molecules and lose 24.4 (B for elastic impacts divided by .027) times as much energy as they would if the impacts were elastic.

The mean free path of the electrons in thermal equilibrium with nitrogen molecules has been found to be .000131 cm at a pressure of 760 mm. This is about 2.7 times the value determined from kinetic theory con-

siderations which give a value $4\sqrt{2}$ times that of the molecular free path. That there is a tendency for the electronic free path to become abnormally long in nitrogen in low fields may be seen from the results of Townsend and Bailey referred to above.

An examination of the mobility curves in Fig. 2 shows that as the voltage increases, the intercepts of the mobility curves become ill defined and the curves tend to approach the voltage axis asymptotically. This

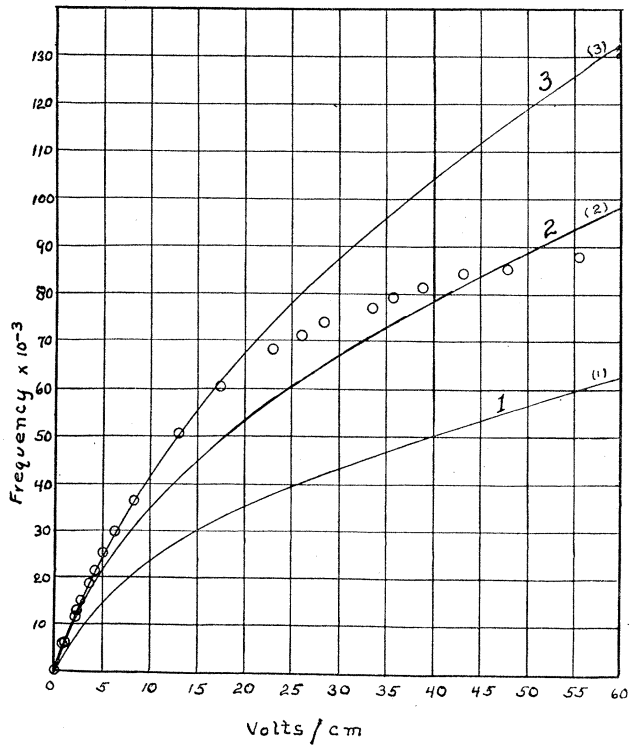

Fig. 5. Comparison of results with theory.

effect was also observed by Loeb[2] who offered as a possible explanation that it might be due to the combination of electrons with impurities in the gas to form ions. These ions would remain in the field and, under a space charge action, would drift across to the collecting plate. Such an explanation will not account for the disappearance of the effect at low frequencies and in low fields. If, however, as seems to be the case, there is a decrease in the mobility due to a decrease in the mean free path with increasing fields and superimposed on this the decrease in the mobility due to the increase in the energy of agitation of the electrons, these effects would combine in such a way as to make the mobility de-

crease more rapidly than it normally would. Consequently, over a small range of field strengths the product of the mobility and the field would be nearly a constant and since this is the velocity of the electrons which in turn is proportional to the space current, the current due to an alternating voltage would tend to become independent of the voltage. This would cause the mobility curves to approach the voltage axis asymptotically. It is significant that this "tailing off" of the curves becomes most marked in fields above 15 volts/cm, and it is above this field strength that the experimentally determined frequencies begin to deviate from the theoretical values appreciably, as may be seen from Fig. 5.

Since the terminal energy of the electrons exceeds that of the molecules by an amount depending on the voltage, the electrons moving under the action of an alternating sine wave field will not reach their terminal energy, but there will be a phase lag between this energy and the applied voltage. This might act in such a way as to change the mobility. If, however, the distance the electrons must travel before the terminal energy is reached is small, the lag would be small. That this distance is small has been shown by Compton[4] who computed that at atmospheric pressure the electrons gain 99 per cent of their terminal energy within .015 cm from the starting point, assuming that they start from rest. As may be seen from Fig. 2, the mobilities determined with a plate distance of 2.4 cm (values represented by crosses) check the values obtained with a plate distance of 1.55 cm to within the limit of experimental error so that any effect due to a non-steady state is slight.

In conclusion, the writer wishes to express his thanks to Professor Compton for the use of the manuscript of the paper referred to above.

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