THE MOBILITIES OF ELECTRONS IN HELIUM

By Leonard B. Loeb

Abstract

Mobility of electrons in helium was determined for fields up to 50 volts/cm and at pressures of 447, 189, 117 and 49 mm of Hg, using, as in previous work, the alternating current method with high frequency oscillations of sine wave form. The He was purified by prolonged circulation over charcoal in liquid air, maintained by a mercury vapor pump. Spectroscopic observation indicated the impurity to be about 1/10,000, chiefly H. The results for the mobility constant, reduced to 760 mm pressure and computed in the ordinary way are well represented by the equation $K'=[11.0\ (10)^8/(2.45+E_0\ 760/p)]^{\frac{1}{2}}$, where $E_0=V_0/d$ is the electric field as read on a static voltmeter. It has been pointed out that since K is a function of E_0, a correction is necessary because of the use of a varying field in the measurements. It is shown that this correction may be made by multiplying the constants in the equation for K' by factors which depend upon the form of the equation. The corrected equation is $K=[7.57\ (10)^8/\ (1.56+E_0\ 760/p)]^{\frac{1}{2}}$. For $E_0=0$, this gives $K_0=22{,}000$, a value close to that given by Townsend's equation. Hence the mean free path seems to have the value given by the Kinetic Theory. Compton's theory agrees well for E_0 above 20 volts/cm, but gives too low values for lower fields.

Mobility of electrons in hydrogen and nitrogen.—*The corrected empirical equations* from previous observations are $K=[3.637(10)^5/(11.9+E_0\ 760/p)]$ for N_2 and $K=\left\{4.32(10)^5/[55.2+E_0\ (760/p)^{\frac{3}{4}}]\right\}$ for H_2.

Introduction

THE results of measurements on the mobilities of electrons in N_2[1] and H_2[2] at atmospheric pressures using the Rutherford alternating current method with sine wave form oscillations, have been given in previous papers. From the results obtained it became of interest to determine the mobilities of electrons in He in which K. T. Compton[3] has shown that the fraction of the energy lost by an electron at each impact with a He atom lies close to the theoretical value $f=2m/M=.000268$, (m being the mass of the electron and M the mass of the He atom). The determination of the electron mobilities in helium made in the same manner as those for the gases cited above form the contents of the present paper.

Experimental Procedure

The greatest difficulty involved in the measurements was the attainment of pure He at from 50 to 400 mm pressure in the type of vessel used

[1] Loeb, L. B., Phys. Rev. **19**, 24-37, Jan. 1922
[2] Loeb, L. B., Phys. Rev. **20**, 398-404, Nov. 1922
[3] Benade and Compton, Phys. Rev. **9**, 187, 1918

for an ionization chamber. The conditions imposed on the ionization chambers by the nature of the measurements were as follows: (1) the walls of the chamber had to be conducting, (2) the insulation of the wire leading out of the chamber to the electrometer had to be of amber, (3) the plate source of photo-electrons had to be quite active, and (4) the two plates had to be parallel. The experimental arrangements fulfilling these conditions precluded "baking out" the apparatus so that the contamination of the He by the occluded gases from the walls became a serious disturbing factor.

The new ionization chamber. The ionization chamber used in the later measurements consisted of a small glass box 14.5×9 cm square and 6 cm deep, which was ground to a heavy glass plate. The inside of the box was heavily silvered by Brashear's solution. A quartz window 3.5 cm in diameter ground onto one of the small ends served to admit the

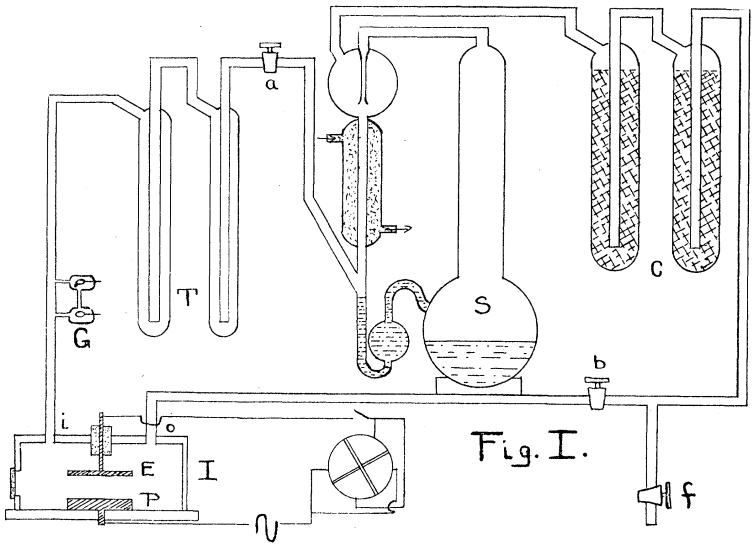

Fig. 1

light from the quartz arc. The photo-electric plate P which consisted of a highly polished cylindrical brass block 7 cm in diameter, was screwed onto a brass rod ground through the thick plate glass base. A brass plug ground through the side enabled contact to be made with the silver coating on the inside, by means of a light copper brush. The tubes i and o were ground into the top of the chamber I. The upper plate E was fastened to a brass rod ground into an amber plug, which in turn was ground into the thick glass top of the chamber. The plates were adjusted to be as nearly parallel as possible, with a plate distance of 1.641 cm.

After adjusting the plates the apparatus was sealed so as to be gas tight, by gradually heating it up in an oven to about 110°C and running "Express" sealing wax around the outside of the ground joints. To insure complete sealing all the joints were gone over with a small jet-like flame. With this procedure gas-tightness was insured without direct exposure of the space inside to any large quantity of wax. In no place did the wax flow into the chamber, so that the vapors from the wax traversed long leakage paths through the ground joints before diffusing into the apparatus.

Purification of the He. Three separate attempts were made to obtain pure He. In all cases the gas initially came from a large tank obtained from the Navy which contained some four to six per cent of impurities. The principle used in the purification was the repeated circulation of the gas over charcoal cooled in liquid air.

The first attempt which was a failure as far as the results were concerned, consisted in the repeated circulation of the gas stored in a large glass reservoir, over three bulbs of some of Professor H. B. Lemon's specially prepared charcoal cooled in liquid air. This was accomplished by the use of a Toepler pump filled with pure mercury. After the purification in this fashion the gas was run through the charcoal bulbs into the ionization chamber which had been evacuated for several hours by a mercury vapor pump.

In the second purification system the gas was continuously circulated from the ionization chamber to the reservoir and back again through the charcoal bulbs. The results at 447 mm pressure, to be described later, were obtained on gas that had made over 30 complete passages over the charcoal before the measurement was begun.

In the third attempt at purification the large ionization chamber having a volume of seven liters was replaced by a smaller chamber with a volume of but 800 cc. The preliminary purification was made as before using the Toepler pump. Final purification was accomplished in a separate system connected to the ionization chamber, by a continuous flow method using a mercury vapor pump suggested by Dr. F. M. Kannenstine. This purification system is shown diagrammatically in Fig. 1. The ionization chamber I has two plates P and E. The tube i served as the inlet for the gas which came from the Stimson[4] pump S, after passing through a stopcock a, through two mercury traps cooled in liquid air T, and through a Geissler tube of the H type G. The outlet tube o of the ionization chamber I carried the gas through the stopcock b, and through the charcoal bulbs C into the intake of the pump s. It

[4] Stimson, H. F., J. Wash. Acad. Sci. **7**, 477-482, 1917

was found that the pump could be made to circulate well below a pressure of 300 mm, if the tube coming from the boiler and leading to the jet was electrically heated. Owing to the irregular ebullition of the mercury the flow was not smooth; the pressure in the ionization chamber thus varied abruptly over a range of 10 mm so that the flowing gas could not be utilized in measurements.

The actual process of purification therefore consisted in running the partially purified He from the Toepler pump into the circulating system described above, which had been thoroughly evacuated and left standing connected to the charcoal bulbs cooled in liquid air for the preceding twenty-four hours. The pump S was then set into operation and the gas circulated for at least two days before measurements were attempted. When the pump was not in operation (i.e., during the night, or while readings were being made), the stopcocks a and b were closed to keep the gases liberated in C due to the slow evaporation of the liquid air, from getting back into I. The gas was circulated for three hours before measurement. Then the pump was shut off and the readings followed in general within half an hour. In no case were measurements made longer than eight hours after the circulation ceased.

Purity of the helium attained. The purity of the gas was tested spectroscopically using a large Schmidt and Haensch direct-vision spectroscope and small induction coil operating on 6 volts. In the first attempt at purification the spectrum was examined at 20 mm pressure. It showed besides the He lines strong mercury lines from the mercury seal in the old ion chamber. It also showed hydrogen lines, the Hα line being in general as strong if not stronger than the red He line of wave-length 6678 A. No bands were observed unless the Geissler tube was run for a minute when the discharge heated the tube to about 50° C. The gases causing these bands doubtless came from the walls and electrodes of the Geissler tube, and gave two prominent bands in the green associated with the spectrum of CO.

The values of the mobilities obtained with this gas indicated contamination from the chamber in a marked degree. If the gas stood in the chamber more than 24 hours the value of the mobility observed increased as much as 10 per cent. Furthermore, measurements made on the freshly purified gas taken a week apart showed a progressive decrease towards lower values, which did not appear to approach a definite limit. It was for this reason that after some seventy measurements made with this arrangement the results were discarded, and the method of purification modified.

In the second attempt the spectroscopic tests at 20 mm pressure showed no marked improvement, and in fact the two very faint CO bands appeared to be slightly more prominent than before. The mobilities were now, however, constant over several days and showed no tendency to change on further purification. The values lay well below those obtained before. They are recorded for a pressure of 447 mm. in curve 1, Fig. 2. The lack of spectroscopic improvement led to the third more radical attempt at purification of the gas.

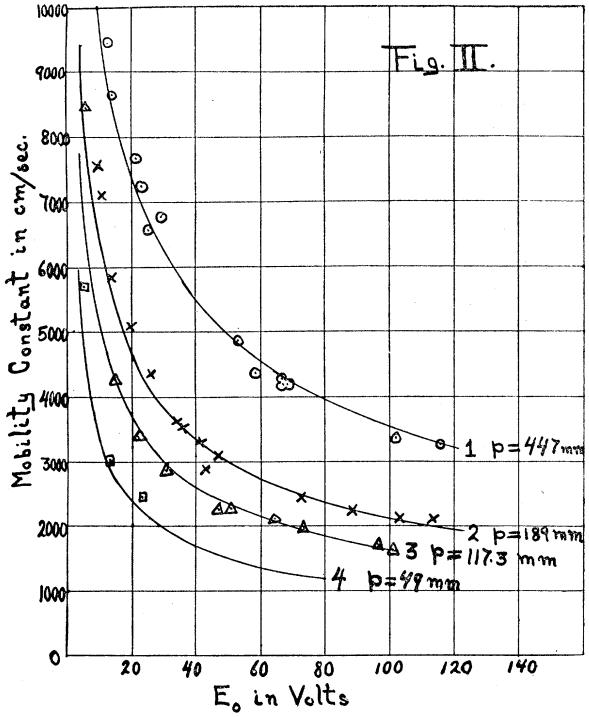

Fig. 2

In this case the gas was examined spectroscopically at 50 mm, 20 mm, and at a pressure less than a mm. In the latter case the gas came directly from the charcoal bulbs immersed in liquid air and nothing but the He lines appeared. The gas studied at 50 mm, while it was circulating, or just after the pump was shut down, showed the green Hg line 5461 A very faintly. The Hα line was about one-fourth as intense as the red He line, 6678 A (sometimes even fainter), and the CO bands were so faint as to be barely perceptible to an eye well accustomed to the dark, even on continued passage of the discharge through the Geissler tube. (From the appearance of the spectrum one might estimate the relative

quantity of impurities in the gas to lie below one part in 10000). The measurements made with this system on several different samples of He showed no change in mobility, and were thoroughly consistent among themselves. The results obtained at 189, 117 and 49 mm are shown in curves 2, 3 and 4 of Fig. 2.

Except for the use of the new ionization chamber in some of the measurements, the apparatus and procedure were precisely that used in the case of the electrons in H_2.[2] In the present experiments, however, the use of retarding fields, suggested in the previous paper, was *occasionally* resorted to. The method for computing the mobility when a retarding field is employed is described in a forthcoming paper on the mobilities of electrons in air.[5] The values found for He with and without such fields were so closely the same that it was impossible to distinguish between them.

Values of the Mobility

The points plotted in the curves marked 2, 3, 4 of Fig. 2 are the mobility constants computed from the relation $K=(\pi\, d^2/\sqrt{2}\, E_0 T)(760/p)$ (i. e. the values of the mobility reduced to atmospheric pressure), plotted against the value of the critical voltage E_0. These were obtained with the small ionization chamber using a plate distance of 1.641 cm. The points on the curve 1 were taken at 447 mm with the plate distance of 1.95 cm in the large ion chamber, during the second attempt at purification.

The smooth curves drawn through the points represent the equation

$$K=\sqrt{\frac{11.0\times 10^8}{2.45+(E_0/d)(760/p)}}$$

with the appropriate values of E_0, d and p inserted. As is seen the curves on the whole fit the points very well. The agreement obtained shows that the mobilities in He may be deduced for any pressure and plate distance by means of this relation. Thus He in contrast to H_2[2] shows the strict proportionality between the theoretical constants a and b (of the previous papers[1,2]) and the quantity $760/p$, which is to be expected from theoretical considerations. It is also to be noted that while N_2 and H_2 do not show the expected theoretical form for the equation for the mobility constant K, (viz. $K=\sqrt{a/(b+E_0)}$) the gas He does. A similar agreement in form between the theory cited and experiment has recently been observed by Wahlin[6] in the case of CO.

[5] Loeb, L. B., Proc. Nat. Acad. Sci., Oct. 1923

[6] Wahlin, H. B., Phys. Rev. **21**, 517-524, 1923

CORRECTION OF MOBILITIES DETERMINED WITH A SINE WAVE

In the preceding papers and up to this point in the present paper, the value of K taken from the experimental points has been assumed to be the true value of the mobility.

In December 1922, H. A. Wilson pointed out that owing to the use of a sine wave alternating potential with the variable electron mobilities, a correction should be made in the value of K deduced from experiment to give the true value. The writer had considered the possibility of such a correction in 1921, but the method of analysis used failed to show the necessity of it. A more careful consideration following Professor Wilson's suggestion showed such a correction to be essential. Wahlin[6] in his recent paper on electron mobilities in CO independently arrived at the same conclusion.

To see how such a correction must be applied one may consider the simple case where the mobility constant K is given by an equation whose form is $K=b-aV$, where V is the voltage and a and b are constants. This is an equation which holds for electrons in air at 66.5 mm pressure.[5] In the discussion it is simpler to leave out the factor reducing the mobility computed at a pressure p, to the mobility constant K, at 760 mm. In what follows one may consider that this reduction has already been made. For a sine wave alternating potential the quantity V takes on the value $E_0 \sin\omega t$ when the electrons can just cross the distance d in one half cycle $T/2$. The condition for electrons crossing the plates may be expressed by the relation:

$$d=\int_0^{\frac{1}{2}T}(E_0/d)K \sin \omega t\, dt$$

where $\omega=2\pi/T$

or

$$\frac{d^2}{E_0}=\int_0^{\frac{1}{2}T}(b-aE_0 \sin\omega t) \sin \omega t\, dt$$

whence

$$\omega d^2/E_0=2\pi d^2/TE_0=2b-\tfrac{1}{2}\pi aE_0.$$

Now the mobility observed, K', is computed from the relation $K'=\pi d^2/TE_0$. Hence $K'=b-\frac{1}{4}\pi aE_0$. But from the observed data we deduce a relation $K'=b-a'E_0$ whereas actually $K=b-aE_0$. Thus to get the true value of K from the observed value we must divide the a' observed by $\frac{1}{4}\pi$, i.e. we must write $a=(4/\pi)a'$.

In the case of the gases H_2 and N_2 where we found K to be expressed by an equation of the form $K=a/(b+V)$ the correction is not as simple as the one above. Putting this value of K into the condition for electrons going a distance d, we get:

$$d=\int_0^{\frac{1}{2}T}\frac{aE_0}{b+V}\sin\omega t\, dt=\int_0^{\frac{1}{2}T}\frac{aE_0 \sin\omega t\, dt}{b+E_0 \sin\omega t}$$

or $$d^2 = a\int_0^{\frac{1}{2}T} \frac{\sin \omega t\, dt}{b/E_0 + \sin \omega t}$$

Call $\omega t = \theta$, $B = b/E_0$, and the equation becomes

$$d^2 = \frac{a}{\omega}\int_0^{\pi} \left(1 - \frac{B}{B+\sin\theta}\right) d\theta$$

This integral has two values depending on the value of B

For $B>1$ $$\int_0^{\pi} \frac{d\theta}{B+\sin\theta} = \frac{2B}{\sqrt{(B^2-1)}}\left(\tfrac{1}{2}\pi - \tan^{-1}\frac{1}{\sqrt{(B^2-1)}}\right)$$

For $B<1$ $$\int_0^{\pi} \frac{d\theta}{B+\sin\theta} = \frac{B}{\sqrt{(1-B^2)}} \log \frac{1+\sqrt{(1-B^2)}}{1-\sqrt{(1-B^2)}}$$

From these functions it is impossible to separate out the variable B in a form so as to permit its being applied to the correction of the observed constants a' and b'. By plotting the integrals as a function of B, it is found that the curve obtained may be very closely approximated by the purely arbitrary equation of the form $1.785/(0.568+B)$. The accuracy of the fit is as close as the errors involved in the mobility measurements permit one to work. Thus one can write that

$$\int_0^{\pi} \frac{\sin\theta\, d\theta}{B+\sin\theta} = \frac{1.785}{0.568+B}$$

and hence one may set

$$d^2 = \frac{a}{\omega}\left(\frac{1.785}{0.568+B}\right) = \frac{2E_0}{\omega}\left(\frac{.893\, a}{b+0.569E_0}\right)$$

or $$\frac{\pi d^2}{TE_0} = K' = \frac{1.57a}{1.766+E_0} = \frac{a'}{b'+E_0}.$$

But the true K is given by $K = a/(b+E_0)$, while the observed K is given by the equation above, hence one can write that the observed a' divided by 1.57 gives the true a, and the observed b' divided by 1.76 gives the true b. So that the equations obtained for N_2 and H_2 must have their constants corrected by multiplying the a' observed by .637, and the b' observed by .568. Hence the electron mobilities in H_2 and N_2 now become:

$$K = \frac{363{,}700}{11.9+(V_0/d)(760/p)} \text{ (for } N_2\text{) and } K = \frac{4.32+10^5}{55.2+(V_0/d)(760/p)^{\frac{1}{2}}} \text{ (for } H_2\text{).}$$

The values of K_0 when $V_0 = 0$ become:

$K_0 = 30{,}500$ cm/sec. (for N_2) and $K_0 = 7820$ cm/sec. (for H_2).

Finally in the case of He the correction is derived from the condition that the electrons cross when

$$d=\int_0^{\frac{1}{2}T}\sqrt{\frac{a}{b+E_0\sin\omega t\,dt}}\;\frac{E_0\sin\omega t\,dt}{d}$$

for in He, K varies as $\sqrt{a/(b+V)}$. Putting $B=b/E_0$ and $\omega t=\theta$ we get the relation:

$$d^2=\frac{\sqrt{aE_0}}{\omega}\int_0^{\pi}\frac{\sin\theta\,d\theta}{\sqrt{(B+\sin\theta)}}$$

The integral is an elliptic integral, which as Wahlin[5] shows on reduction takes the form:

$$\int_0^{\pi}\frac{\sin\theta\,d\theta}{\sqrt{(B+\sin\theta)}}=2\sqrt{2}\{2E(k,\Phi)-F(k,\Phi)\} \qquad (2)$$

where

$$\Phi=\sin^{-1}(1+B)^{-\frac{1}{2}}\text{ and }k=\sqrt{\tfrac{1}{2}(1+B)},$$

for values of B less than unity only. For values of B above unity the relation between B and the value of this integral may be computed graphically. The curve represented by plotting the integral in Eq. (2) against B can be very closely approximated to by the following empirical expression:

$$\int_0^{\pi}\frac{\sin\theta\,d\theta}{\sqrt{(B+\sin\theta)}}\cdot=\cdot\sqrt{\frac{3.69}{0.638+B}}$$

and hence

$$d^2=\frac{\sqrt{a\,E_0}}{\omega}\sqrt{\frac{3.69}{.638+B}}$$

whence

$$\frac{\omega d^2}{2E_0}=\frac{\pi d^2}{E_0T}=K'=\sqrt{\frac{1.445a}{1.566\,b+E_0}}$$

Thus the a' and b' observed experimentally are to be multiplied by .688 and .638 respectively to give the true values of the constants of the mobility equation for electrons in He. The mobility of the electrons in He are accordingly best represented by the equation

$$K=\sqrt{\frac{7.57\times10^8}{1.565+(V_0/d)\left(\frac{760}{p}\right)}}.$$

Discussion of the Results and Interpretation of Equation

From this value of K, the value of K_0, the mobility for vanishingly small fields, comes out to be 22,000 cm/sec. Substituting the value of L the mean free path taken from the kinetic theory of gases, into Townsend's equation for electronic mobility viz. $K=.815eL/mc$, where c is the velocity of thermal agitation, one obtains the value of K_0 as 20,300 cm/sec. On this basis it may be concluded that the electron in He has about the mean free path to be expected from the kinetic theory.

Since the form of the mobility equation for He observed experimentally coincides with the theoretical equation deduced by the writer it is of interest to compare the absolute magnitudes of the observed and computed mobilities. According to the writer[1] the mobility is given by

$$K = .815 \frac{e}{m} \frac{L}{c\sqrt{1+.575\, EeL/mdc^2\sqrt{f}}}$$

Putting $d=1$, $p=760$, $f=.000268$, and L proportional to $4\sqrt{2}\times$ that for atoms in He, the equation reduces to

$$K = \sqrt{54.6\times 10^8/(13.2+E)}$$

The curve given by this equation is shown in curve 2, Fig. 3. Curve 1 is the experimental curve, taken from the results cited above. It is obvious that there is no agreement between the two curves except near the point

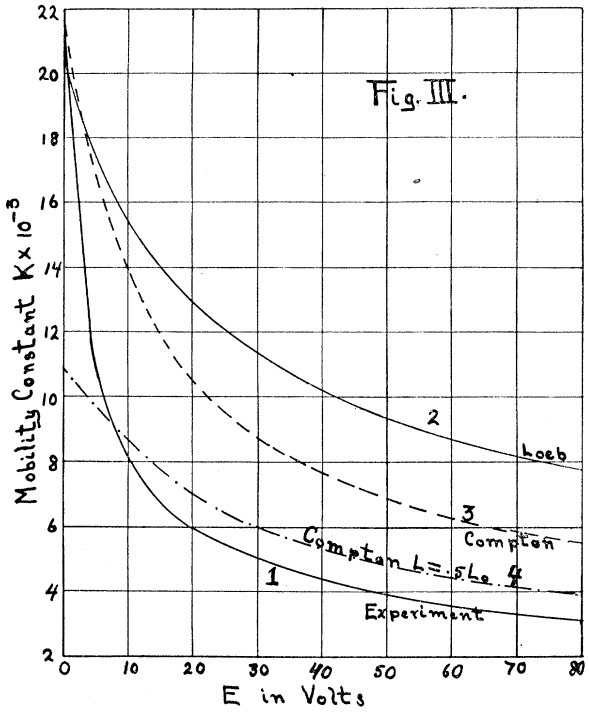

Fig. 3

$E=0$, and except in the fact that their general form is similar. The discrepancy results from the fact that the value of the computed term $.575eL/mc^2\sqrt{f}$ is .0756 which is much smaller than the corresponding factor .64 in the experimental equation. The only uncertain quantities in the theoretical factor are the $\sqrt{f}$ and the L. If the $\sqrt{f}$ were 0.119 times its theoretical value the equations would agree fairly well. This is,

however, impossible as the lower limit that f can have is .000268. It is possible that by lowering L the curves can be made to coincide at higher fields. In doing this the discrepancy is merely shifted to the lower values of E, for the L is in the numerator of the equation for K as well as in the factor discussed above.

In a recent paper K. T. Compton[7] has deduced a slightly different equation for K, in which the averaging of the energy of thermal agitation of the electron and its energy acquired in the field is probably carried out in a more legitimate fashion than in the writer's case. Compton's equation may be written as follows

$$K=\frac{254{,}000\ l_0}{[1+\{1+1.355\times10^6\ Ml_0^2\ (E/p)^2\}^{\frac{1}{2}}]^{\frac{1}{2}}},$$

where l_0 is the electronic free path at 1 mm, p is the pressure in mm, M is the atomic weight of the gas relative to hydrogen as unity, and E is the electric intensity. Putting M equal to 4, l_0 equal to 0.1205 cm, p equal to 760 mm and d equal to unity, this equation becomes comparable to those above, in the form

$$K=\sqrt{\frac{9.37\times10^8}{1+(1+0.1365E^2)^{\frac{1}{2}}}}$$

On plotting this equation the curve 3 of Fig. 3 is obtained. It is seen that while it lies nearer to the observed curve 1 than the writer's equation, the agreement is far from good. If the value of l_0 be taken as 0.32 the kinetic theory value used before, the two curves will lie fairly close together above 20 volts, but differ radically below this. Curve 4 is obtained from Compton's equation putting $l_0=0.0602$ cm.*

The conclusion to be drawn from these results is that while the theory is able to account for the variation of the electronic mobility in its broad outlines, it is in its present form unable to predict the numerical values

[7] Compton, K. T., Phys. Rev. **22**, 333, 1923

* Since this paper was written there has appeared an article on electron mobilities in He by J. S. Townsend (Phil. Mag. **46**, 274, Oct. 1923), in which are given the results of the measurement of electron mobilities in He at pressures up to 240 mm, using his well known lateral diffusion method (Phil. Mag. Dec. 1921). The values obtained by him lie well along the curve 4 (Fig. 3) obtained by putting $l_0=0.0602$ cm in Compton's equation. Since it is probable that the method of Townsend is more capable of giving correct values for K than the writer's owing to the difficulties mentioned in the paper, one may consider this a partial verification of Compton's equation if l_0 have the value assumed. It is desired to point out that the less certain method used by the writer was the first method to give any estimate of the value of the electron mobilities above 100 mm pressure, and is today the only one capable of going above this pressure in most gases. The results given by it are certainly correct in order of magnitude and indicate the variation of these mobilities with field strength and pressure with fair accuracy at the higher field strengths.

obtained for the mobility. It is quite possible that the quantity *assumed to be the mobility in the measurements, is not the true electron mobility*, and thus should not be compared too closely with the theory. The reason for this is that the mobility computed from the measurements is the average of a mobility varying with the field, measured in a variable field. Thus this mobility is an average in which owing to the complexity of the phenomena the averaging may not have been properly performed. There is to be included among the factors making the interpretation of the results uncertain, the added one suggested by Compton[7] that the electrons may travel a finite distance in the electric field before they achieve their terminal speeds. At the lowest pressure used in the present experiments the distance traversed before nine tenths of the terminal speed is acquired is at a maximum seven per cent of the distance between the plates. The error introduced by this effect then is not serious, but should be considered. Thus allowance must be made for the uncertainties in the true values of the mobilities when an attempt is made to compare them with the theory.

National Research Fellowship,
Ryerson Physical Laboratory,
August 7, 1923.