ON THE COLOR OF THE SEA[1]

BY WAS. SHOULEJKIN

ABSTRACT[2]

Color of seas and lakes cannot be explained either by (1) the intrinsic color of water due to selective absorption $f(\lambda)$, or by (2) the Rayleigh scattering by microscopic particles or bubbles of gas $\frac{1}{2}\alpha/\lambda^4$, alone. A combination of these two causes, however, together with (3) selective reflection $\varphi(\lambda)$ by larger particles (clay, plankton, etc.) and (4) light reflected from the sky, seems to be adequate, if the effect of waves is taken into account. The *general theory* is developed, and then, using experimentally determined values of the selective reflection $\varphi(\lambda)$ for a brown clay suspension, curves are computed for various values of the proportion of such particles β, and for the scattering coefficient α, which give resultant colors varying from green to brown, as determined by optical synthesis. By the use of a colloidal solution of rosin in alcohol colored with Redulin blue, a *model of the sea* was made which gave spectrophotometric curves agreeing closely with the theory. The *theory of the Secchi disk* is given and the relation between the coefficient of diffusion α and the maximum depth at which the disk is visible. Coefficients computed from disk observation for various lakes agree with those obtained from the spectral distribution curves of Aufsess.—G. S. F.

THE numerous attempts to explain the color of seas and lakes have usually proceeded from two principal assumptions. One group of authors ascribe the origin of this color to the intrinsic color of water due to the strong absorption in the water of rays of the yellow and red parts of the spectrum and to the feeble absorption of the blue and violet rays, noted by Bunsen and finally definitely established by Spring. Then after the work of Lord Rayleigh[3] on the color of turbid media was published, other authors tried to explain the color of the seas entirely by the scattering of light by fine particles suspended in the water as a result of which they thought the diffused light coming from the water acquires a blue color. But such an explanation has not given satisfactory results, quantitatively nor even qualitatively, and O. v. Aufsess,[4] who applied methods of spectrophotometry to such researches, was obliged to discard it.

[1] Communicated to the Scientific Institution of Moscow, October 8 and 15, 1921.
[2] This abstract was prepared by an editor and has not been verified by the author.
[3] Lord Rayleigh, Phil. Mag (4) **41,** 1871; (5) **12,** 1881; (5) **47,** 1899.
[4] O. v. Aufsess, Physikalische Eigenschaften der Seen; Die Farbe der Seen (Dissertation) München.

From observations of the coefficient of absorption in different parts of the spectrum for a series of specimens of water taken from different lakes, Aufsess concludes that the diffusion of light by the suspended particles is a phenomenon of the second order, and that all gradations of color in seas and lakes are caused by coloring materials mixed with the water, which produce the dependence of the coefficient of absorption upon the wave-length shown by curves calculated by him.

It is easy to show that neither the theory of Rayleigh in its pure form, nor the supposition of Aufsess, is justified by the facts. The first would lead to white coloring for the sea and the second to perfectly black, in case we should try to get quantitative results.

This work is an attempt at a quantitative investigation, as far as possible, of all the fundamental phenomenona connected with the transmission of light through sea water. Such an investigation permits us to obtain the spectrum of the diffused light emitted by the sea, for a series of typical cases.

1. *Diffused interior light.* Let us suppose that upon the surface of the sea there falls a certain quantity of radiation, depending, generally speaking, upon the height of the sun and illumination of the sky. Let us denote the quantity of energy flowing in through a unit of surface by J_0. Let us consider an elementary horizontal stratum, dz, the distance of which below the surface of the water is z. The quantity of energy flowing in from above will be J and the quantity of energy flowing out downwards $J+dJ$, where dJ is negative. The decrease of energy dJ is caused by two factors, (1) the diffusion of energy in the elementary stratum dz and (2) absorption of energy by the water of this stratum.

If we denote by φ the angle between the ray falling upon an element of volume of the turbid medium and one of the diffused rays emitted in all directions, the energy of this diffused ray, according to Rayleigh, is proportional to $(1+\cos^2\varphi)$. Consequently all particles enclosed inside the stratum dz, will diffuse an equal quantity of energy upwards and downwards from this stratum. Therefore, if $(\alpha/\lambda^4)Jdz$ is the whole quantity of energy emitted by the particles of the stratum, the quantity of energy radiated upwards will be equal to $\frac{1}{2}(\alpha/\lambda^4)Jdz$ where λ is the wave-length and α is the coefficient of diffusion.

The energy absorbed in the water of the stratum dz is also a function of the wave-length and will be equal to $Jf(\lambda)dz$. The total decrease of the flow of energy through unit surface downwards, will be, therefore,

$$dJ = -J\left[\frac{1}{2}\frac{\alpha}{\lambda^4}+f(\lambda)\right]dz \tag{1'}$$

We will take into account the absorption of energy by the suspended particles themselves later, in section 4. From this equation it is easy to determine the quantity of energy flowing from above on unit surface of the stratum with the coordinate z

$$J = J_0 e^{-[\frac{1}{2}(\alpha/\lambda^4)+f(\lambda)]z} \qquad (1)$$

Remembering that the quantity of energy, radiated by the stratum upwards is equal to $\frac{1}{2}(\alpha/\lambda^4)Jdz$, and denoting it by dI_z, we obtain from Eq. (1)

$$dI_z = \tfrac{1}{2}(\alpha/\lambda^4) J_0 e^{-[\frac{1}{2}(\alpha/\lambda^4)+f(\lambda)]z} dz$$

The light coming out of the sea is emitted by these elementary strata dz, but of course between each stratum dz and the surface of the sea, there lies a stratum of water with the thickness z, the absorption and diffusion in which is determined by formulas analogous to (1′) and (1).

If dI_1 is the quantity of energy that has reached the surface of the sea from the stratum dz, then

$$dI_1 = dI_z e^{-[\frac{1}{2}(\alpha/\lambda^4)+f(\lambda)]z} = \tfrac{1}{2}(\alpha/\lambda^4) J_0 e^{-[\frac{1}{2}(\alpha/\lambda^4)+f(\lambda)]2z} dz \qquad (2)$$

As the visible rays of the spectrum do not, practically, penetrate depths of even several hundred meters, the integration of Eq. (2) can be carried out without paying attention to the bottom of the sea, that is within limits from 0 to ∞

$$I_1 = \int_0^\infty \tfrac{1}{2}(\alpha/\lambda^4) J_0 \cdot e^{-[\frac{1}{2}(\alpha/\lambda^4)+f(\lambda)]2z} dz = \frac{J_0}{2} \frac{\frac{1}{2}(\alpha/\lambda^4)}{\frac{1}{2}\alpha/\lambda^4+f(\lambda)} \qquad (3)$$

It is evident that I_1 forms only one part of the whole energy coming out of the sea. For on the way from the stratum dz to the surface of the sea, part of the element of energy dI_z is scattered downwards. Applying here the same methods as we used with reference to the incident energy J_0, and carrying out the integration, we find the second part I_2 of the light energy reflected upwards to be

$$I_2 = \frac{J_0}{4} \left[\frac{\frac{1}{2}\alpha/\lambda^4}{\frac{1}{2}\alpha/\lambda^4+f(\lambda)} \right]^2$$

In the same way can be obtained the other parts of the total energy I radiated upwards, and we find the sum $I = I_1+I_2+I_3+\ldots$

$$I = \sum_{n=1}^{n=\infty} J_0 \left[\tfrac{1}{2} \cdot \frac{\frac{1}{2}\alpha/\lambda^4}{\frac{1}{2}\alpha/\lambda^4+f(\lambda)} \right]^n = J_0 \frac{\frac{1}{4}\alpha/\lambda^4}{\frac{1}{4}\alpha/\lambda^4+f(\lambda)} \qquad (4')$$

Evidently Eq. (4) allows us to obtain the spectrum of light "reflected" by sea water in all its thickness from the surface to the bottom, if α and $f(\lambda)$ are known. Before passing to the consideration of these quantities however, let us reduce this formula to a somewhat different form.

For simplicity, let us suppose that the sea is illuminated from above by parallel rays falling perpendicularly on its surface. This illumination is composed of two parts, the light of the sun (S_0) and the illumination of the sky (H_0), which over our seas, even the southern ones, is very feebly colored, and therefore we shall not take this coloring into account.[1] Therefore, $I_0=H_0+S_0$. As the light I emitted by the sea, is diffuse, the intensity of interior illumination M_0 will be $M_0=I/\pi$. The final form of Eq. (4′) will therefore be

$$M_0=\frac{H_0+S_0}{\pi}\cdot\frac{\frac{1}{4}(\alpha/\lambda^4)}{\frac{1}{4}(\alpha/\lambda^4)+f(\lambda)} \tag{4}$$

From this we also see that neither the diffusion of rays by the suspended particles, nor their absorption by the water itself (the intrinsic color of the water) should be disregarded. In the first case ($\alpha=0$) we should obtain a perfectly black surface ($M_0=0$), and in the second ($f(\lambda)=0$) a perfectly white one, $M_0=(H_0+S_0)/\pi=\text{const.}$, independent of λ. The sea would reflect the light in the same way as a white diffusing surface, for which the albedo is one.

2. *The determination of the coefficients of diffusion and absorption.* The absorption of visible rays of light in optically clear water has been studied sufficiently well. In our calculations according to our main formula (4) we have used the numerical values of the coefficient of absorption $f(\lambda)$ found in the above mentioned book of O. v. Aufsess. It appears that this book also contains in a concealed form the data for the determination of the coefficient α. In fact, if we examine the curves that represent the coefficient of absorption in the water of different lakes investigated by Aufsess, as a function of the wave-length, it will be obvious that the characteristic irregularity of these curves shows the composite character of this "coefficient of absorption" which the author ascribes to coloring materials mixed with the water. We have only to subtract from the ordinates of these curves the corresponding ordinates of the curves $f(\lambda)$ (the coefficient of absorption in optically clear water) and the remaining part will come out inversely proportional to the fourth power of the wave-length. The experimental curves of Aufsess appear, then, to express the dependence of the sum of the coefficients of absorption and of diffusion upon the wave-length, i.e. their ordinates are equal to $m=\frac{1}{2}(\alpha/\lambda^4)+f(\lambda)$.

By changing the parameter α, we can get a series of curves for m.

In Fig. 1 are shown 4 curves of this series, curves 1, 2, 3 and 4 corresponding to four different values of α, zero, .004, .020 and .070 respec-

[1] See section 5.

tively. No. 1 shows the coefficient of absorption in optically clear water [$f(\lambda)$]. No. 5, taken from the above mentioned book of Aufsess, was experimentally obtained by him for Staffel Sea.

3. *Calculation of the spectra, neglecting the absorbing effect of suspended particles.* If the function $f(\lambda)$ be known and if the order of magnitude of coefficients α as actually found, is also known, we can calculate a series of typical spectra from formula (4), corresponding to definite values of α. The spectra obtained are represented in Fig. 1, curves 6, 7, and 8 cor-

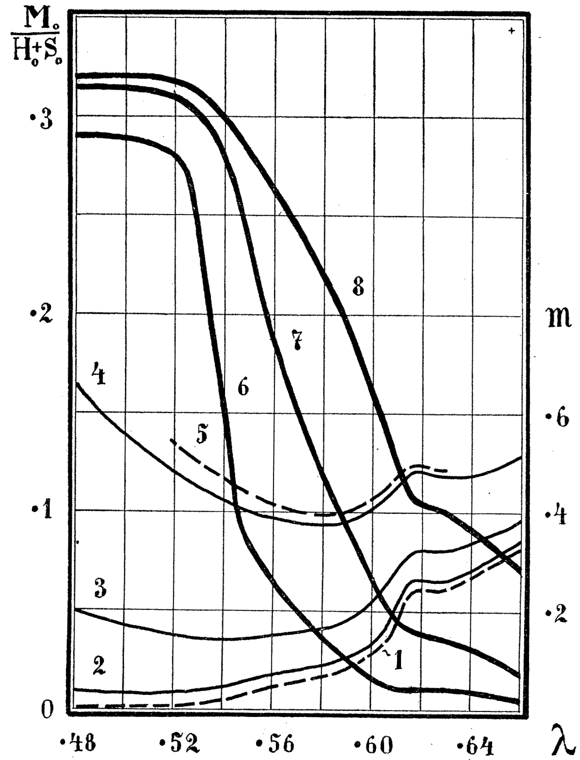

Fig. 1

responding to $\alpha = .004$, .020, and .070, respectively. The figure shows how abruptly the curve 6 falls in the green part of the spectrum and how much absorption there is in the yellow and red. As the coefficient of diffusion becomes larger the curve becomes less steeply inclined and the color of the sea consequently greener and considerably less saturated, i.e. with a large mixture of white.

4. *Influence of suspended particles showing selective reflection.* In deriving Eq. (4), it was supposed that the suspended particles diffused

light without absorbing it. In reality besides such particles (possibly small bubbles of air) the water may contain, especially after a gale, a large amount of finely divided dust or clay particles, plankton, etc., capable of selective reflection and absorption. As these particles are of a comparatively large size, we can, without large error, assume simple reflection and absorption of light by them and restrict the Rayleigh scattering to the smallest suspended particles of which we spoke in §3. When we take into consideration the action of the particles of both kinds and the absorption of light in the water itself, we can obtain results completely analogous to those above.

If the coefficient of reflection of the particle, corresponding to the wavelength λ is $\varphi(\lambda)$, then $\psi(\lambda)=1-\varphi(\lambda)$ will be the corresponding coefficient of absorption. The decrease of light by a stratum of a thickness equal to dz will be

$$dJ=-J\{(1-\beta)\cdot \tfrac{1}{2}(a/\lambda^4)+f(\lambda)+\beta[\varphi(\lambda)+\psi(\lambda)]\}dz$$

where β is the probability of the ray encountering a particle of the second kind, in a path of one meter. Upwards there will be reflected a quantity of energy

$$dI_z=J[(1-\beta)\tfrac{1}{2}a/\lambda^4+\beta\varphi(\lambda)]dz$$

Continuing the analysis in the same way as in section 3 we obtain

$$I=J_0\frac{(1-\beta)\tfrac{1}{4}(a/\lambda^4)+\beta\cdot\tfrac{1}{2}\varphi(\lambda)}{(1-\beta)\tfrac{1}{4}(a/\lambda^4)+f(\lambda)+\beta[1-\tfrac{1}{2}\varphi(\lambda)]} \tag{5'}$$

and finally,

$$M_0=\frac{H_0+S_0}{\pi}\cdot\frac{(1-\beta)\tfrac{1}{4}(a/\lambda^4)+\beta.\tfrac{1}{2}\varphi(\lambda)}{(1-\beta)\tfrac{1}{4}(a/\lambda^4)+f(\lambda)+\beta[1-\tfrac{1}{2}\varphi(\lambda)]} \tag{5}$$

As an example I determined $\varphi(\lambda)$ for brown clay experimentally. From formula (5), which is the most general, were then derived the spectral curves 9, 10 and 11 in Fig. 2. They correspond to different values of β, the concentration of particles of the second kind. The coefficient a is in all cases supposed equal to 0.004 as for the curve 6 of Fig. 1; this curve is also given, for comparison, on Fig. 2.

5. *Influence of the reflected light of the sky.* The diffused light emitted by the sea was investigated in §§3 and 4. This light can be readily observed by looking downward from a steep coast into a deep place or from ship board. But to the observer looking at the sea at an acute angle the coloring of the surface appears always changing, depending greatly upon a number of exterior conditions, chiefly upon the state of the surface, and the form and dimensions of the waves.

Let the ray of vision which comes to the eye from some element of surface, make the angle φ with the perpendicular to this element. Evidently, it is easy to divide it into two component rays I_M and I_H. The first one belongs to the diffused interior light. It reached the element of the surface at some angle ψ with the perpendicular. The second ray

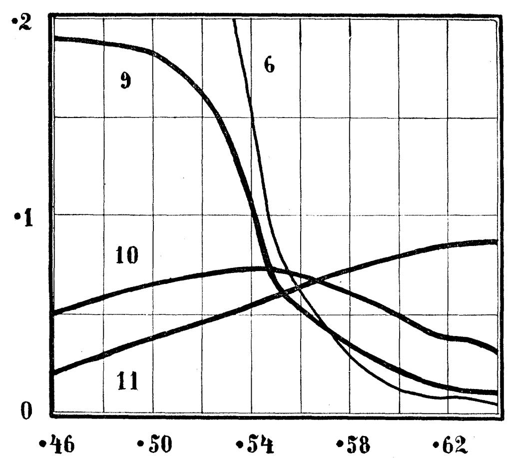

Fig. 2

Curve 9 corresponds to $\beta=0.02$ (color greenish)
Curve 10 corresponds to $\beta=0.20$ (color green)
Curve 11 corresponds to $\beta=1.00$ (color brown).

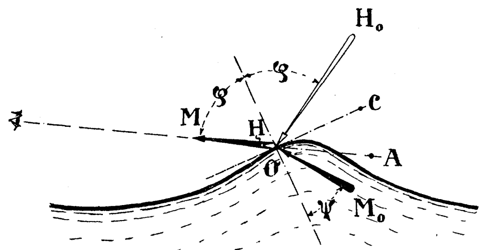

Fig. 3

is the ray of light reflected from the sky, i.e. which has fallen on an element of the surface at the same angle φ with the perpendicular. The intensity of the rays I_M and I_H will depend upon the angle φ according to the well known formulae of Fresnel.

Let us introduce the quantities M and H, the visible brightness of illumination of the surface by the diffused interior light and the reflected light of the sky, respectively, instead of the intensity of the rays I_M and I_H.

Evidently the dependence M upon M_0 and of H upon H_0 will be, by the Fresnel formulae, as follows:

$$M = \frac{1}{2}\frac{\sin 2\varphi \cdot \sin 2\psi.}{\sin^2(\varphi+\psi)}\left[1+\frac{1}{\cos^2(\varphi-\psi)}\right]\cdot M_0 \qquad (6)$$

$$H = \frac{\frac{1}{2}\sin^2(\varphi-\psi)}{\sin^2(\varphi+\psi)}\cdot\left[1+\frac{\cos^2(\varphi+\psi)}{\cos^2(\varphi-\psi)}\right]\cdot H_0 \qquad (7)$$

Now, taking into account the fact that for water: $\sin\varphi/\sin\psi = 1.32$ it is possible to calculate M and H for different angles φ. Fig. 4 shows curves I, II, III sketched according to Eqs. (6) and (7) in polar co-ordinates. Curve I represents the brightness of reflected light of the sky, together with the diffused light of the sea; curve II is the brightness of the diffused light of the sea in the blue part of the spectrum (see curve 6 in the Fig. 1); curve III is the brightness of the diffused interior light in the yellow-green part of the spectrum. The relative lengths of the

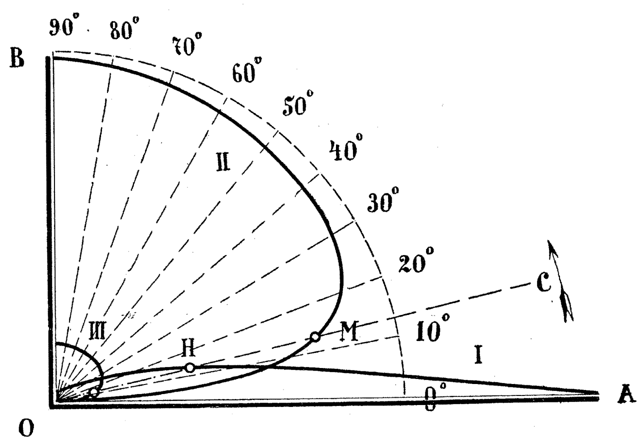

Fig. 4

limited vectors $OA = H_0$ and $OB = M_0$ correspond to the case where $H_0 = S_0$, that case being the usual one according to Bunsen and Roscoe, Pernter and others.

The rays of sight are directed from O towards A. The angle AOC is the angle at which the element of the surface is inclined towards it. From this diagram it is apparent that in calm weather when the surface is horizontal, the angle AOC is most acute and the sea has the same color as the sky (M is nearly zero and H is a maximum). But with comparatively small waves the vector representing H begins to decrease rapidly and the vector M begins to grow, and the sea soon acquires a blue tint. Usually an increase of the intensity of the coloring does not depend so much upon large waves as upon small secondary ones that run over the large ones.

The sea acquires the deepest blue with so called forced waves that differ greatly from the trochoidal free waves and have a very steep front from the lee side. This difference in color can be very well noticed by any one observing the sea from a ship whose course is perpendicular to the direction of the wind; from the lee side the sea seems much more intensively colored than from the weather side where the surfaces of the waves are more steeply inclined.

6. *Approximate determination of the coefficient of diffusion with the Secchi disk.* The so called Secchi disk is usually employed for measuring the transparency of water. This disk (ordinarily of 30 cm diameter), painted white, red or any other color, is sunk into the sea until it disappears in the surrounding background. The corresponding depth of sinking is taken as a measure of transparency for the corresponding color.

Let us try to define, according to Eq. (4), the correspondence between the depth of disappearance and the coefficient α, upon which it must depend. We must notice that all our results can be applied only to a disk with a small diameter, although, as Secchi showed, a considerable increase of diameter beyond a certain amount does not much increase the depth of sinking, a disk with a diameter of 1.37 metres disappearing at a depth only 15 percent greater than a disk with a diameter one-fifth as great. Having made such a limitation and disregarding the shadow which the disk throws upon the surface of the sea as a result of illumination by the interior light, we will find that the disk disappears at a depth when

$$\frac{(M_0+D_z)-M_0}{M_0}=\frac{D_z}{M_0}=\Delta \qquad (8)$$

where D_z is the additional intensity of light which depends upon the depth of sinking of the disk z, and Δ is the least perceptible relative difference in illumination. M_0 is derived from Eq. (4) and D_z can also be obtained with sufficient approximation. In fact, if there were no diffusion, the intensity of light, emitted by the disk at a distance r from it (in a vertical direction), would be:

$$J_r=\mu\frac{J_1\sigma\cos i}{r^2}=\frac{J_1\mu\sigma}{r^2}$$

where J is the intensity of incident light, σ the area of the disk, and μ the albedo. Denoting the compound coefficient of absorption of water $\frac{1}{2}(\alpha/\lambda^4)+f(\lambda)$ by m, we shall find that the decrease of light dJ_r in the distance dz, is

$$dJ_r=-\left[\frac{2J_1\mu\sigma}{r^3}+J_r m\right]dr$$

or
$$\left[\frac{2J_1\mu\sigma}{r^3}+mJ_r\right]dr+dJ_r=0$$

The integral of this differential equation between the limits 0 and z is

$$D_z=J=-2J_1\mu\sigma e^{-mz}\int_0^z\frac{e^{mr}}{r^3}dr$$

Or, as $J_1=(H_0+S_0)e^{-mz}$

$$D_z=-2(H_0+S_0)\mu\sigma e^{-2mz}\int_0^z\frac{e^{mr}}{r^3}dr \tag{9}$$

The value of this integral, reduced to a logarithmic integral, can not be found exactly. Applying methods of approximate calculation, and supposing $\Delta=1/133$ (according to Helmholtz) it is not difficult to get the desired relation between the quantity α and the critical depth of sinking of the disk, z. It is obvious that such a calculation would be justified only when the disk as well as the surrounding background emitted monochromatic light. Then calculating the quantities $f(\lambda)$ and α/λ^4 corresponding to this spectral color for every α we could find m and hence, from Eqs. 8, 4, and 3, get z. From this it can be seen, that first of all the blue disk disappears, and last the red one. As, after a great many experiments, it seemed that the white disk disappears at nearly the same depth as the red one, we thought it possible to take $\lambda=0.65\ \mu$ for an approximate calculation, supposing that the white disk disappears nearly at the same depth as the one which could reflect monochromatic rays with $\lambda=0.65$.

Calculations according to Eqs. (8), (4), and (9) furnish the curve in Fig. 5. In the above mentioned book of Aufsess, a curve of the compound coefficient of absorption is shown for Walchensee. Applying this curve we calculate the coefficient α (see §2) to be 0.0142. On our curve a depth of sinking of the Secchi disk of nearly 20 meters corresponds to such a value of α. This is nearly the same as found by Aufsess in the Walchensee, for he observed the disk to disappear at 17 meters in summer and 24 meters in winter. Comparing the value of α given by the curve in Fig. 5 with the depths of disappearance of the disk in different seas,[5] we arrive at the conclusion that of the curves on Fig. 1, No. 6 corresponds to the Black Sea, No. 7 to the Baltic, No. 8 to the White Sea. In the case of the last two seas, there probably exists in them a large quantity of particles of the second kind.

This summer I hope to make an experimental examination of this theory, using specially constructed instruments. However I have al-

[5] J. Shokalsky, Oceanography., Petrograd, 1917.

ready been able to complete several controlling laboratory experiments and calculations.

7. *Synthesis of colors. Perceptible color of diffused radiation.* It appeared possible to effect such a synthesis experimentally with the aid of a very simple method, which may prove to be useful in other cases where it is desired to synthesize colors according to a given spectral curve.

The spectrum of white light, obtained in the camera of a precisely graduated spectograph was converged by a lens and directed on a screen where it formed a white image. Into the camera (at the place where the

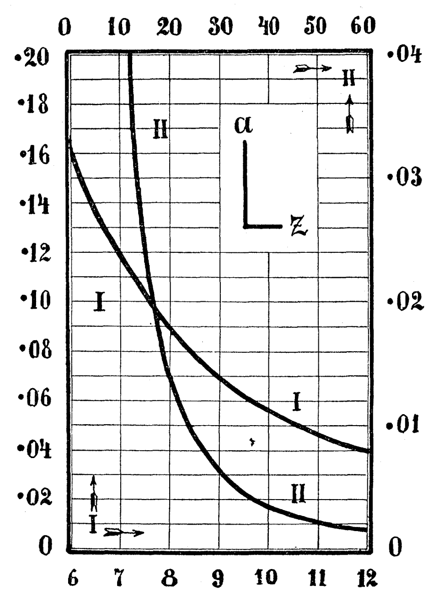

Fig. 5

real image of the spectrum was obtained) a partition was set, in which a slit precisely fitting one of the calculated curves Nos. 6, 7, 8, 9, 10 and 11 was made. The scale of the abscissae was of course different in different parts of the spectrum. Due to such a slit the relative brightness in any part of the spectrum, depending on the breadth of the slit in the corresponding place, was made proportional to the corresponding ordinate of the theoretically obtained curve. By observing the alteration of colors upon the screen beyond the converging lens, it was easy to ascertain that the colors obtained corresponded closely to the actual ones. For the purpose of demonstrating the synthesis of colors, this experiment was somewhat altered. Upon a rotating glass cylinder was placed a gelatin film on which a spectrum occupying half the circumference of the cylinder, the other half remaining transparent, was painted with transparent colors. The spectrum was covered with an opaque template in which a slit corresponding to one or other spectral curve was cut out. The cylinder was placed between the condenser and the object glass of a projection lantern, the focus of the condensor lying on the axis of rotation of the cylinder which was perpendicular to the optical axis of the lantern. Such a construction easily allowed a synthesis of colors; for the intensity of light in each part of the spectrum was proportional to the corresponding ordinate of the theoretical curve (Nos. 6-11).

Results fully concordant with the described experiment can also be obtained by means of a synthesis of colors, based upon the ion theory of color vision of P. P. Lasareff.[6] In accordance with this theory, there were sketched, in each part of the spectrum, three curves characterising the quantity of pigments sensitive to 1) red, 2) green and 3) violet, which are decomposed in the eye by the influence of white light. Multiplying the ordinates of these three curves by the corresponding ordinates of our spectral curve (Nos. 6, 7, 8) there are drawn three new curves for each spectral curve, showing the quantity of all the three pigments affected in the eye by colored light. By evaluating the relative areas of these curves, we obtain the resultant color perceived and its saturation value. The results of synthesis for four of the most typical spectral curves, are given in the following table.

TABLE I

No. of Curve	α	β	Particles of 1st kind	Particles of 2nd kind	By theory of P. P. Lasareff		Color obtained by direct synthesis
					λ percept.	Saturation	
6	0.004	0	Few	None	0.48μ	3.0	Blue, saturated
7	0.020	0	Many	None	0.50μ	1.5	Greenish-blue (pale)
10	0.004	0.2	Few	Few	0.55μ	0.45	Green
11	0.004	1.0	Few	Many	0.61μ	0.39	Brown (pale)

It is clear that the whole scale of colors from saturated blue to light brown is easily explained without introducing the hypothesis of colors added to the water, as did v. Aufsess.

Here it has been supposed that there are suspended in the water, particles of clay; of course particles of other origin, plankton, etc., can produce, according to formula (5), a great variety of colors.

[6] P. P. Lasareff. Bulletin de l'Académie des Sciences de Russie, p. 1051, 1918, Petrograd, and Isvestija Fizicheskago Instituta I, 113, 1920, Moscow.

8. *Photometry of artificial turbid media in reflected light.* As Fig. 1 shows, the total coefficient of absorption and diffusion for natural water is a quantity of the order 0.1—1.0, if it is applied to a wave-length 1μ and a stratum of water 1 meter thick. It is evident that laboratory experiments with photometry in reflected light cannot be made with such liquids and it is therefore necessary to use some liquid more intensively colored than water, and in which the particles are suspended in a greater concentration. It was very tedious choosing colors and suspensions, as conditions had to be created to prevent suspended particles from being themselves colored and a number of other undesirable phenomena had to be avoided. After a number of tests it was found that an acceptable color, because of its curve of absorption and resemblance to water, is Rodulin blue and the most suitable suspension is the colloidal solution of rosin in alcohol, drops of which were added to water colored by Rodulin. With the aid of a König-Martens spectra-photometer (1) the coefficient of absorption for Rodulin-colored water and (2) the total coefficient of absorption and diffusion for the same liquid

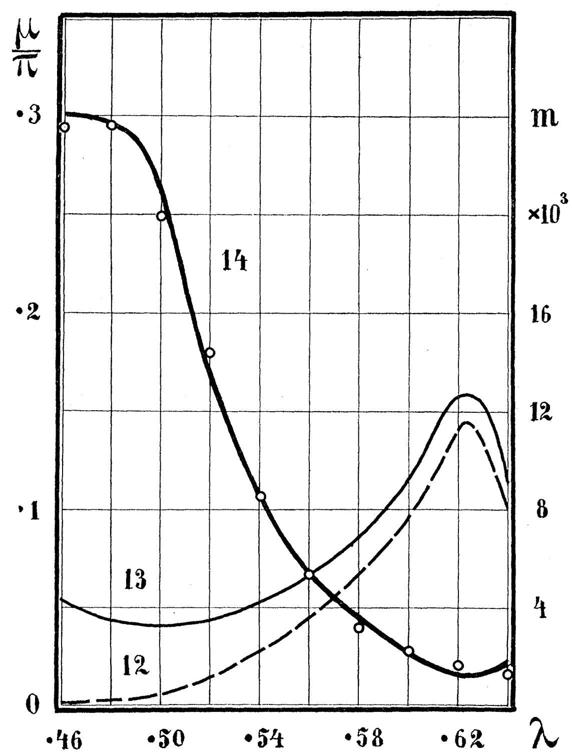

Fig. 6

with an admixture of suspended particles were determined and also for verification, (3) coefficients of diffusion for clean water in which the same quantity of suspension was added as in case (2). These results made possible a calculation of the spectrum of light reflected by our "model of the sea" according to formula (4).

In Fig. 6, curves (12) and (13) accordingly represent the coefficients (1) of absorption for Rodulin solution and (2) of absorption and diffusion for the same solution with suspended particles of rosin (the coefficients relate to a wave length of 1μ and to a layer thickness of 1cm); curve (14) represents the spectrum of reflected light for my "model of the sea," calculated according to our formula (4), that is the ratio of the intensity of the reflected spectral ray to that of the corresponding incident ray, or the coefficient of reflection of the sea divided by π. Then,

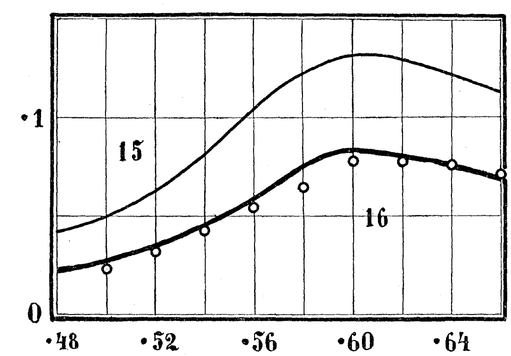

Fig. 7

with the aid of a spectrophotometer, the light reflected by the model of the sea, was compared with the incident light and the experimental points obtained were marked on Fig. 6. It is easy to ascertain that they fully coincide with the curve 3, calculated theoretically.

To verify the formula (5), which takes into account the effect of particles of the second kind, a similar experiment was made in clean water to which there was added a certain quantity of ochre, which produces large-grained particles that do not sink very rapidly. In the first place, with the aid of photometry, the coefficient of reflection, $\varphi(\lambda)$, for dry ochre was obtained, and then it was possible to calculate the spectrum of reflected light according to formula (5), which here takes the simpler form,

$$M_0 = \frac{H_0 + S_0}{\pi} \cdot \frac{\frac{1}{2}\varphi(\lambda)}{1 - \frac{1}{2}\varphi(\lambda)}$$

or: $\frac{\mu}{\pi} = \frac{\frac{1}{2}\varphi(\lambda)}{1 - \frac{1}{2}\varphi(\lambda)}$, because $\alpha = 0$ and $f(\lambda) = 0$, where

μ is "the coefficient of reflection" of our yellow "sea." Both curves $\varphi(\lambda)$ and μ are drawn in Fig. 7, where they are denoted by Nos. 15 and 16.

Experimental points on the same diagram were obtained by direct photometry in reflected light. They lie sufficiently close to the calculated spectral curve (16) although they do not coincide with it as well as in Fig. 6.

9. *On the applicability of the theoretical considerations of §7 to observation with the Secchi disk.* The experimental material collected by O.v. Aufsess, proves just the opposite of his conclusions with regard to the causes of color of lakes and wholly supports my assumptions in deriving formula (4). It is sufficient to look over the last column of Table II which contains the results for Arbersee, calculated by me in accordance with Aufsess's dissertation, to be convinced of the existence of Lord Rayleigh's effect.

TABLE II

λ :	0.52	0.54	0.56	0.58	0.60	0.62	0.64
$m-f(\lambda)$:	1.0	0.86	0.72	0.65	0.56	0.50	0.45
$[m-f(\lambda)]\lambda^4$:	0.0731	0.0731	0.0708	0.0735	0.0726	0.0739	0.0755

It is obvious that the product of the fourth power of the wave-length by the difference between the coefficient of absorption for lake water and for clean water remains constant and near to the mean value 0.0732. So the water of the lake can be looked upon as Rayleigh's turbid medium. The value of the coefficient of diffusion for Arbersee will, obviously, be $a=2\times 0.0732=0.146$.

By extrapolating the curve in Fig. 5, it is easy to calculate the depth of sinking of the Secchi disk in this lake; the depth appears to be equal to $z=6.3$ meters; Aufsess' experiments showed that in reality in the Arbersee the Secchi disk disappears at a depth of 6 meters, which agrees with our calculated quantity.

By calculating tables analogous to that of II, for other lakes for which the observations by Aufsess gave a gradual change of coefficient of absorption with the change of wave-length, it can be demonstrated that for them also our conclusions are justified. For Staffelsee, calculations gave $z=8.5$ meters, and the real depth at which the Secchi disk disappeared was 7.5; for Kochelssee calculations gave 14.7 meters and observations by Aufsess 16(?) meters; for Walchensee our calculations gave $z=19.6$ meters and observations by Aufsess 20.2. Taking in account the roughness of observations with the Secchi disk and the approximations introduced into the calculations, this agreement between theoretical and experimental results can be considered satisfactory.

Conclusions

1. The coloring of the sea depends on four factors: (a) Selective absorption of light by water; (b) diffusion of light by tiny suspended particles (probably small bubbles of air, or of other gas contained in it; (c) selective reflections of light by particles of the second order (particles of fine dust, plankton etc.); (d) admixture of the reflected light of the sky.

2. In the present work the spectrum of the diffused light emitted by the sea is calculated for typical cases.

3. The relation is found between the inclination of waves running over the surface of the sea, and the intensity of its color.

4. A method is given by which an approximate calculation can be made of the coefficient of diffusion, with the aid of an ordinary Secchi disk.

5. Fig. 4 allows us to explain the intense color of mountain lakes where the surrounding mountains cover part of the sky near the horizon. The zones of the sky are reflected in water only at a considerable angle. The admixture of white color is therefore not great and the observer sees the color of the lake only as it is colored by the diffused interior light.

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