MOBILITIES OF ELECTRONS IN GASES

By K. T. Compton

Abstract

Motion of electrons through gases.—The theoretical equation for mobility constant obtained in a recent paper, is put in the form

$$K=2.54(10^5)l_0\left\{1+[1+1.355(10^6)Ml_0^2(E/p)^2]^{\frac{1}{2}}\right\}^{-\frac{1}{2}}$$

where l_0 is electronic mean free path at 1 mm pressure, M is molecular weight relative to H, E is the electrical field in volts/cm, and p is pressure in mm Hg; and it is tested by comparison with recent experimental data obtained by Loeb and by Townsend and co-workers. Although the equation contains no arbitrary constant if l_0 is taken from kinetic theory, the agreement with experiments is good in the case of H_2 and within a small factor in the case of He, N_2 and A, for values of E/p less than the critical value which is characteristic of each gas and equal to about 20 for H_2, 1.3 for N_2, 0.5 for A, and $>.4$ for He. *Elasticity of collisions.* In explanation it is suggested that when electrons collide at speeds greater than the critical for the gas, the collisions are no longer perfectly elastic, as assumed by the theory. In the region of elastic collisions, the equation may be used to compute the actual equivalent *mean free paths* for elastic spheres from the measured values of K, for comparison with the kinetic theory values of l_0. This free path in H_2 is close to the kinetic theory value, in He it is a little less, while in N_2 and A it is greater and varies with the speed, being greater at small speeds. These free paths are compared with values determined in other ways and the differences are attributed to differences in the meaning attached to the term collision. The data for O_2 and CO_2 are insufficient to give the values of the critical E/p but indicate that these are lower than for the other gases.

IN a preceding paper[1] the nature of the motions of electrons in gases in an electric field was considered with reference to the energy lost at collisions, the terminal speed acquired, the number of collisions made and the average rate of drift in the direction of the field. Particular attention was given to the special case in which collisions between electrons and molecules are treated as if between elastic spheres. The purpose of the present paper is to compare the consequences of this theory with results of experimental determinations of electron mobilities in gases.

Owing to the large values of the mobility and complicating results of temporary attachments to gas molecules, it is only recently that these mobilities have been measured with any claim to accuracy. Loeb[2, 3] has successfully applied a modification of Rutherford's[4] alternating current

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[1] Compton, Phys. Rev. **22**, 1923

[2] Loeb, Phys. Rev. **19**, 24, 1922

[3] Loeb, Phys. Rev. **20**, 397, 1922

[4] Rutherford, Proc. Camb. Phil. Soc. **9**, 401, 1898

method and Townsend[5,6] has developed a new method depending on the deflection of the direction of average drift by a transverse uniform magnetic field. Loeb's method is the more direct, but is subject to uncertain corrections due to a.c. wave form, persistence of ions between cycles and lag of terminal speed behind impressed e.m.f., from which Townsend's method is free.

The equation for the electron mobility derived in the preceding paper is

$$\mu=\frac{0.815el}{\sqrt{m}[\alpha T+(\alpha^2T^2+l^2ME^2e^2/1.134m)^{\frac{1}{2}}]^{\frac{1}{2}}},\tag{1}$$

in which e and l are the electronic charge and mean free path, m and M are the masses of electron and molecule, αT is the mean energy of thermal agitation of gas molecules, and E is the electric intensity. This equation may be considered as a corrected form of an equation recently proposed by Loeb.[3] All mobilities considered in this paper are reduced to standard conditions, being expressed as mobility constants K, given by

$$K=(p/760)(273/T)\mu.\tag{2}$$

Since the mean free path l varies inversely as the pressure, we may put $l=l_0/p$, where l_0 is the mean free path at 1 mm pressure. We thus see that K is a function of E/p, which takes the following form when numerical values are substituted for certain constants:

$$K=\frac{254000l_0}{[1+(1+1355000Ml_0^2\,(E/p)^2)^{\frac{1}{2}}]^{\frac{1}{2}}},\tag{3}$$

in which E is expressed in volts per cm and M is the molecular weight on the basis $M_H=1$.

HYDROGEN

Fig. 1 shows the experimental data and the theoretical curve (1) obtained by substituting $M=2$ and $l_0=0.0784$ cm in Eq. (3). This is the kinetic theory value of l_0 given by $l_0=4\sqrt{2}L_0$, where L_0 is the molecular mean free path at 1 mm pressure. Townsend and Bailey's[6] values fall nearer the theoretical curve than do Loeb's.[7] When it is noted that Eq. (3) contains no adjustable constants, the agreement is rather striking. If the electronic mean free path were 0.58 of the kinetic theory value, Eq. (3) would give curve (2) which falls fairly well through the mean of Loeb's values of K. The failure of all of Loeb's data, taken at different

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[5] Townsend and Tizard, Proc. Roy. Soc. **88**A, 336, 1913

[6] Townsend and Bailey, Phil. Mag. **42**, 873, 1921

[7] In this connection it may be remarked that Loeb's results have all been corrected slightly for a.c. wave form by a factor which has not yet been published, but which Dr. Loeb kindly communicated to me.

pressures, to be expressed as a single function of E/p suggests that some one of the corrections noted above should be applied to them.

NITROGEN

Fig. 2 presents Loeb's data on nitrogen and one determination by Townsend and Bailey.[6] Curve (1) is the theoretical curve of Eq. (1), taking the kinetic theory value of $l_0 = 0.0405$ cm. The actual values of K are considerably larger than the predicted values.

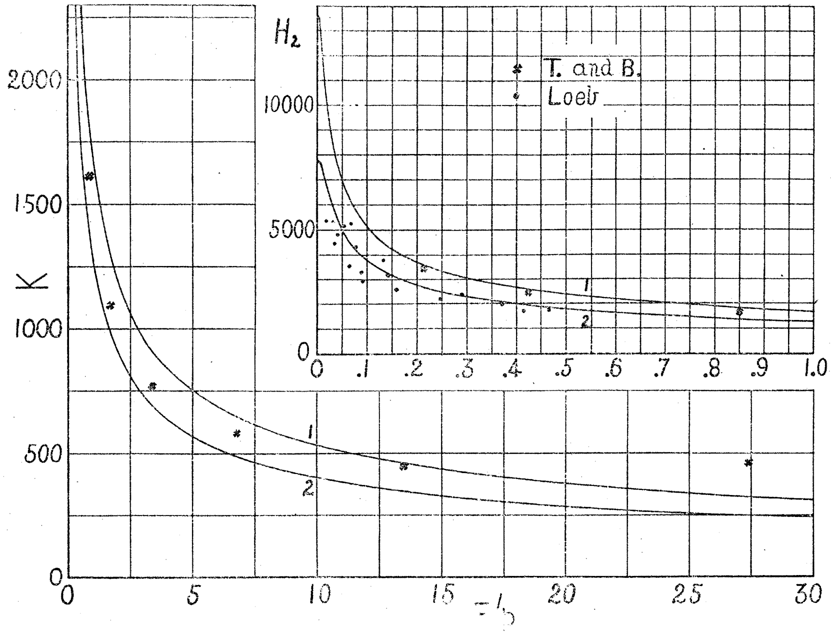

Fig. 1

There are two ways, indicated by the theory, whereby abnormally large values of K may be accounted for. (1) The electronic mean free path l_0 may be larger than the value predicted by kinetic theory, as is not unreasonable since the phenomenon of electron impact may be quite different from that of molecular impact. (2) The electron impacts may result in a greater loss of energy than that characteristic of perfect elasticity assumed in Eq. (3). It is shown in the previous paper that the effect of a *small* departure from perfect elasticity is to decrease the last term in the denominator of equation (3), whereas *large* departures from perfect elasticity lead to values of K which, in the limiting case of complete energy loss at impacts, are given by

$$K_i = 7.01(10)^4\sqrt{(l_0 p/E)}. \qquad (4)$$

Which of these two variations from the conditions assumed in Eq. (3) is responsible for the large values of K in nitrogen may be inferred by examining the shape of the theoretical curves on the two hypotheses. Increasing the value of l_0 increases K relatively more for small values of E/p than for large ones, thus leading to a steeper curve. This is illustrated by curve (4) which is calculated on the assumption of a free path l_0 four times the kinetic theory value. On the other hand, an increase in the energy loss at impacts tends to flatten the curve, increasing K relatively more for large values of E/p. This is shown by curve (2), which is calculated on the assumption that the electrons lose on the average 0.05 of their energy at a collision, instead of the fraction 0.000038 characteristic of perfect elasticity.[8] If the impacts were completely inelastic, the electrons not even retaining their thermal energy of agitation after impacts, the mobility constant would be given by curve (3) from Eq. (4). It is evident that the assumption of an abnormally long mean free path gives best agreement with experiment. To secure perfect agreement, however, it is necessary to assume that the free path varies somewhat with the speed, being longer at the smaller speeds.

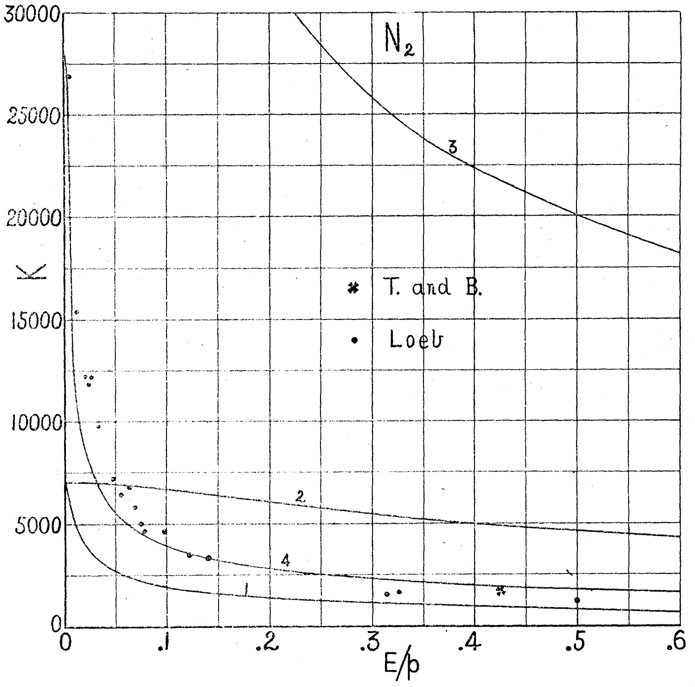

Fig. 2

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[8] Compton, loc. cit.[1]

Fig. 3 shows values of K determined by Townsend and Bailey for larger values of E/p, and curves (1) and (4) extended from

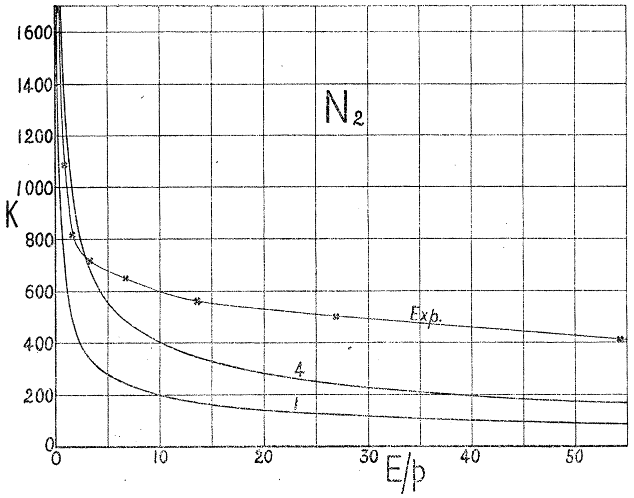

Fig. 3

Fig. 2. There is a notable increase of K above the theoretical values at large values of E/p. This increase is noted also in hydrogen and espe-

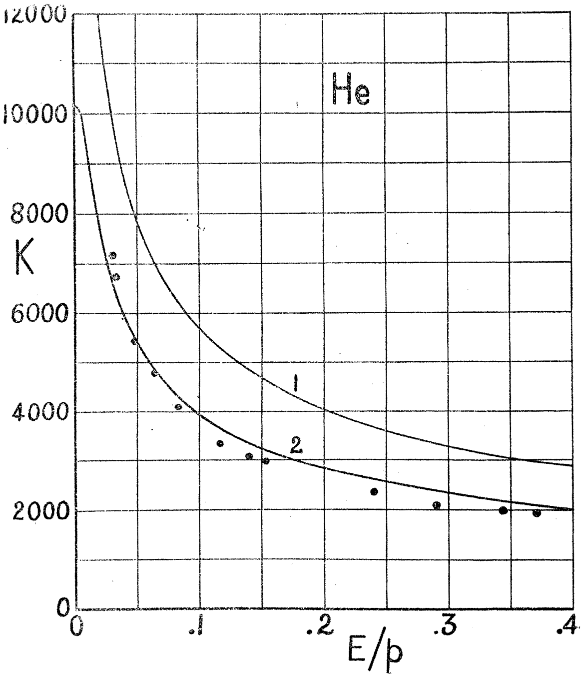

Fig. 4

cially in argon and carbon dioxide, and is evidently due to the setting in of inelastic impacts at the larger values of E/p. This is discussed later in the paper.

HELIUM

The only available data are those of a preliminary nature kindly communicated to me by Dr. Loeb. If they are in error it is probable that they are too large, since it was found that K was increased by the presence of impurities. Curve (1) represents Eq. (3) using the kinetic theory value $l_0 = 0.1223$ cm and curve (2), which fits the data well, is calculated on the assumption that $l_0 = 0.5$ of the kinetic theory value.

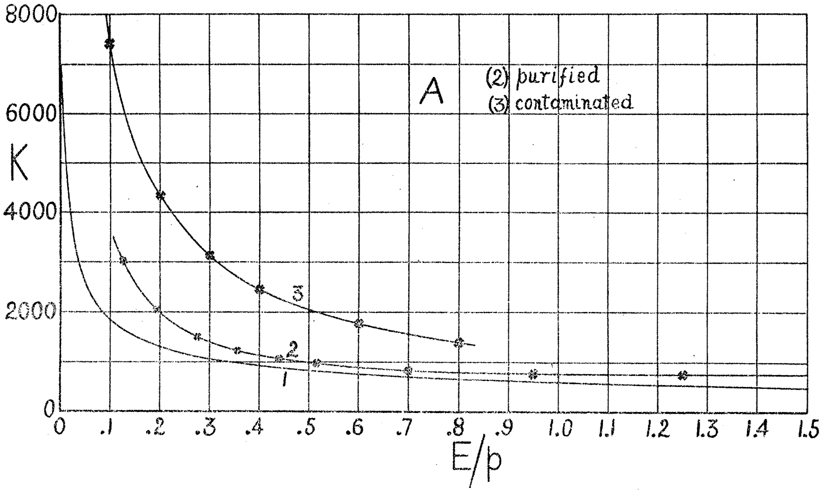

Fig. 5

ARGON

Curve (1), Figs. 5 and 6, represents the theoretical K for argon, using the kinetic theory value of $l_0 = 0.043$ cm. Curve (3) gives the early experimental results of Townsend and Bailey,[9] and curve (2) shows their later results with argon more carefully purified.[10] The indication is that in argon the mean free path is larger than the kinetic theory value. The effect of inelastic impacts beyond $E/p = 1$ is especially prominent.

OXYGEN

Eq. (3) with $l_0 = 0.0428$ cm gives curves (1), Fig. 7, whereas Townsend and Bailey's[6] results lie on curve (2). The shape of this curve suggests that the large values of K are, in this case, due to imperfect elasticity of impacts rather than to abnormally long free paths.

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[9] Townsend and Bailey, Phil. Mag. **43,** 593, 1922

[10] Townsend and Bailey, Phil. Mag. **44,** 1033, 1922

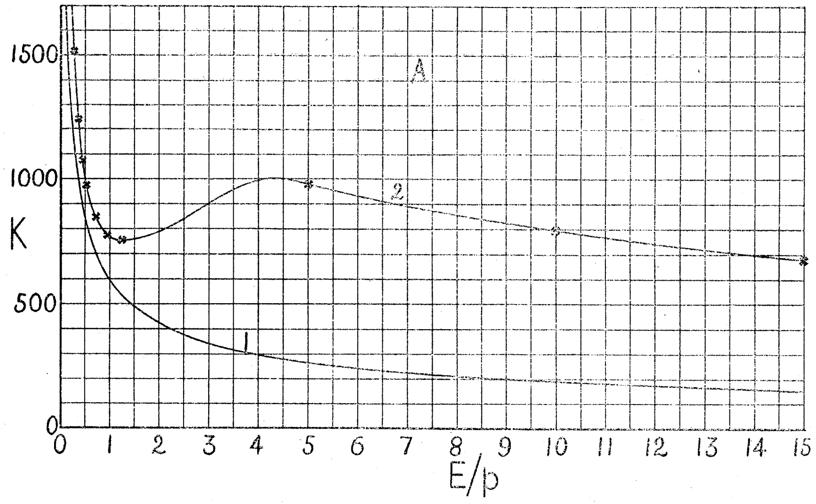

Fig. 6

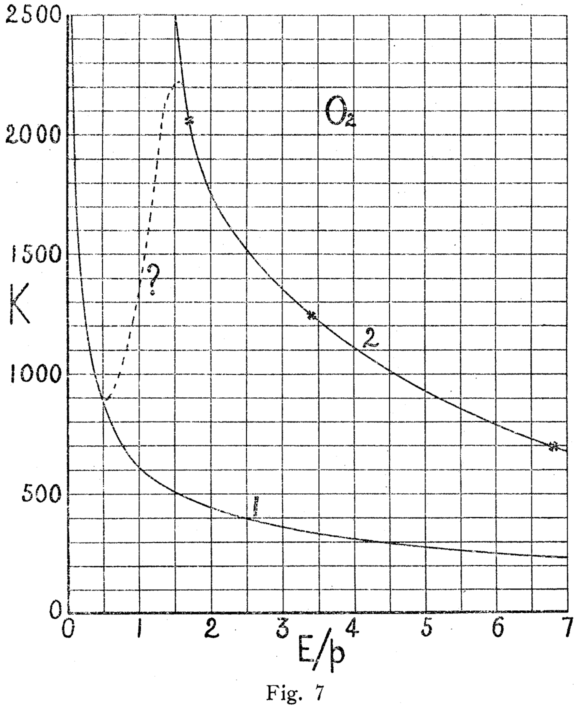

Fig. 7

Carbon Dioxide

Eq. (3), with the kinetic theory value of $l_0 = 0.0272$ cm, gives curve (1), Fig. 8, while the experimental values obtained from the work of

Skinner[11] are shown on curve (2). Here there is evidence that inelastic collisions set in at much smaller values of E/p than in the other gases except possibly oxygen. Apparently collisions are elastic up to some value of E/p below 0.3. The upper portion of the experimental curve probably corresponds to curve (2) for oxygen, suggesting that experiments in oxygen with lower values of E/p would show lower values of K characteristic of elastic impacts.

We conclude, in agreement with other lines of evidence, that electron impacts are generally less elastic in CO_2 than in the other gases, except possibly oxygen, but we must interpret this as meaning simply that inelastic impacts set in at smaller energies of impact. Rough calculations, based on methods discussed in the next section, indicate a minimum energy for inelastic impact not greater than one tenth the value in the other gases.

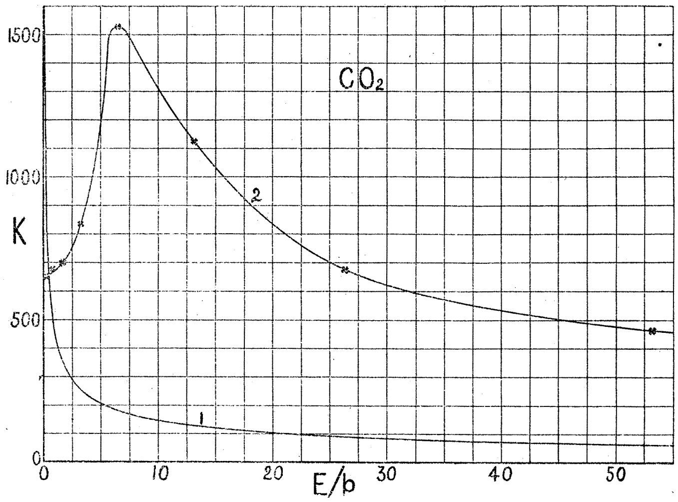

Fig. 8

Free Paths of Electrons

There is evidently some reason to accept the general correctness of Eq. (3) for the preceding gases in the range of values E/p below which there is no evidence of inelastic impacts. If we assume its correctness, we may use it to compute the values of the electronic mean free path l_0 at 1mm pressure, and the way in which this free path varies with the speed of the electrons. Solving Eq. (3) for l_0 gives

$$l_0 = 5.38K^2[1.236(10)^{-12}/K^2 + 0.159(10)^{-16}M(E/p)^2]^{\frac{1}{2}}. \tag{5}$$

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[11] Skinner, Phil. Mag. **44**, 994, 1922

Substitution of experimental values of K leads to values of l_0 given in Table I.

The value of mean energy U is calculated by Eq. (12) of the preceding paper.

TABLE I

l_0 = electronic m. f. p. at 1 mm pressure;
l_K = value of l_0 predicted by kinetic theory;
l_T = value calculated by Townsend;
U = average kinetic energy of electrons, in equivalent volts.

Data	Gas	E/p	U (volts)	l_0 (cm)	l_0/l_K	l_T (cm)
Loeb	H_2	0 to 0.5	0.0372 to 0.249	0.0452 approx.	0.577	
T. & B.	H_2	0 to 15	0.0372 to 6.90	0.0784	1.000	0.036 to 0.028
Loeb	N_2	0.05	0.109	0.296	7.30	
		0.1		0.201	4.96	
		0.2	to	0.132	3.26	
		0.3		0.099	2.44	
		0.4		0.090	2.21	
		0.5	0.907	0.082	2.02	
T. & B.	N_2	0.425	1.30	0.137	3.38	0.038
		0.850	2.15	0.114	2.82	0.033
Loeb	He	0 to 0.4	0.0372 to 0.222	0.0611 approx.	0.50	
T. & B.	A	0.0	0.0372			
(2)		0.1	0.599	0.222	5.16	0.22
		0.2	0.591	0.109	2.54	0.147
		0.3	0.683	0.084	1.97	0.110
		0.5	0.902	0.068	1.57	0.091
*		0.7	1.120	0.062	1.4	0.085
*		1.0	1.478	0.056	1.3	0.086

*Extrapolated values of K into region where K is affected by inelastic impacts

These values of l_0 do not agree very well with directly determined values recently published by Mayer[12] and by Ramsauer.[13] Their "specific absorbing powers," converted into terms of l_0, give the following average results:

Gas:	H_2	N_2	He	A(U=0.7)	A(U=1.2)
l_0	0.022	0.031	0.053	0.227	0.132
l_K:	0.078	0.041	0.122	0.043	0.043

l_K is the kinetic theory value in each case, included for comparison. It should be said that the value of l_0 will depend somewhat on the

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[12] Mayer, Ann. der Phys. **64,** 451, 1921

[13] Ramsauer, Ann. der Phys. **64,** 513, 1921

definition of a collision. Certainly Mayer's experimental method, and probably also Ramsauer's, do not measure exactly the same quantity as enters into Eq. (3) and they involve different speeds at impact. The experimental results of Akesson[14] on nitrogen agree much better with Table I, but there is some question regarding their correct interpretation.[15]

Townsend's values l_T and those calculated by Eq. (5) agree fairly well in the case of argon, and also in other cases the agreement is better than with the values of Mayer and Ramsauer. Townsend's l_T has more nearly the same significance as the l_0 used in the present paper. His values should be nearer the true values, since his method depends less on assumptions regarding the nature of impact than do those calculated by Eq. (5).

In this connection it should be recalled that we have, for simplicity, treated collisions as if between elastic spheres. This puts a definite interpretation on the meaning of l_0, and also justifies a neglect of the correction due to "persistence of velocities" at impacts. Recently H. A. Wilson[16] has shown that, if the molecule be considered as a positive core surrounded by a sperical shell of negative electricity, the correction for persistence of velocities takes an interesting form, being less than unity in a certain range and greater than unity elsewhere. Possibly this correction may account for at least part of the variation in l_0 shown in Table I. Our value of l_0 should therefore be defined as the "equivalent mean free path for elastic spheres," and will depart from the true value of l_0 for any gas to an extent depending on the degree to which this conception of an impact is in error.

EFFECT OF INELASTIC IMPACTS

An examination of the values of the kinetic energy U of the electrons in Table I shows that the increase in K observed in the figures at large values of E/p begins when E/p is large enough to give such a mean energy U that an appreciable number of electrons have energies exceeding the critical value above which inelastic impacts may occur. This critical energy is lower in carbon dioxide and probably also in oxygen than in the other gases. An approximate estimate of the effect of these inelastic impacts on the resulting mobility may be made as follows.

Out of N electrons, let F represent the fraction which have speeds greater than V_0, which is the critical speed above which impacts with

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[14] Akesson, Lunds Univ. Archs. **12,** no. 11

[15] Cf. Mayer[12]

[16] Wilson, Proc. Roy Soc. **103** A, 53, 1923

molecules may be inelastic. If this fraction is small, we may neglect the influence of inelastic collisions of this group in reducing the average speed of all the electrons, and consider the remaining electrons to have the same average speed as if no inelastic impacts occurred. We may then consider the total current $Ne\mu E$ as due to the sum of the currents carried by the two groups, NF electrons making inelastic impacts, and moving with mobility μ_i and $N(1-F)$ electrons making elastic impacts and moving with mobility μ_e. Actually, of course, these groups do not remain distinct, but statistically they may be treated separately provided F is small. Thus

$$Ne\mu E = NeEF\mu_i + NeE(1-F)\mu_e.$$

In terms of mobility constants, this is equivalent to

$$K = FK_i + (1-F)K_e. \tag{6}$$

K_i is given by Eq. (4) and K_e by Eq. (3). In most cases inelastic impacts set in at energies considerably greater than the mean energy of thermal agitation, so that for the present purpose the terms αT of Eq. (1) may be neglected and Eq. (3) used in the form

$$K_e = 7440\sqrt{\frac{l_0}{\sqrt{M}} \cdot \frac{p}{E}}. \tag{7}$$

The fraction F is obtained from Maxwell's distribution equation as

$$F = 4\pi\left(\frac{h}{\pi}\right)^{3/2}\int_{v_0}^{\infty} v^2 \epsilon^{-hv^2} dv,$$

where $h = 3/(2c^2)$ and c^2 is the mean square speed. Since v_0 is in general much larger than c, we may integrate in series, thus obtaining

$$F = \sqrt{\frac{6}{\pi}} \cdot \frac{v_0}{c} \epsilon^{-(3/2)(v_0^2/c^2)}\left(1 + \frac{c^2}{3\ v_0^2} - \frac{c^4}{9\ v_0^4} + \frac{c^6}{9\ v_0^6} - \frac{5c^8}{27\ v_0^8} + \cdots\right) \tag{8}$$

Similarly, since the energy of thermal agitation is negligible, we may express the mean terminal speed by Eq. (15) of the preceding paper, recalling that the mean speed $\bar{v} = 0.922c$, and reducing to

$$c = 2.68(10)^8\ (\sqrt{M} \cdot l_0 E/p)^{\frac{1}{2}} \text{ cm/sec.}, \tag{9}$$

where E is in volts per cm and M is on the basis of 1 for hydrogen. In terms of equivalent volts, the mean terminal energy is

$$U = 20.2\ l_0 \sqrt{M} \cdot E/p. \tag{10}$$

We now write Eq. (6) in the form

$$K = K_e + (K_i - K_e)F = \sqrt{(l_0 E/p)}[7440/M^{\frac{1}{4}} + (70100 - 7440/M^{\frac{1}{4}})F]. \tag{11}$$

We may thus calculate K for any gas by substituting the molecular weight M and the value E/p in Eq. (11), evaluating F by Eq. (8), in which the ratio $c^2/v_0^2 = U/V_0$, where U is given by Eq. (10) and V_0 is the minimum potential of inelastic impact.

The following table shows, as examples, the effect of applying Eq. (11) to hydrogen and argon, the effect of inelastic impacts being expressed by $\Delta K/K$, which is the fraction by which the mobility constant is increased as a result of inelastic impacts. The equation predicts effects due to inelastic impacts at about the right values of E/p, the discrepancies being in the nature of predicting too large an effect rather than too little. This is not surprising since Eq. (11) involves the most favorable assumptions, such as an inelastic impact at *every* collision with speed above v_0 and an impact so inelastic that not even thermal energy of agitation is retained. It is therefore to be expected that Eq. (11) should overestimate the increase of mobility due to inelastic impacts. It is quite likely that these inelastic impacts do not result in complete loss of energy, but only of an amount equal to eV_0, any excess of energy being retained as energy of agitation of the electrons. In view of the uncertainties involved it does not seem worth while to attempt a more precise treatment at present. It is sufficient to have shown that inelastic impacts are the probable cause of the increase in K at large values of E/p. Eq. (3) is applicable only at smaller values of E/p.

	Hydrogen			Argon		
E/p	0.5	1	2	0.25	0.5	1
$\Delta K/K$	.000054	.032	.73	.003	.23	2.73

CONCLUSIONS

The general results of this study may be summarized as follows. The support of the theoretical mobility equation is sufficiently good to give confidence in the approximate correctness of equations for terminal speed, number of collisions while advancing unit distance, etc., which were deduced in the preceding paper. The departures of the theory from experimental results suggest the ways in which electron impacts in various gases and at various speeds differ from impacts between elastic spheres. The mean free path employed in the theory may be looked upon as the "equivalent mean free path of elastic spheres." Its value is determined by Eq. (5) from actual values of the mobility constant K. If these "equivalent values" are used instead of the kinetic values of l_0 in the equations of the preceding paper, these equations should be quite accurate in the case of those gases in which impacts are elastic.

In this connection it is interesting to note the strong evidence that slow speed electrons make nearly or perfectly elastic impacts with the multiatomic molecules. This is in agreement with the observations of Foote and Mohler and their collaborators, who found just as strong evidence for elastic impacts in these gases as in monatomic gases. It appears that the older view that electrons make quite inelastic impacts in multiatomic gases, must be replaced by the view that impacts in most or all such gases are quite elastic provided the energy is less than that required, according to the quantum relation, to produce radiation or dissociation in the gas.

We must then look to another explanation of the different behaviors of monatomic and multiatomic gases as regards ionization and arc characteristics. Evidence is accumulating to show that this difference is due to the production of resonance radiation with resulting high concentration of excited atoms because of the strong absorption of this radiation by the gas. In multiatomic gases the result of inelastic impacts is either dissociation or the excitation of elements of band spectrum, which are not strongly absorbed by the gas and do not, therefore, tend so much to facilitate ionization of the gas.

In my conclusion, I wish to express my indebtedness to Dr. Loeb for placing data at my disposal and for suggestions and criticisms.

Palmer Physical Laboratory,
Princeton, New Jersey,
May 19, 1923.