CONTINUOUS SPECTRAL ENERGY DISTRIBUTION WITHIN THE X-RAY TUBE

By Paul Kirkpatrick

Abstract

Continuous spectral energy distribution of x-radiation as emitted from the target of a Coolidge tube at right angles to the beam of electrons, has been determined from the distribution curve obtained with a rock salt crystal spectrometer and a methyl iodide ionization chamber, by applying the following *corrections:* (1) For superposed higher orders (determined from polychromat curves and also from the absorption of the complex rays in aluminum); (2) for incompleteness of absorption in the ionization chamber (determined by absorption experiments); (3) for absorption by glass wall, by air, and by aluminum window (determined by observation or by computation); (4) for reflecting power of the rock salt crystal (obtained from curve derived from results of various investigators). The final curves show maxima at 0.445 and 0.503A for 71 and 51 kv respectively. Comparison of these curves with those given by the theories of Behnken, March, and Davis, shows agreement with none. The maxima correspond to the equation $\lambda_m = k + k'\lambda_o$, where λ_o is the quantum wave-length.

Intensities of various orders of x-ray spectra simultaneously reflected from a crystal, determined experimentally as stated above, do not correspond to Dauvillier's assumption. Families of spectra may be corrected for order effects by a graphical method based upon the linearity of isochromats.

Production of constant potential x-ray spectra with alternating voltage by use of a synchronous shutter between tube and spectrometer is described.

Introduction

ORDINARY x-ray spectrum curves (curves of ionization against wave-length) do not represent the actual energy distribution in x-ray emission, the curves being affected by superposition of orders, by absorption in the target, tube wall, ionization chamber window, air, etc., and by the wave-length variation of crystal reflectivity, ionization chamber absorption, and energy required for ionization. Some or all of these effects have been recognized by various workers and attempts have been made to apply the required corrections and so to learn the true form of the spectral energy curve at the origin of emission, a matter of importance.

Dauvillier[1] shows curves corrected for order, ionization chamber, and tube wall effects. March[2] has applied a tube wall correction to his (theo-

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[1] Dauvillier, Ann. de Physique **13**, 49, 1920

[2] March, Phys. Zeit. **22**, 429, 1921

retical) curves. Kustner[3] shows curves corrected for the calculated effects of various crystals. Wagner and Kulenkampff[4] and Kulenkampff[5] have recently published corrected spectra for low voltage ($\lambda_0 = 1.17$A).

The present paper describes the correction of spectra for from 50 to 70 kv, and the final curves set forth the wave-length energy distribution within a Coolidge x-ray tube, the measure of the energies being the amount of L, M, etc. ionization produced on complete absorption in methyl iodide vapor. The paper further presents a comparison of the corrected curves with curves resulting from three theories of continuous x-ray emission.

APPARATUS AND OPERATION

The experimental equipment consisted of a seven-inch, medium-focus, tungsten-target, Coolidge tube energized from a 2 kw transformer and 1 kw motor-generator set, the motor being operated from storage batteries at about 100 volts. The spectrometer was furnished with an accurate ten-inch circle and with microscopes reading to minutes, and carried a 41 cm ionization chamber, capable of rotation on ball bearings about the axis of the spectrometer and having a string electrometer rigidly fixed at its outer end. A rock salt crystal was used, being mounted for face reflection from the (100) planes. The x-ray beam was defined by a tube slit 18 cm from the crystal axis and an ionization chamber slit at a like distance. A third slit 4 cm from the crystal on the tube side further limited the breadth of the beam, requiring it to fall at all times within the limits of the crystal face.

Although alternating potential was applied to the tube, constant potential conditions were simulated through the introduction between the tube and the spectrometer of a *synchronous shutter* carrying a slit capable of adjustment so that only x-rays generated by a potential band of the desired width and phase should pass to the spectrometer. The shutter was of pendulum type operated by a taut steel wire attached to a pin mounted eccentrically in the end of the motor-generator shaft. The operation of the shutter at thirty oscillations per second was extremely steady. Its phase relationship to the generating potential was readily checked by observing with the spectrometer the shortest wave-length transmitted. In obtaining the spectra the phase of the shutter was so adjusted as to transmit only rays generated by the peak of the secondary voltage wave, and the width of its slit was such that the maximum possi-

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[3] Kustner, Zeit. für Phys. **7**, 2, 1921

[4] Wagner and Kulenkampff, Ann. der Phys. **68**, 369, 1922

[5] Kulenkampff, Ann. der Phys. **69**, 548, 1922

ble variation in the generating voltage, assuming a sinusoidal wave-form, was one per cent. The current through the tube was held constant throughout the spectra observations, by means of a mercury rheostat of special design[6] in the filament circuit. In securing a given spectrum curve observations were taken at wave-length intervals of about 0.1A. Care was taken to avoid the line spectra of the target. The radiations investigated were those leaving the face of the target at an angle of 45° and making an angle of 90° with the cathode stream.

Reflected intensities were measured by fixing the position of the ionization chamber and rotating the crystal slowly through the proper position for reflection, observing the total ionization occasioned. This method of measurement is fully discussed by Bragg, James, and Bosanquet.[7] Crystal rotation in steps of one minute of arc each second was accomplished by a metronome-controlled electrical device. Results of these measurements are plotted as ordinates in Fig. 1. The tube potentials for the two curves are 70.7 and 51.4 kv. The circles denote observed points with the exception of those on the axis, which were computed from the known tube voltages. The spectrometer slit widths were such that a maximum wave-length range of approximately .05A was to be expected for each angle θ. A crystal rotation of a 1.5° was found sufficient in most cases to cover the full range of reflection.

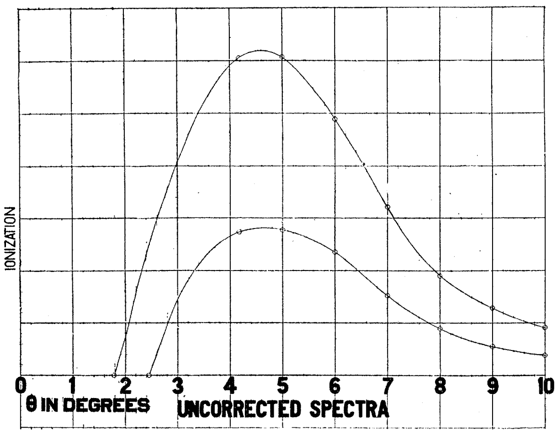

Fig. 1.

Order correction. The first correction to be applied to the observed spectra is the elimination of higher order effects. Dauvillier[1] has supposed that the intensities of the several orders simultaneously reflected

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[6] Kirkpatrick, Jour. Am. Opt. Soc. etc., Feb. 1923

[7] Bragg, James, and Bosanquet, Phil. Mag. No. 243, p. 309, 1921; No. 247, p. 1, 1921

from a crystal in a common direction are connected through the relation

$$I = \Sigma(I_n/n^3) \qquad (1)$$

I being the combined intensity of reflection and I_n the intensity which a wave-length now reflected in the order n would possess if reflected in the order one. The summation extends over all values of n which may be present in the complex reflection.

The writer has tested this assumption by two experimental methods. The first comprised a study of the type of curve known as the isochromat; a curve giving intensity of x-ray reflection at a fixed angle θ as a function of generating potential. This curve is straight at potentials only slightly greater than that necessary to elicit a reflection and continues straight until twice the starting potential is reached, when the slope of the line abruptly increases; the curve then continues straight until three times the starting potential is attained, when a further, though slight, increase in slope may be observed. Each change in slope results from the entrance of a new higher ordered wave-length into the reflection. By extrapolating the portion of the curve due solely to first order intensity, it is at once seen from the graph how the total ionization at any potential is divided between first and higher orders. A large number of curves of this type, which might more properly be called polychromats, were constructed from observed data, and spectra correction factors were computed from them. A sample curve is shown in Fig. 2. The ratio of the ordinates inclosed by the curved brackets gives the fraction of the total reflected intensity which, at 65 kv, is ascribable to first order reflection.

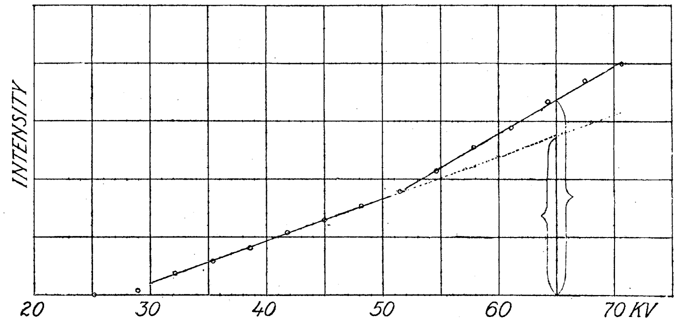

Fig. 2. Dichromat. Variation of intensity of reflection with generating potential.

The second method of investigating the strengths of different ordered reflections was the following. The reflected ray under examination, known to contain, say, two orders, was intercepted by a layer of aluminum and the diminution of intensity noted. The absorption coefficients of the two wave-lengths in aluminum were known and hence their relative strengths in the duplex beam could be computed. Given a beam made up of two wave-lengths having the respective intensities I_1 and I_2, and given K_1 and K_2 as the respective fractions transmitted by a given absorber, then we have the equation $I_1/I_2=(K_3-K_2)/K_1-K_3)$, where K_3 is the observed fraction of the *entire* beam transmitted by the same absorber. The procedure is readily extended to beams of three or more constituent wave-lengths, although beyond three the errors of observation become important, and the accuracy is apt to be low. The following table shows the agreement between this method and the preceding.

Angle θ	*Ratio of first order intensity to total intensity* By absorption	 By polychromats
3°	1.00	1.00
4°	.99	1.00
5°	.82	.825
6°	.72	.67
7°*		
8°	.45	.47
9°	.335	.30

*Discarded because of mistakes in absorption measures

By comparison with these results, Eq. (1) appears to be unreliable, ascribing too large a value to the first order in the shorter wave-length region and too small a value (frequently negative) in the longer wave-length region. Furthermore the subtractive nature of the corrections by Eq. (1) makes it particularly unlikely to give an accurate result, for the portion of the total intensity discarded by subtraction at long wave-lengths, is much greater than the portion retained, so that it is easily possible for a 5 per cent error in the estimate of the correction to result in a 50 per cent error in the remaining intensity.

The effect of removal of higher-ordered reflections from the spectra is shown in B of Fig. 3. In this figure A is the upper curve of Fig. 1. A new ordinate scale has been adopted and the abcissae have been changed to denote wave-length instead of crystal angle.

Ionization chamber correction. As a measure of the energy of the reflected rays the amount of ionization resulting from complete absorption in methyl iodide vapor has been taken, with the provision that the K absorption of iodine be excluded from consideration. The reason for this exclusion will soon appear. The ionization chamber used in this work did not actually absorb completely the radiations entering it, but it was possible to determine how far toward completeness the process of absorption went and so to adjust the spectra to the form which complete absorption would have produced. The chamber had an interior length of 41 cm, was closed at the crystal end by an aluminum window .01 cm thick, and at the other end by a plug of hard rubber. A lead slit limited the entering ray so that within the chamber its minimum distance from the chamber wall was 1.8 cm. The absorption by methyl iodide vapor under the conditions existing in the chamber was determined from spectrometer measurements upon a volume of the vapor contained in a glass cylinder 14 cm long, closed by light cork ends. The absorptions of this cylinder of vapor were determined for five different wave-lengths by the usual method for determining absorption coefficients, and a curve was plotted from the data, showing the fractions at various wave-lengths which should be absorbed by the ionization chamber of the spectrometer. For all wave-lengths dealt with the absorption by the chamber was above 80 per cent. The correction of the spectra was effected by multiplying each ordinate by the reciprocal of the corresponding fraction absorbed. The effect of this correction is seen in curves B and C of Fig. 3.

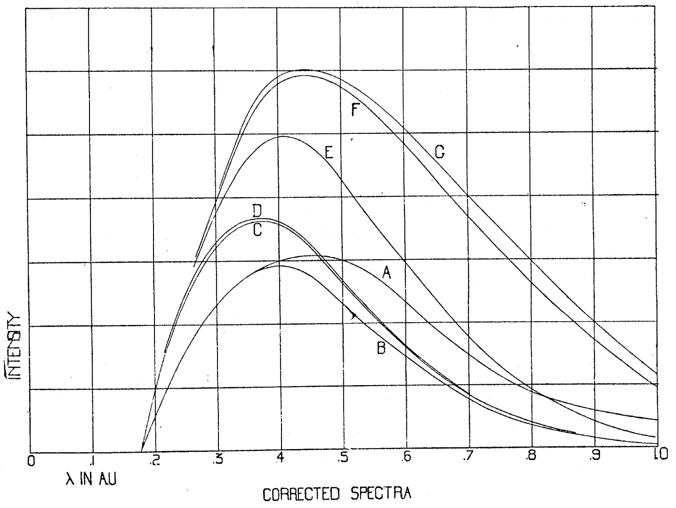

Fig. 3. A. Observed spectrum.
B. Corrected by removal of orders of reflection higher than first.
C. Further corrected for partial absorption by ionization chamber.
D. Further corrected for absorption by chamber window.
E. Further corrected for variation of reflecting power of crystal.
F. Further corrected for absorption by x-ray tube wall.
G. Further corrected for absorption by atmosphere. This curve depicts the (ionizing) energy distribution within the x-ray tube at 70.7 kv.

Considering now only wave-lengths greater than about .37 A, calculation shows that practically all the energy abstracted from the entering beam within the chamber is actually absorbed by the gas, only a minute portion, of the order of one per cent, reaching the chamber walls. Wave-lengths shorter than .37 A, however, will excite in the chamber the penetrating K-radiation of iodine, which would dissipate appreciable and unknown amounts of energy in the chamber walls. Furthermore the K discontinuity in the absorption of iodine appears at about the same wave-length, and the absorption data referred to in the preceeding paragraph become inapplicable as they pertain only to the region of L, M, etc. absorptions. For these reasons no spectral energy measurements were made in this short wave-length region, the gap between the observed points and the known quantum wave-length being interpolated.

Window correction. A computation based upon the measured absorption coefficients of aluminum permitted the spectra to be corrected for the small loss at the aluminum chamber window. In Fig. 3 a comparison of curve C and the corrected curve D shows the effect to be relatively unimportant. The correction has nevertheless been made for all the curves. It is assumed that no important amount of the energy absorbed by the window is re-emitted in any ionizing form within the chamber.

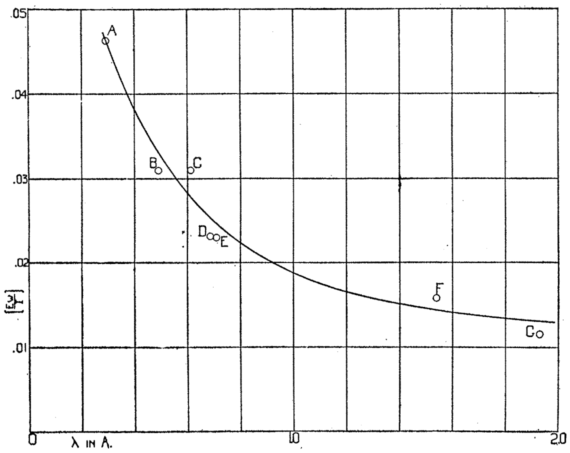

Fig. 4. Points A, B and D by Davis and Stempel;[9] C by Bragg, James, and Bosanquet;[7] E by A. H. Compton;[8] F and G by Wagner and Kulenkampff.[4]

Crystal correction. Bragg and other workers have denoted the reflecting power of crystals by the expression Ew/I where E is the total energy of a given wave-length reflected by the crystal during one sweep through its reflecting position with angular velocity ω, and I is the energy obtainable per second from the same beam before reflection. The absolute

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[8] A. H. Compton, Phys. Rev. **10**, 95, (1917)

[9] Davis and Stempel, Phys. Rev. **19**, 504 (1922). The evaluation of $(E\omega/I)$ however does not appear in this reference but was kindly furnished by Professor Davis from an unpublished manuscript.

values of the quantity $E\omega/I$ for seven particular x-ray wave-lengths have been determined by others and are shown in Fig. 4. All pertain to (100) faces of rock salt, prepared, save one, by grinding. ω is in degrees per second.

The curve drawn through these points has been employed for correcting the spectra of this paper. We desire to plot in the corrected spectra the quantity I or at least something proportional to it. From the spectrometric observations we have, or should have if the resolving power of the spectrometer were high, the quantity E. Actually, since the slits are of considerable width and the resulting spectra impure, we have instead $E\,d\lambda$.

Starting with the identity

$$I \equiv \omega \frac{E\,d\lambda}{(E\omega/I)\,d\lambda}$$

and substituting $d\lambda = 2d \cos\theta\, d\theta$, we obtain, k being a constant,

$$I = k\,\frac{(E\,d\lambda)\sec\theta}{(E\omega/I)}$$

which is the quantity desired for plotting. Hence to effect the crystal correction we multiply the ordinates of the given spectrum by the secants of the corresponding angles of reflection and divide by the corresponding coefficients of reflection from Fig. 4. Applying this correction to D of Fig. 3 gives E.

Tube wall correction. Direct measurement of the thickness of the tube wall was attempted by the use of a traveling microscope. The uncertainty in focusing upon the inner surface was considerable and the index of the glass was unknown. The attempt served to show however that the thickness was not the same at all parts of the equator, measurements varying by as much as twenty per cent. For this reason it seems important to make measurements with respect to the particular spot through which the rays to be employed emerge.

The x-ray tube used in the foregoing measurements was removed from its place before the spectrometer and a second tube substituted. The first tube was mounted between the crystal and the ionization chamber of the spectrometer so that it could be readily moved into or out of the path of the reflected ray. This ray in traversing the tube passed along a side of an imaginary equilateral triangle inscribed in the equator of the tube. The tube was turned about its geometrical axis so that one of the points of intersection of the reflected ray with the tube wall fell at that portion of the wall through which rays had been received throughout all the above described work.

The absorptions of the tube for radiations of seven different wavelengths within the range from .2 to .8A were observed. The resulting values of μx, where x is the glass thickness in the direction of the ray and μ the absorption coefficient, pertain to oblique transmission through two thicknesses of glass wall. They were adjusted so as to apply to single normal transmission through the appropriate spot of the wall by a procedure based upon further absorption measurements with respect to the other two sides of the above mentioned equilateral triangle. The process did not assume uniformity in the thickness of the tube wall. The resulting values of μx when plotted against the 2.8 power of the wave-length fell along an extremely good straight line. The intensity at any wavelength within the tube was readily deduced from these data and a knowledge of the intensities without. See Fig. 3.

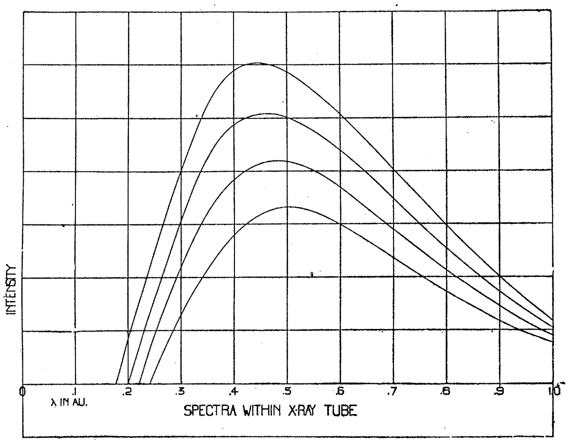

Fig. 5

Air correction. It remains to consider the effects of atmospheric air, which composed 65 cm of the total path length of the radiations. A few direct measurements of the absorption coefficients of air were attempted as well as several computations based upon knowledge of the absorptions of the constituents. In the final corrections, however, Wagner and Kulenkampff's value ($\mu = 3.1 \times 10^{-3} \lambda^{2.8}$) has been followed. The effect of the air correction appears in *G* of Fig. 3.

DISCUSSION

Curve G in Fig. 3 should show the ionizing energy distribution at the potential 70.7 kv, within the x-ray tube. Fig. 5 repeats curve G, and gives also the corresponding curves at three other potentials. The upper and lower curves of Fig. 5 are the final forms of the spectra of Fig. 1.

The two intermediate curves of Fig. 5 have no independent value as they were not obtained from separate observations. However the observed fact that the isochromats are linear, permits these intermediate spectra to be drawn in with perfect definiteness, that is, with an accuracy as great as that of the observed curves upon which they are based. The rule is that a family of spectrum curves (potential being the parameter and tube current constant throughout) cut off on any ordinate (except quite near the feet of the curves) segments proportional to the differences of their generating potentials. This holds independently of whether the curves have been corrected for absorption, ionization, or reflectivity effects, but requires that the correction for superposition of orders shall have been carried out. Where several members of a family of observed spectrum curves are given, as in the often-cited curves of Ulrey,[10] this rule furnishes an easy means of correcting for the order effects simply from inspection of the graphs. This process is fundamentally the same as the more elaborate procedure applied to the curves of this paper.

As before stated no observations were taken in the region to the left of the 0.4 A ordinate of Fig. 5, so the curvature near the feet of the curves may be slightly other than that shown.

Inspection of the maxima of the curves of Fig. 5 shows that their positions are not in agreement with either the displacement rule of Ulrey or that suggested by Dauvillier. According to the former the product of the wave-length at the maximum by the square root of the generating potential should be constant, while the latter makes the maximum wave-length proportional to the minimum or quantum wave-length. Denoting these wave-lengths by λ_m and λ_o respectively, the present results are found to conform to the relation $\lambda_m = k + k'\lambda_o$, both constants being positive.

It would be very desirable to correct these curves further for absorption of the rays by the target, and so have an expression of the distribution of energy *as emitted*. Knowledge of these effects is not complete enough to allow this. Fortunately the comparison of the present results with theory is not prevented by this limitation, since theories of the continuous spectrum contain their own assumptions about the absorptions of the emerging rays. In Fig. 6 are drawn the proposed energy distributions of three different theorists, all the curves corresponding to voltages near 51 kv and setting forth the distributions within the tube. With them appears the corresponding experimental curve of this work. The forms of these curves only, not the relative heights, are of signifi-

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[10] Ulrey, Phys. Rev. **11**, 401 (1918)

cance. The Davis curve results from a new (unpublished) theory of Bergen Davis which considers the radiation to result from the impact of the cathode electrons with the orbital electrons of the target atom. The energy transferred from a cathode electron to an orbital electron is radiated as a single quantum. Behnken[11] gives a distribution equation which includes a term expressing the absorptions of the tube wall. Neglecting this term we may write as Behnken's distribution within the tube;

$$I = \frac{1}{\lambda^{2.8}}\left[1 - e^{-3.6(\nu_0-\nu)/\nu^{2.8}}\right]$$

This equation is plotted in Fig. 6. The March curve is plotted from the equation

$$I = \lambda^{-4.5}\, e^{-4.5\lambda m/\lambda}$$

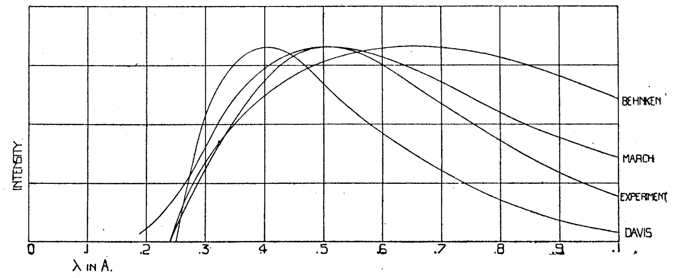

Fig. 6

The discrepancies between the theoretical and experimental curves are wide. Probably no real agreement is to be expected until more knowledge is obtained relative to the penetration of cathode electrons and the wave-length variation of the mechanical equivalent of ionization.

The writer desires in conclusion to express his thanks to Professor Elmer Dershem of the University of California for advice and help throughout this work and to record his appreciation to Professor Bergen Davis of Columbia University for his interest and for the data supplied.

Physical Laboratory,
University of California,
January 17, 1923

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[11] Behnken, Zeit. für Phys. **4**, 241, 1921