ON THE MOTIONS OF ELECTRONS IN GASES

By K. T. Compton

Abstract

Motion of electrons through gases when electric field and pressure are such that collisions are elastic.—For electron speeds below a critical value characteristic of each gas, the collisions with molecules are probably as between elastic spheres, since neither ionization nor radiation is produced. The average fraction of its *energy lost in elastic collision* with a molecule of mass M, by an electron of mass m, is $f=2(m/M)(1-\Omega/U)$, where Ω and U are the average kinetic energies of molecules and electrons. But while, if $U>\Omega$, the electrons lose energy at collisions, they gain energy from the field between collisions, so the kinetic energy tends to increase or decrease to a *terminal energy*. Assuming Langevin's equation for mobility, we find U_t (in volts) $=.66\ El/\sqrt{f}$ where El is the product of field strength and mean free path. In the case of plane parallel electrodes the number of collisions per cm and also the ratio of the speed at any point to the terminal speed, are both found to be independent of the field strength. In the case of a straight filament cathode surrounded by a coaxial cylindrical anode, the electrons may acquire their maximum speed near the cathode and subsequently lose speed while approaching the anode, their speed being greater than the terminal speed for this region. Curves are given for various cases. Substituting the value of terminal speed in Langevin's equation leads to the following expression for *electron mobility:* $\mu=0.815l\sqrt{(\frac{1}{2}e/m)}/[\frac{1}{2}\Omega+(\Omega^2/4+W^2)^{\frac{1}{2}}]^{\frac{1}{2}}$ where the energy due to the field is $eW=leE/1.506\sqrt{f}$. This expression is somewhat similar to that given by Loeb (corrected for numerical error) but leads to values 0 to 10 per cent higher. When the collisions are inelastic the mobility is shown to be greater.

To a certain degree of approximation, which varies with conditions, collisions of electrons with gas molecules may be regarded as if between elastic spheres, provided the relative velocities do not exceed a value characteristic of the particular gas. The motions of such electrons in gases may then be treated by the ordinary methods of the Kinetic Theory of Gases. For many practical purposes such a treatment is sufficiently accurate. In this paper are presented some characteristics of electron motions in a gas in an electric field, with some applications to experimental results and a discussion of the conditions in which the method is inapplicable. We shall consider principally cases in which the gas pressure is sufficiently large, in comparison with the electric field, to make the average speed gained between successive collisions small compared with the total speed.

Consider first electrons each of charge e, mass m, speed $\bar{v}$, moving with a mean free path l in a gas under the influence of an electric field E.

If the speed gained between successive collisions is small compared with the total speed $\bar{v}$, all directions of motion are statistically equally probable after an impact and the average distance advanced during each interval between collisions is

$$s=\tfrac{1}{2}a\bar{t}^2=\tfrac{1}{2}(eE/m)\ (l/\bar{v})^2,$$

where $a=eE/m$ is the acceleration and $l/\bar{v}$ is the mean free time $\bar{t}$.

The mean rate of advance of the electrons, which may be written as the mobility μ multiplied by the electric field E, is

$$\mu E=s/t=\tfrac{1}{2}eEl/m\bar{v}.$$

Hence the electron mobility is

$$\mu=\tfrac{1}{2}el/m\bar{v}.$$

This simple relation is inaccurate for several reasons. The effect of a distribution of free paths and of speeds cannot be averaged simply by use of average values of paths and speeds, and the dynamical effect of the collisions is such as to make the average initial speed after a collision different from zero. Townsend[1] gives an approximate treatment of the kinematical correction, which is more thoroughly treated by Lorentz;[2] Maxwell[3] considered the dynamical correction but not the complete kinematical one, while Langevin[4] and Lenard[5] considered both. Langevin's equation for μ may be written

$$\mu=\frac{3\sqrt{(3\pi)}}{8\sqrt{2}}\frac{el}{mc}\sqrt{\frac{m+M}{M}}, \tag{1}$$

where c is the square root of the mean square speed and M is the mass of a molecule. A critical discussion of these equations is given by Mayer.[6]

Boltzmann[7] and Langevin showed that the electrons acquire a Maxwell's distribution of speeds about their mean velocity of drift, so that $c=1.086\ \bar{v}$. Since m is very small compared with M, we may write (1)

$$\mu=0.75\ el/m\bar{v}, \tag{2}$$

in which the factor 0.75 differs from that quoted by Townsend, 0.815, because of our substitution of $\bar{v}$ for c.

Pidduck[8] has recently shown that the electrons do not acquire exactly a Maxwell distribution of speeds, and that the departure from such a distribution may increase the numerical factor by about eleven per cent.

[1] Townsend, Electricity in Gases, p. 83
[2] Lorentz, Theory of Electrons, p. 68
[3] Maxwell, Scientific Papers, **1,** 398, 1890
[4] Langevin, Ann. de Chim. et de Phys., (**8**), **5,** 245, 1905
[5] Lenard, Ann. der Phys. **60,** 329, 1920
[6] Mayer, Jahr. Radioakt. Elek. **18,** 201, 1922
[7] Boltzmann, Gastheorie, vol. 1, p. 114
[8] Pidduck, Proc. London Math. Soc. **15,** 89, 1916

In the following treatment, we shall neglect this departure and assume the correctness of equations (1) and (2).

Since μE is the average distance advanced in the direction of the electric field in a second, and $\bar{v}/l$ is the average number of collisions made during the same time,

$$s = l\mu E/\bar{v} = 0.75\ l^2 eE/m\bar{v}^2 \tag{3}$$

is the average advance between successive collisions. If we put $\bar{v}^2 = 0.849\ c^2$, as in a Maxwell's distribution, and $eU = \frac{1}{2}mc^2$, we obtain

$$s = 0.441\ l^2 E/U. \tag{4}$$

Thus the average number of collisions $1/s$ made while advancing unit distance varies inversely as l^2 and hence directly as the square of the gas pressure p.

If ν is the number of electrons per unit volume, the current density j of electrons is

$$j = \nu e\mu E = 0.75 le^2 E\nu/m\bar{v} \tag{5}$$

and the density of negative space charge ρ is

$$\rho = \nu e = jm\bar{v}/0.75\ leE. \tag{6}$$

Number of Collisions made while Advancing a Distance from the Cathode

Consider first electrons emitted with negligible speed from a plane cathode and drawn through a gas toward a plane parallel anode. The total average number of collisions per electron in distance d is

$$N = \int_0^d \frac{1}{s}\, dx = \int_0^d \frac{Ex\, dx}{0.441\ l^2 E} = 1.134\,\frac{d^2}{l^2}. \tag{7}$$

This differs by the factor 1.134 from a similar expression derived by Hertz.[9]

If the electrons are emitted from a cylinder of radius r_c, such as a wire filament, and are drawn toward a coaxial cylindrical anode, the average number of collisions made by an electron while attaining a distance r from the axis is

$$N = \int_{r_c}^{r} \frac{U}{0.441 l^2 E}\, dr = \int_{r_c}^{r} \frac{r}{0.441 l^2} \log\frac{r}{r_c}\, dr$$

since, if σ is the charge per unit length of the cathode, $E = 2\sigma/r$ and $U = \int E dr = 2\sigma \log r/r_c$. This integral gives

$$N = 1.134\frac{r^2}{l^2}\left\{\log\frac{r}{r_c} - \tfrac{1}{2} + \tfrac{1}{2}\frac{r_c^2}{r^2}\right\}. \tag{8}$$

[9] Hertz, Physics, **2**, 15, 1922

This reduces, of course, to Eq. (7) if $r = r_c + d$, where d is small compared with r_c, i.e., when the distance from the cathode is so small compared with its radius that it can be considered as a plane surface.

Eqs. (7) and (8) are inaccurate in that they assume no loss of energy by the electrons as they advance through the field. Such losses actually exist, for even if the collisions are perfectly elastic, some momentum is transferred to the molecules at each impact. At high gas pressures these energy losses, though individually very small, are in the aggregate quite large and limit the speed which the electrons attain in any given field. The consequences of these energy losses are considered in the following two sections.

Energy Losses at Collisions

If an electron collides with a molecule, initially at rest, and is thereby deflected through an angle φ with a change in speed from v to v_1, the molecule receives a velocity ω_1 in the direction θ. Assuming perfect elasticity (as for electron impacts below the critical speed for inelastic impacts) we have the relations

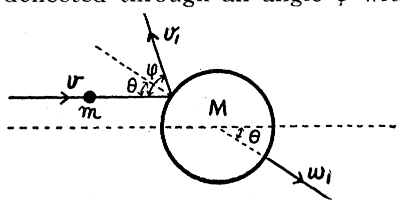

Fig. 1

$$mv + mv_1 \cos \varphi = M\omega_1 \cos \theta$$
$$mv_1 \sin \varphi = M\omega_1 \sin \theta$$
$$\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}M\omega_1^2,$$

from which

$$f_\theta = \frac{\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_1^2}{\tfrac{1}{2}mv^2} = 4\frac{Mm}{(M+m)^2}\cos^2\theta$$

gives the fraction of its energy lost by the electron at the collision. Actually electrons may strike the molecule at any angle θ between 0 and $\pi/2$, and the average fraction of the energy lost is

$$f = \frac{1}{\pi}\int_0^{\pi/2} f_\theta \, 2\pi \sin\theta \cos\theta \, d\theta = 2\frac{Mm}{(M+m)^2}.$$

Since M is much larger than m, we may put

$$f = 2m/M. \tag{9}$$

We have assumed, thus far, that the molecules are initially at rest. We may take into account the effect of initial molecular velocities by averaging the effects of two types of collisions, between electrons and molecules moving in opposite directions with average speeds c and ω and between those moving in the same direction. Out of N collisions, there are $\frac{1}{2}N(c+\omega)/c$ of the first type, resulting in an average energy loss equal to $M[(m/M)^2c^2+(m/M)c\omega]$, and $\frac{1}{2}N(c-\omega)/c$ collisions of the second type, with an average energy loss equal to $M[(m/M)^2c^2-(m/M)c\omega]$. We can therefore obtain the total loss of energy by the electrons in all N collisions, and thus the average loss per collision. When this is divided by the average energy before collision we obtain.[10]

$$f=2\left(\frac{m}{M}-\frac{\omega^2}{c^2}\right)=2\frac{m}{M}\left(1-\frac{\Omega}{U}\right). \tag{10}$$

If the speed c of the electrons is less than that which would make them in thermal equilibrium with the gas, f is negative and the electrons gain energy at encounters, on the average. If, as is the case in an electric field, their mean kinetic energy exceeds that of the gas molecules, they lose energy at collisions. Eq. (9) may be used with sufficient accuracy except under conditions, such as high gas pressures and small fields, in which the mean energy of the electrons is not much greater than that of the molecules.

Eq. (9) has been experimentally verified for helium and it is shown in a later paper that it is probably true for impacts in nitrogen and hydrogen. We may assume it to be correct for impacts of electrons with monatomic molecules at speeds below their minimum radiating speeds, and for at least some multiatomic molecules under similar conditions.

Terminal Speeds

In advancing a distance dx in a field E each electron gains energy $eEdx$, and loses an average of feU at each of the dx/s intervening collisions. The net gain of energy per electron is

$$edU=e(E-fU/s)dx.$$

Substituting for f and s from (10) and (4), we have

$$dU/dx=E-4.536\ mU(U-\Omega)/l^2ME. \tag{11}$$

The terminal speed of steady drift is found by solving this equation for U when $dU/dx=0$, giving

$$U_t=\tfrac{1}{2}\Omega+\sqrt{\frac{\Omega^2}{4}+\frac{l^2ME^2}{4.536m}} \tag{12}$$

in equivalent potential drop. In terms of average speed, since $\frac{1}{2}mc^2=eU$ and $c=1.086\bar{v}$, and the mean energy $e\Omega$ of the molecules is given in terms of absolute temperature by αT,

$$\bar{v}_t=\frac{1}{1.086\sqrt{m}}\left[\alpha T+\left(\alpha^2T^2+\frac{l^2ME^2e^2}{1.134\ m}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}. \tag{13}$$

[10] Benade and Compton, Phys. Rev. **9**, 187, 1918

In most practical cases, Ω or αT are so small compared with the other terms that they can be neglected. This is equivalent to using Eq. (9) in place of Eq. (10) for f. In such cases Eqs. (12) and (13) for terminal speed reduce to

$$U_t = \frac{lE}{2.13}\sqrt{\frac{M}{m}} \tag{14}$$

$$\bar{v}_t = 0.891\left(\frac{lEe}{m}\sqrt{\frac{M}{m}}\right)^{\frac{1}{2}}. \tag{15}$$

We may obtain an approximate idea of the magnitudes of these terminal speeds under various conditions by putting the mean free path $l = 4\sqrt{2}\cdot L$, where L is the mean free path of a gas molecule. This relation is known to be approximately correct, although there are deviations from it, which will be discussed in a later paper. From Table I the terminal energies in equivalent volts may easily be computed for various fields and gas pressures in the case of some common gases.

TABLE I

Terminal energies in various gases

l_1 = electronic free path at 1 mm pressure and 20°C.
$U_t = A\ E/p$, where E is in volts/cm and p in mm.

Gas	$\sqrt{(M/m)}$	l_1(cm)	U_t(volts)
He	85.9	.1313	5.30 E/p
A	271.5	.0461	5.86
H_2	60.8	.0842	2.40
N_2	227.4	.0435	4.65
CO_2	285.0	.0290	3.88
Hg	608.0	.0135	3.85

These values of U_t are meaningless (1) if E and p are so chosen as to make U_t comparable with the mean energy of thermal agitation $\Omega = 0.0372$ volts, in which case Eq. (12) must be used in place of Eq. (14); (2) if E and p are so chosen as to make U_t comparable with the critical energy at which electron impacts become inelastic. Between these values Table I should be approximately correct.

When in this terminal state of speed, the average number of collisions made per centimeter advance is given by Eqs. (4) and (14) as

$$\frac{1}{s} = \frac{1.065}{l}\sqrt{\frac{M}{m}}. \tag{16}$$

Mean Energy of Electrons at any Distance from Cathode

(a) *Plane parallel electrodes.* The rate of gain of energy at any point is given by Eq. (11). Putting

$$4.536\ m/l^2M = 4.536\ p^2m/l_1^2M = a^2 \tag{17}$$

and integrating from $U=0$ at the cathode $x=0$ to $U=U$ at the distance $x=x$ from the cathode, we find the mean energy at distance x to be

$$U=\tfrac{1}{2}\Omega+E\sqrt{\frac{1}{a^2}+\frac{\Omega^2}{4E}\,\frac{\epsilon^{2a^2\sqrt{(1/a^2+\Omega^2x/4E^2)}}-1}{\epsilon^{2a^2\sqrt{(1/a^2+\Omega^2x/4E^2)}}+1}}. \qquad (18)$$

If the field $E=0$, this reduces to $U=\Omega$, i.e., the energy of the electrons is simply that of thermal agitation. If the energy due to the field is so large that Ω may be neglected, Eq. (18) becomes

$$U=\frac{E}{a}\,\frac{\epsilon^{2ax}-1}{\epsilon^{2ax}+1} \qquad (19)$$

which is identical, except for a small correction in the value of a, with an equation previously derived.[11] When x is very large, U approaches the terminal value $U_t=E/a$, which is identical with Eq. (14).

The mean energies of the electrons at any distance from the cathode for various values of field and gas pressure with different gases are shown in Figs. 2 and 3. Fig. 2 gives U as a function of x for various fields and values of a, and Fig. 3 indicates the type of gas and its pressure corresponding to any value of a.

The average number of collisions made while going a distance x from the cathode is given by (4) as

$$N=\int_0^x \frac{1}{s}\,dx=\int_0^x \frac{U}{0.441\, l^2E}\,dx.$$

If we substitute for U from Eq. (18) and integrate we obtain the correct number of collisions. If we substitute from Eq. (19) we get a number which is quite accurate except in very weak fields. From Eq. (19) we find readily that

$$N=(M/4m)\,[\log\,(2+\epsilon^{-2ax}+\epsilon^{-2ax})-\log 4]. \qquad (20)$$

This equation reduces to $N=0$ if $x=0$. If x is very large N approaches x times the value of $1/s$ given by Eq. (16) for the case of terminal speed. If energy losses are negligible, which means M/m has a very large value, N takes the value given in Eq. (7).

From Eq. (19) we can readily compute the distance d in which the electrons acquire a fraction φ of their terminal energy to be

$$d=\frac{1}{2a}\log\frac{1+\varphi}{1-\varphi}, \qquad (21)$$

and the average number of collisions made while acquiring this fraction of the terminal energy is

$$N=\frac{M}{4m}\log\frac{1}{1-\varphi^2}. \qquad (22)$$

[11] Benade and Compton, loc. cit.[10]

Similarly, the average time to go a given distance or acquire a given fraction of the terminal energy may easily be computed from these equations. It is interesting that the number of collisions made in any given

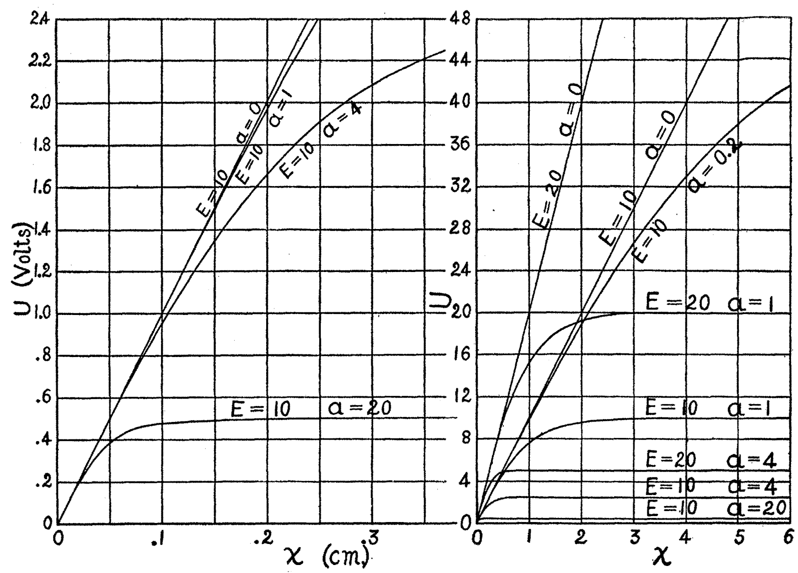

Fig. 2. Energy of electrons as a function of distance from a plane cathode for certain values of the field E and pressure factor a.

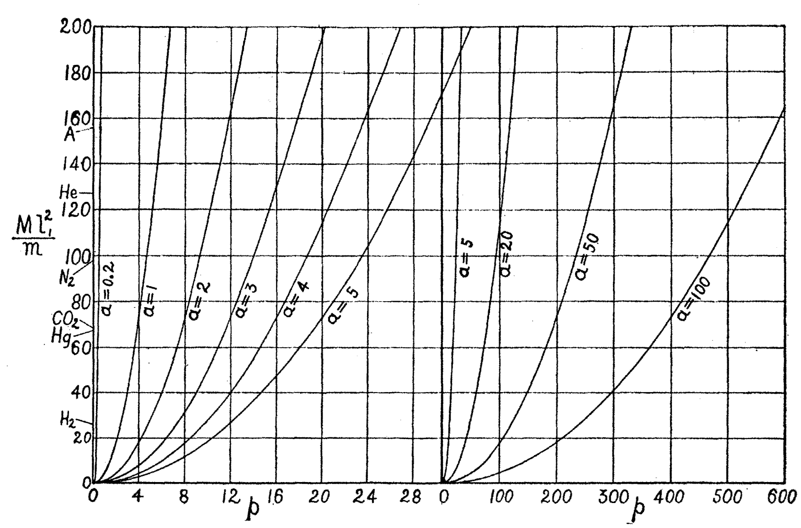

Fig. 3. Pressures corresponding to various values of a for various gases.

distance is independent of the strength of field; the distance in which a fraction φ of the terminal energy is acquired is independent of the field;

and the number of collisions made while acquiring any fraction of the terminal energy is independent of both the field strength and the mean free path.

The terminal speeds are acquired surprisingly quickly in gases at considerable pressure, as illustrated in Table II.

TABLE II

Distance in which fraction φ of terminal energy is acquired (0°C)

p	φ	Helium d (cm)	Nitrogen d (cm)	Mercury d (cm)
760	0.1	0.00064	0.00057	0.00047
	0.2	0.00131	0.00115	0.00096
	0.5	0.00356	0.00313	0.00262
	0.9	0.00953	0.00840	0.00708
	0.99	0.01710	0.01507	0.01260
10	0.1	0.049	0.043	0.036
	0.9	0.724	0.638	0.533
1	0.1	0.49	0.43	0.360
	0.9	7.24	6.38	5.33

(b) *Coaxial cylindrical electrodes.* This case is of particular interest because it corresponds to the common arrangement of a hot wire filament source of electrons situated at the axis of a surrounding cylindrical anode.

The rate of gain of energy by an electron at any point between the electrodes is given by Eq. (11), with

$$E = 2\sigma/r \text{ and } 2\sigma = (V_a - V_c)/\log\,(r_a/r_c), \tag{23}$$

where V_a, V_c and r_a, r_c are the potentials and radii of the anode and cathode, respectively. Introducing a from Eq. (17), we have

$$\frac{dU}{dr} = \frac{2\sigma}{r} + \frac{a^2\Omega U r - a^2 U^2 r}{2\sigma}. \tag{24}$$

I have been unable to find a solution of this equation, but have evaluated the integral by a laborious step-by-step process, as follows.

If r_1 and r_2 are very nearly equal, we may consider the field in the interval $r_2 - r_1$, to be constant and given by $E = \sqrt{(E_1 E_2)}$ and calculated from Eqs. (23) in terms of the applied potential difference and the dimensions of the electrodes. We may then use

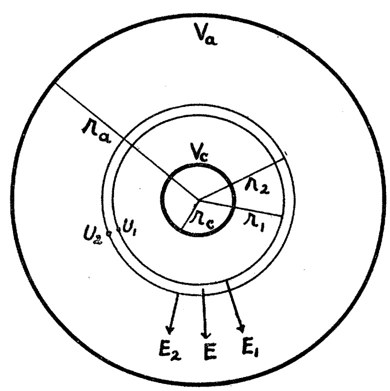

Fig. 4

$$dU/dr = (E^2 + a^2\Omega U - a^2U^2)/E$$

and integrate to find the increase in U through this range from r_1 to r_2

$$\int_{U_1}^{U_2} \frac{dU}{E^2+a^2\Omega U - a^2U^2} = \int_{r_1}^{r_2} \frac{dr}{E}.$$

I have solved the special case where the term in Ω is negligible, as is the case when electron energies are larger than about 0.05 of the minimum ionizing energy of any gas. We then find the mean energy U_2 at the end of the interval in terms of the energy U_1 at the beginning of the interval to be

$$U_2 = \frac{E(E+aU_1)\epsilon^{-2a(r_2-r_1)} - (E-aU_1)}{a(E+aU_1)\epsilon^{-2a(r_2-r_1)} + (E+aU_1)}. \tag{25}$$

This process may be carried out over each of the small intervals into which

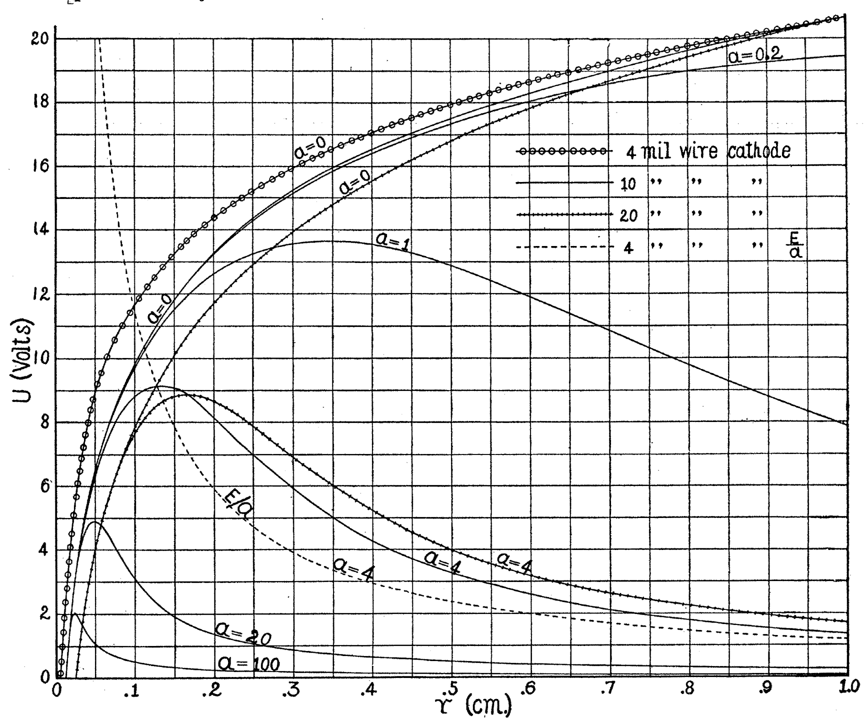

Fig. 5. Energy of electrons at various points between cylindrical electrodes.

the space between the electrodes is subdivided, and the energy at every point thus determined.

Examples of the way in which electrons gain energy between cylindrical electrodes in gases are given in Fig. 5. The gas pressure for any type of gas corresponding to each value of a may be found in Fig. 3. The curves are drawn for cases of 4, 10 and 20 mil wires ($r_c = 0.005$, 0.0125, 0.0250 cm, respectively) with an anode of 1 cm radius and an applied potential difference $V_a - V_c = 20.66$ volts. The same curves may be made to apply to other cases by altering the units according to the relations shown in Eq. (23).

It is seen that even at quite moderate gas pressures, as low as 1 mm, the aggregate loss of energy at impacts is quite large and the electrons fail to acquire the energy corresponding to the applied potential difference. At the higher pressures the electrons acquire their greatest energy quite close to the filament, and lose energy as they proceed toward the anode into the regions of less electric intensity. The maximum value of U for each curve is that which is equal to the terminal speed for the electric intensity at that particular point, as given by $U = E/a$, by Eq. (14). To the left of this maximum the electrons have less energy than the terminal value corresponding to their location, while to the right they have greater energy than the corresponding terminal value. This is illustrated by the curve of theoretical terminal energy drawn for one particular case ($a = 4$).

These facts have a bearing on the design of ionization tubes for work at low voltages. For instance, the theory of cumulative ionization[12] indicates that the effect of imprisoned resonance radiation in promoting ionization should increase with increase of gas pressure and thus an increase of gas pressure should facilitate the striking of a low voltage arc. This is true up to about 5 mm (depending on the electrode distance), but above this pressure the arc strikes with increasing difficulty and higher voltage is required. This is to be expected in view of the large energy loss by impacts at the higher pressures.

Since, at these gas pressures, the electrons have approximately a Maxwell's distribution of speeds, some may ionize even though the mean energy is below the minimum ionizing energy. The greater part of the ionization will occur in the region of the maximum value of U.

The above considerations still hold in principle, but the equations and numerical values must be altered, if the electron current density is so great as to appreciably alter the potential distribution between the electrodes. In the case of a large current from a filament, for instance, the negative space charge shifts the regions of large electric intensity toward the anode, so that curves like those in Fig. 5 would reach a lower maximum value of U and this maximum would be shifted toward the anode. The correct curves giving the mean energy at every point

[12] Compton, Phys. Rev. **20**, 283, 1922; Phil. Mag. **45**, 1923

could be obtained by the method of Eq. (25) if the actual distribution of potential between the electrodes were known.

Electron Mobilities in Gases

The electron mobility in electric fields is given by Eq. (2) provided the mean speed $\bar{v}$ is so small that there are no appreciable number of ionizing or otherwise inelastic impacts. (According to Pidduck[8] the numerical factor may be as large as 0.84.) Substitution for the average speed from Eq. (13) gives, for the mobility,

$$\mu=\frac{0.815el}{\sqrt{m}[\alpha T+(\alpha^2T^2+l^2ME^2e^2/1.134m)^{\frac{1}{2}}]^{\frac{1}{2}}}. \tag{26}$$

If the speed due to the field is negligible compared with that due to thermal agitation, this equation reduces to the familiar mobility equation[13]

$$\mu_0=0.815\ el/\sqrt{(2m\alpha T)}=0.815\ el/mc_0=0.75\ el/m\bar{v}_0. \tag{27}$$

If, on the other hand, the ratio E/p is so large that the speed of thermal agitation is negligible compared with that due to the field,

$$\mu_1=0.842\sqrt{\frac{el}{E\sqrt{(Mm)}}}=0.707\sqrt{\frac{el\sqrt{f}}{Em}}. \tag{28}$$

When E/p is small, the mobility is therefore independent of the field; when E/p is large, the mobility decreases as the field increases. The mobility is evidently greater the less elastic are the collisions, as is shown by the presence of the quantity f in the numerator of Eq. (28). In terms of f, the complete mobility Eq. (26) takes the form

$$\mu=\frac{0.815\ el}{\sqrt{m}[\alpha T+(\alpha^2T^2+1.76\ l^2E^2e^2/f)^{\frac{1}{2}}]^{\frac{1}{2}}}. \tag{29}$$

The so-called "mobility constant" K, is defined as the mobility of electrons in the gas at 760 mm pressure and 0°C, and is given by

$$K=(p/760)\ (273/T)\mu. \tag{30}$$

Loeb[14] was the first to derive a mobility equation in which the effect of both thermal agitation and the electric field were considered in estimating the mean velocity of drift to be substituted in Eq. (2). His equation, when expressed in the symbols of the present paper, is

$$\mu'=\frac{0.815\ el}{\sqrt{m}[2\alpha T+0.575\ lEe/\sqrt{f}]^{\frac{1}{2}}}. \tag{31}$$

The difference between Loeb's equation and Eq. (29) may be seen most easily by putting $\alpha T=e\Omega$ and $lEe/1.506\sqrt{f}=eW$, where $e\Omega$ is the energy of thermal agitation and eW is the energy which the electrons would have

[13] Townsend, Phil. Mag. **40**, 505, 1920
[14] Loeb, Phys. Rev. **19**, 24, 1922

acquired from the field had the thermal energy of the molecules not been contributed. In this notation Eqs. (29) and (31) take the forms

$$\mu=\frac{0.815\ el}{\surd(2me)[\frac{1}{2}\Omega+(\Omega^2/4+W^2)^{\frac{1}{2}}]^{\frac{1}{2}}},\tag{29a}$$

$$\mu'=\frac{0.815\ el}{\surd(2me)[\Omega+0.433\ W]^{\frac{1}{2}}}.\tag{31a}$$

The presence of the factor 0.433 in Loeb's equation is due (1) to his use of a value of the terminal energy which I derived in an earlier paper[15] and which is less accurate than the value used in the present paper and (2) to his use of this energy as a maximum rather than a mean value, thereby erroneously dividing it by 2.

Apart from this numerical inaccuracy, which is accidental, the two expressions differ in form. Loeb takes the resultant terminal energy of electrons to be the sum of the energies which would be due to thermal agitation and to the field if each agency contributed additively. Thus his term in brackets, after correctmg the numerical error, is $U'=\Omega+W$ whereas the correct value is

$$U=\tfrac{1}{2}\Omega+(\Omega^2/4+W^2)^{\frac{1}{2}}\tag{32}$$

If we put k equal to the ratio of the actual energy U to the energy of thermal agitation Ω, we have from (32)

$$U=k\Omega,\qquad W/\Omega=\surd[k(k-1)],\tag{33}$$

whereas on Loeb's assumption we would have

$$U'=k'\Omega,\qquad W/\Omega=k'-1,\tag{34}$$

which is the same as an equation derived as an approximation by Pidduck and frequently used by Townsend.[16]

It is found that the equations of Loeb and Pidduck are correct at both extremes, when $k=1$ and when $k=\infty$, but that there is a small error at intermediate values, reaching its maximum at $k=4/3$, when $W=(2/3)\Omega$. Here k' is 25 per cent larger than the true value k, and the mobility μ' calculated by Loeb's corrected equation is about 10 per cent less than the theoretical value μ. It is likely that some of the conclusions regarding small departures from perfect elasticity of electron impacts in gases, which Townsend has based on Eq. (34), should be reconsidered in the light of Eq. (33).

Special Case of Completely Inelastic Impacts

This case cannot be considered as a special case of elastic impacts, using Eq. (31) with $f=1$, since that equation was derived on the assump-

[15] Benade and Compton, loc. cit.[10]

[16] It should be noted that we have used the symbols W and Ω as energies, whereas Pidduck and Townsend use them as speeds, so that they appear squared in their equation corresponding to (34).

tion that the speed gained between impacts is small compared with the mean speed. In the case of perfectly inelastic impacts we assume that the electron starts from rest after each encounter. The mean speed of advance is easily found as follows.

Consider the N electrons which cross the plane $x=0$ in any long interval of time. Ndx/l of these collided previously in the layer between x and $x+dx$. Of these, the fraction $\epsilon^{-x/l}$ has arrived at the plane without making an intervening collision. Thus $(N/l)\epsilon^{-x/l}dx$ electrons arrive directly from distances between x and $x+dx$. Their kinetic energy is $\frac{1}{2}mv^2=Eex$, whence $x=mv^2/2Ee$ and $dx=(mv/Ee)dv$. Substituting above, we have

$$F(v)dv=(mv/lEe)\epsilon^{-(mv^2/2lEe)}dv \tag{35}$$

as the fraction which arrives with speeds between v and $v+dv$. The average speed is

$$\bar{v}=\int_0^\infty v\ F(v)\ dv=\sqrt{\frac{\pi}{2}\frac{lEe}{m}}. \tag{36}$$

The mobility is $\mu=\bar{v}/E$, or

$$\mu=\sqrt{\frac{\pi}{2}\frac{le}{mE}}. \tag{37}$$

It is readily seen that the mobility in these circumstances is considerably greater than it would be if the impacts were elastic. Probably impacts in all gases are elastic below critical velocities characteristic of each gas. These critical velocities are known to be much higher than the mean thermal velocity in most cases, although there are probably some gases in which electron impacts are inelastic at speeds below ordinary speeds of thermal agitation.

A test and discussion of these mobility equations will be presented shortly in another paper.

Palmer Physical Laboratory,
Princeton, New Jersey,
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