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ARE QUANTA UNIDIRECTIONAL?

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Abstract

Theory of the Brownian motion of a Planck radiator in black body radiation.—*In the case of a classical resonator* it is shown that the motion takes place even though the emission and absorption is not unidirectional provided the radiation is not strictly monochromatic. This theory is then extended to *the case of a quantized atom.* Interference between the waves emitted by the atom and the surrounding radiation provides for the recoil actions without the assumption of unidirectional quanta. Statistically these have the same effect as Einstein's unidirectional quanta. This theory removes the contradiction postulated by Einstein between the Rubinowicz-Sommerfeld considerations and the requirements of classical Brownian motion. It leaves the wave-theory of light intact and is in agreement with Bohr's principle of correspondence.

Introduction

EXPERIMENTS on the development of photographic plates, on the photoelectric effect as well as the theoretical considerations of Einstein and Schroedinger[1] support the belief that quanta are darts of light, i.e. that the electromagnetic momentum of a quantum is in the same direction for all of its energy. Neither of these is a case of conclusive evidence. The photographic and photoelectric experiments indicate only the places where energy is absorbed and it appears that the only considerations dealing with the momentum of the quantum are the theoretical ones just cited. The fundamental point is contained in the paper of Einstein. This is verbally quoted as a proof of the unidirectional nature of quanta. In the paper there appears to be no statement to this effect though it shows that if it be granted that quanta are darts of energy

[1] A. Einstein, Phys. Zeits. **18**, 121, 1917
E. Schroedinger, Phys. Zeits. **23**, 301, 1922
L. Silberstein and A. H. Trivelli, Phil. Mag. **44**, 956, 1922
L. Silberstein, Phil. Mag. July 1922, p. 257
Svedberg and Andersson, Phot. Journ. August 1921, p. 325
Svedberg, Phot. Journ. 1922, pp. 183-186

the conditions of correct Brownian movement are satisfied. The legitimacy of the converse proposition is not evident—nor does the writer think it is true—though at first sight it is difficult to see in what way the conditions will be satisfied unless the hypothesis of light darts is made.

In this note an alternative to Einstein's treatment is presented which makes it unnecessary to assume the unidirectional nature of quanta.

Similar Problem in Classical Theory

This problem was first discussed by Einstein and Hopf[2] who showed that the requirement of correct Brownian movement of a classical resonator necessarily leads to the Rayleigh-Jeans formula. The discussion given below is not essentially different from that of Einstein and Hopf. It calls attention however to some features of the problem which are of importance in connection with the quantum case.

Let us first see how classical electrodynamics explains the Brownian movement of a Planck resonator in a black body radiation.

For simplicity let the resonator have an axis the orientation of which is fixed, and let its motion of translation be confined to that in a straight line which we choose as the axis of X (OX) and which we take as the resonator's axis. We shall suppose the resonator attached to a particle of large mass M so that the translational velocity of the resonator is always small compared with that of light. The calculation will be similar to that used by Einstein.

We consider the beginning and the end of a fixed time interval τ and we change the beginning and the end at random keeping τ constant. We take the mean square of the momentum for the beginning and for the end and we express the fact that the mean square is the same for both. If the velocity at the beginning of the interval is v, if the systematic resisting force is Rv and if the summation of the accidental random impulses is Δ

$$\overline{(Mv-Rv\tau+\Delta)^2}=\overline{(Mv)^2}$$

and

$$\overline{\Delta^2}/\tau=2RkT \tag{1}$$

The origin of the resisting force Rv lies in the fact that there is a relative motion between the resonator and the radiation (the velocity v). The force Rv is of the same nature as radiation pressure and will have essentially the same expression in Einstein's theory and this.

The origin of Δ is not quite so simple. It lies in the spectral width of the band absorbed by the resonator because the absorption of one frequency

[2] Einstein and Hopf, Ann. der Phys. **33,** 1105, 1910. I am indebted to Professor Sommerfeld for this reference.

by a resonator put in the well known cube of Jeans' does not give any recoil action on account of the stationary character of the waves.

The resonator responds however not to a single frequency but to a band. Consider stationary waves of frequencies ν_1, ν_2. The electromagnetic momentum of the sum depends on $[E_1H_1]+[E_2H_2]+[E_1H_2]+[E_2H_1]$ where E, H are the electric and magnetic intensities corresponding to ν_1 and ν_2. The first two terms of the above expression correspond to the waves acting separately and give no flux during a time large in comparison with the period. The last two terms represent the interference effect of the waves. They give terms of the form sin $(\omega_2-\omega_1)t$ where $\omega=2\pi\nu$ and thus correspond to long period oscillations in the electromagnetic momentum.

Calculation of Δ. For this we avail ourselves of a result of Planck.[3] Planck is concerned with an oscillator having a charge e, a mass m, exposed to radiation of energy density $6T_\nu d\nu$ in the frequency range $(\nu, \nu+d\nu)$. The energy of the oscillator itself is denoted by ϵ and the amount of energy absorbed by the oscillator from the field is n_e. Planck considers n_e^2 for a time τ and takes its average value. The result is

$$\overline{\eta_e^2}=\frac{e^2}{2m}T_\nu\epsilon\tau \tag{2}$$

Now the energy absorbed by an oscillator is the work done on it by the external field. In the time between two instants t_1, t_2 this work is

$$\eta_e=\int_{t_1}^{t_2} eE_z\dot{\xi}dt \tag{3}$$

where E_z is the component of the external electric field taken along the axis of the resonator and $\dot{\xi}$ is the velocity of the oscillator's electrified particle having the charge e.

The force on the oscillator due to the action of the magnetic field on its moving electrified particle is perpendicular to the resonator's axis and gives rise to the radiation pressure. The value of this force resolved along the line of motion of the resonator is $(eH_y/c)\dot{\xi}$ [OY being an axis perpendicular to OX and OZ] and using (2) and (3)

$$\overline{\Delta^2}=\frac{e^2}{2mc^2}T_\nu\epsilon\tau \tag{4}$$

since statistically the behaviour of E and H is the same (for justification see Note 1).

Calculation of R. For this we use formula (260) of Planck[3]

$$\overline{\eta_e}=\tfrac{1}{4}(e^2/m)T_\nu\tau \tag{5}$$

[3] M. Planck, Wärmestrahlung, 4th edition p. 153, formula (261)

This gives the average absorbed energy in the time τ. It may be shown that the radiation pressure is such as corresponds to the absorption of an electromagnetic momentum which is v'/c^2 times the absorbed energy where v' (see Note 2) is $v[3-(\nu/\rho)(\partial\rho/\partial\nu)]$. In the case of the Rayleigh-Jeans radiation formula $v'=v$ and by (5)

$$Rv=(e^2T_\nu/4mc^2)v \tag{6}$$

Using (4) and (6) it is seen that (1) is satisfied if

$$\epsilon=kT \tag{7}$$

which is true in the classical theory.

If the axis of the resonator forms an angle φ with the line of motion the terms $\overline{\Delta^2}/\tau$, R are found to be $\sin^2\varphi$ times the values found above. Equation (1) is therefore again satisfied in virtue of (7).

Resumê of treatment of classical case. The above derivation shows how a classical resonator reacting with the radiation by the ordinary laws of electromagnetism is maintained in a state of Brownian movement with the correct mean kinetic energy per degree of freedom $(kT/2)$. The derivation is based on the similarity between the expressions for the energy absorbed and the radiation pressure. It also implies a statistical equivalence of E_2 and H_y in the calculation of Δ.

The above picture does not involve the hypothesis of unidirectional emission though it is true that this hypothesis also satisfies (1). It is clear that unless some additional fact is given the mere validity of (1) will not enable one to decide between these two possibilities and we are thus inclined to doubt the possibility of such a decision in quantum theory.

Discussion of Quantum Case

Very little is known about the mechanism of absorption and emission of radiation. The discussion of the physical reality is of necessity only tentative.

It will be satisfying to make the basic phenomenon of the quantum and the classical treatment the same. Interference between the field of the resonator and the surrounding field may be thought of as fundamental in the classical theory and will be presented below as the basis of a quantum theory of Brownian movement. By so doing the apparent contradiction between the view of Rubinowicz and Sommerfeld, that the wave emitted by a Bohr atom may be spherical, and Einstein's postulate of correct Brownian movement, is removed because the spherical wave hypothesis may be conceived as satisfying Einstein's postulate in a manner analogous to that of the classical resonator.

For simplicity let us deal with an atom emitting linearly polarized light, (e.g. a quantized Planck resonator). It is feasible to suppose that the recoil action on the atom due to its emission or absorption is the same as if the same energy changes took place from a classical resonator.

We can now calculate the terms Δ and R. For the latter the calculation of Einstein can be interpreted to mean that $v' = v[3-(\nu/\rho)(\partial\rho/\partial\nu)]$ (see Note 2). Applying this to Einstein's model and preserving the relation $c^2\overline{\Delta^2} = \overline{\eta_e^2}$ (see (2), (4)) it is found that both $\overline{\Delta^2}/\tau$ and R can be obtained from his values by multiplying them by the factor 3. The relation (1) is therefore still satisfied.

The fundamental postulates of this theory are then:

(1) *The systematic resisting force Rv has the same ratio to the absorbed energy as if the quantized atom were a classical resonator moving in the actual radiation field,* here determined by Planck's formula.

(2) *The relation $c^2\overline{\Delta^2} = \overline{\eta_e^2}$ is true not only in the classical but also in the quantum theory.*

In the above an atom emitting linearly polarized light, say a quantized Planck resonator was discussed. By interchanging electric and magnetic singularities the result is extended to circularly polarized light.

The writer has spoken about this subject to several people whose ideas influenced his own. He first erroneously believed that one could think consistently of a quantized atom not exchanging momentum with the radiation. In a conversation Prof. Einstein was kind enough to point out that a classical resonator experiences radiation pressure and that it would be therefore inconsistent not to introduce an equivalent of this in the quantum theory. Thus it appears that the classical analogy has been kept in mind by Einstein but that he preferred to build the theory along revolutionary lines rather than to modify the classical theory to the least extent. The subject was also discussed with Prof. Ehrenfest of Leyden, Prof. Ames of Johns Hopkins, Prof. Epstein of Pasadena, Dr. Kemble and Dr. Van Vleck of Harvard.

Note 1. Statistical Equivalence of E_z and H_y in the Calculation of $\overline{\Delta^2}/\tau$

Since E_z affects $\dot{\xi}$ and H_y does not, a justification is needed for the interchange of these in $\int E_z\dot{\xi}dt$ in the derivation of $\overline{\Delta^2}/\tau$.

The classical theory of black body radiation may be developed by considering the resonator as damped by its radiation resistance. To within a good approximation this damping may be taken into account by a frictional term in its equation of motion[4] which we write

$$\ddot{\xi}+\kappa\dot{\xi}+\omega_0^2\xi=(e/m)E_z$$

$\kappa\dot{\xi}$ is here the frictional term. Let

$$\frac{e}{m}E_z=\int_0^\infty A(\omega)\cos(\omega t-\varphi_\omega)d\omega.$$

Then

$$\dot{\xi}=\int_0^\infty\frac{A(\omega)\cos\{(\omega t-\varphi_\omega-\tan^{-1}[(\omega^2-\omega_0^2)/\kappa\omega)]\}}{\sqrt{\kappa^2+[(\omega^2-\omega_0^2)/\omega]^2}}d\omega$$

We are interested in the average square of the integral from t_1 to t_2 of $(e/m)E_z\dot{\xi}dt$. Replacing integrations as to ω by summations, multiplying the expressions and integrating with respect to the time, we obtain terms of the types $A(\omega_1)^2$ and $A(\omega_1)A(\omega_2)$. The latter type is subject to double summation as to ω_1 and ω_2 and it contains a set of terms having in the denominator $\omega_1-\omega_2$. When the result is squared these terms give the main contribution to the average if $\tau=t_2-t_1$ is a short time in comparison with that necessary for the development of E_z into the Fourier series. These terms give

$$\sum{}'\frac{A(\omega_1)^2\,A(\omega_2)^2}{4}\;\frac{\cos^2\psi\sin^2(\omega_1-\omega_2)\tau+4\sin^2\psi\sin^4\frac{1}{2}(\omega_1-\omega_2)\tau}{(\omega_1-\omega_2)^2\,\{\kappa^2+[(\omega_1^2-\omega_0^2/\omega_1)]^2\}}$$

where $\psi=-\varphi(\omega_1)-\tan^{-1}[(\omega_1^2-\omega_0^2)/\omega_1]+\varphi(\omega_2)$ and the summation is taken over all values of ω_1, ω_2 excluding $\omega_1=\omega_2$. It is clear now that if $A(\omega_2)$ be replaced by $B(\omega_2)$ and if $\overline{A(\omega_2)^2}=\overline{B(\omega_2)^2}$ the result of the summation is unchanged on account of the random nature of $\varphi(\omega_2)$ $\varphi(\omega_1)$. This and a random change in the φ's is all that is required in order to replace E_z by H_y. Thus the validity of the interchange is justified. Taking the average and replacing the sum by an integral the average value becomes $(\pi^2/8)(A^2/\delta\omega)^2(\tau/\kappa)$ where $\delta\omega$ is the difference between two successive ω's in the summation. This is readily seen to be identical with the expression of Planck.

The occurrence of the terms $\omega_1-\omega_2$ shows the importance of the interference between adjacent frequencies for $(\overline{\Delta^2}/\tau)$.

NOTE 2. THE CALCULATION OF THE RESISTING FORCE ON A CLASSICAL RESONATOR MOVING IN BLACK BODY RADIATION

Let E_z', H_y' be the components of the electric intensity along the resonator's axis and of the magnetic intensity perpendicular to the

[4] Kretschmann, Ann. der Phys. **65**, p. 310, 1921

resonator's axis and to the axis of motion—both measured in a frame of reference moving with the resonator. The time integral of the radiation pressure is the time integral from t_1 to t_2 of $(e/c)H_y'\dot{\xi}$, where $\dot{\xi}$ is the velocity of the oscillatory particle. The absorbed energy is the corresponding integral of $e^2E_z'\dot{\xi}$. If the average over a long interval of time is desired the various frequency terms can be considered independently. The ratio of two terms of corresponding frequencies in both expressions is $(\int_{t_1}^{t_2}(1/c)\, H_y'(\omega)\cdot E_z'(\omega)dt)/(\int_{t_1}^{t_2} E_z'(\omega)dt)$ where the sign (ω) after a symbol indicates that only the term in the particular frequency discussed is taken and where it is supposed that $H_y'(\omega)$, $E_z'(\omega)$ are in phase. [The field is analyzed into a number of plane waves of different frequencies.] This ratio is the ratio of three times the density of electromagnetic momentum, resolved in the negative direction of OX, to that of the electromagnetic energy. Since resonance is sharp only a narrow band of frequencies need be considered. For this the ratio is the same throughout.

In calculating the ratio we may consider a number of independent plane waves within the band sweeping by the resonator. No interference action between these waves must be considered because their frequencies are affected by Doppler's principle to a different extent on account of the difference in direction. Thus we may use Eq. (19) of Einstein[1] which gives for the energy in the frequency range $d\nu$ per unit solid angle in a direction making an angle φ with the line of motion

$$\frac{1}{4\pi}\left(\rho + \frac{v\nu}{c}\cdot\cos\varphi\cdot\frac{\partial\rho}{\partial\nu}\right)\left(1 - \frac{3v}{c}\cos\varphi\right)d\nu$$

where $\rho d\nu$ is the energy density with respect to the container. Multiplying this by $-\cos\varphi/c$, integrating, and dividing by $\frac{1}{3}\rho d\nu'$ the ratio of radiation pressure to energy absorbed per unit time is

$$(v/c^2)\,[3-(\nu/\rho)(\partial\rho/\rho\nu)] = v'/c^2$$

where $v' = v[3-(\nu/\rho)(\partial\rho/\partial\nu)]$. In the classical theory ρ is of the form $a\nu^2$ and $v' = v$.

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