THE MEASUREMENT OF MAGNETIC FIELDS OF MEDIUM STRENGTH BY MEANS OF A MAGNETRON

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ABSTRACT

Use of magnetron to measure magnetic fields of strength 0.1 to 500 gauss.— The magnetron is a special symmetrical type of kenotron, with a straight axial filament and cylindrical anode. When this is placed in a magnetic field parallel to its axis, the radial paths of the electrons are curved into arcs of circles, and for fields stronger than the critical for the given voltage, do not reach the anode at all; or, for a given field, as the voltage is decreased, the current suddenly begins to decrease rapidly at a critical voltage. *Two methods* are described. (1) The magnetron is connected in series with a high resistance and a source of voltage, and in parallel with a standard voltmeter. Then for a field H at the center of the magnetron, parallel to its axis, there is a definite reading V of the voltmeter, given by the equation $H=6.72\ V^{\frac{1}{2}}/R$, when R is the radius of anode. This method may be used for 20 to 500 gauss. (2) For weak fields, $H<20$ gauss, the magnetron is connected in series with a milliammeter, (which may be the voltmeter used in (1) with its resistance short-circuited) and a source of voltage, and surrounded by a close-fitting solenoid. The solenoid is connected, through tungsten filament lamps, to the same source of voltage as the anodes, and the lamp resistance adjusted so that the magnetic field of the solenoid is the "critical" field of the magnetron. The combination of magnetron and solenoid is then calibrated, and the calibration is found to be practically independent of voltage fluctuations. The accuracy of the first method is $\frac{1}{4}$ per cent with calibration or for relative values over a small range without calibration, using commercial power supply and portable meters. The accuracy of the second method is about 1 per cent under good conditions.

INTRODUCTION

ALL methods in use at the present time for measuring magnetic fields of weak or medium intensity have two practical limitations: (1) They require a delicate, non-portable current measuring instrument; (2) they measure the effect of a *change* of field, not the field itself. Hence, it is necessary either to vary the field that is to be measured, as by annulling or reversing, or to move the measuring coil with respect to the field. In the majority of cases the field is due in part to magnetism in iron, and cannot be varied in a known manner, so that the movement of the coil is the only practical method.

The method here described is free from these limitations. The measurements can be made with a standard portable voltmeter, without motion or variation of field, and with an accuracy fully equal to that of the galvanometer methods.

The Magnetron

All electron tubes, especially those of the high vacuum type, are susceptible to the influence of magnetic field. Any such tube could therefore be used, with proper calibration, for the measurement of magnetic field intensity. In general, however, the effect of the field depends on the construction and orientation of the tube in a way that is not easily specified, so that careful precautions would be necessary in operation. These precautions are unnecessary when a symmetrical tube of the type here described is used. Even calibration may be dispensed with, except when great accuracy is desired. In addition, it becomes possible to extend the measurements to fields much weaker than can be detected with ordinary tubes.

The magnetron has been fully described elsewhere.[1] It is a special form of two-electrode kenotron, whose cathode is a straight filament, and whose anode is a circular cylinder concentric with the cathode (Fig. 1). The effect of a magnetic field parallel to the cathode is as fol-

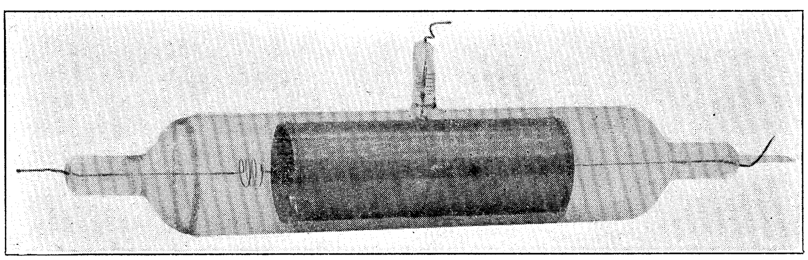

Fig. 1. Photograph of typical magnetron.

lows: (a) In the absence of magnetic field the electrons travel radially in straight lines, from cathode to anode (Fig. 2a). (b) A weak magnetic field parallel to the cathode curves their paths slightly, in planes normal to the cathode (Fig. 2b). (c) If the voltage between cathode and anode is kept constant the curvature of paths will increase with increasing magnetic field, and at a certain field strength the electrons will strike the anode almost tangentially (Fig. 2c). They will still all reach the

[1] Hull, Phys. Rev. **18**, 31-57, 1921; J.A.I.E.E. Sept., 1921, pp. 715-723. My attention has recently been called to the fact that H. Greinacher (Verh. D. Phys. Ges. **14**, 856, 1912) had previously suggested the use of such a tube for measuring e/m, and worked out part of the mathematical theory here given. His failure to obtain experimentally the predicted results was due to the combination of poor vacuum and insufficient electron emission.

anode, however, so that no change in current will be observed if the emission from the cathode is limited by its temperature, and only a slight decrease, due to the greater length of path of the electrons and correspondingly increased space charge, when the current is limited by voltage (space charge limitation). (d) A further increase in magnetic field causes

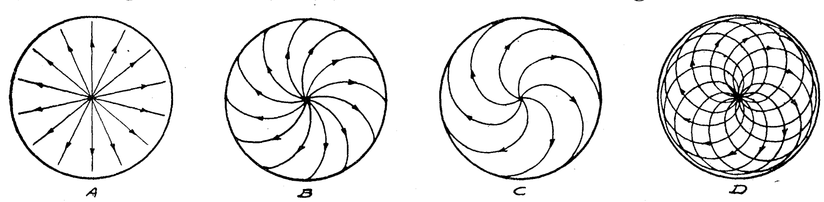

Fig. 2. Cross-section view of magnetron, showing effect of a magnetic field parallel to the axis on the paths of the electrons.

the electrons to miss the cylinder entirely and return to the cathode (Fig. 2d). The current to the anode falls abruptly to zero, in the case of perfect vacuum and perfect symmetry. Under practical conditions of vacuum and symmetry, an increase of about 10 per cent in magnetic field is required to reduce the current from its maximum value to essentially zero, and the reduction is not quite to zero, but to a small residual value corresponding to the number of electrons that are deflected from

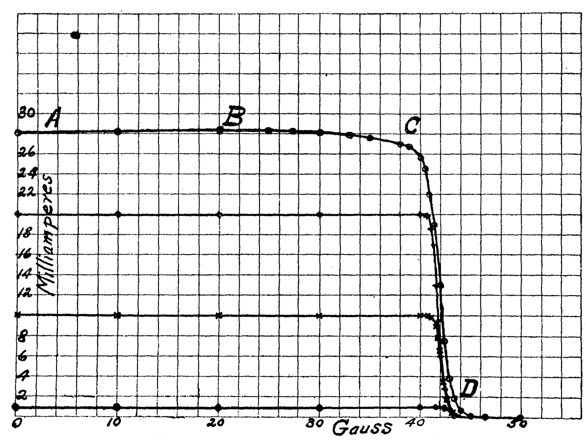

Fig. 3. Effect of magnetic field on current through magnetron. In the upper curve the current is limited by space-charge, in the other three by filament temp.

their paths by collision with gas molecules or unsymmetrical parts of the tube, and hence can be kept from the anode only by a magnetic field much stronger than the "critical" value. This residual current is of the order of a tenth of a milliampere at 250 volts, but may be as high as 50 milliamperes in good vacuum at 10000 volts. In using a magnetron to measure magnetic field strength it is obviously necessary to use currents larger than this residual value.

An example of the electrical characteristics just described is shown in Fig. 3. The stages represented in Fig. 2 are designated by the corresponding letters A, B, C and D. In the upper curve the current is limited by space charge. In the lower ones it is limited by filament temperature. The portion c—d becomes more nearly vertical the higher the voltage and the more perfect the symmetry.

The critical value of magnetic field which is just sufficient to prevent electrons from reaching the cylinder is given[2] by the equation

$$H_0=\sqrt{(8m/e)V}/R=6.72V^{\frac{1}{2}}/R \qquad (1)$$

where R = radius of cylinder in cm;

V = potential difference between cathode and cylinder in volts;

H_0 = magnetic lines per cm^2 (gauss) parallel to axis of tube.

It is evident from Eq. (1) that for a given magnetron, H_0 is a function of voltage only.

Circuit and Operation

Method 1. Medium fields. For measuring fields of medium strength the simplest method of operation is that shown in Fig. 4. The cathode of the magnetron is connected to the negative terminal of a generator or other source of voltage E_0, and the anode is connected through a high resistance r to the positive terminal. The same source of voltage may be used to furnish heating current for the cathode, through a resistance r_1, as shown in Fig. 4, or a separate source of either direct or alternating

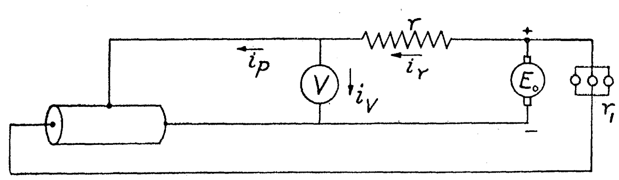

Fig. 4. Circuit for measuring magnetic fields between 20 and 500 gauss by means of magnetron.

current may be used. The temperature of the cathode is adjusted once for all at a value high enough so that under operating conditions the electron emission will never be limited by temperature. This adjustment is easily made in practice by gradually increasing the current through the filament until further increase has no effect on the voltmeter reading, and then adding about 10 per cent, e.g. by decreasing r_1 by 10 per cent, to allow for voltage fluctuation.

[2] Hull, Phys. Rev. **18**, 34, 1921.

A standard portable D. C. voltmeter V, preferably of low current type, is connected in multiple with the magnetron, and measures the potential between cathode and anode during operation.

The operation consists in placing the magnetron in the field to be measured, with its axis parallel to the direction of the field, if this is known, and reading the voltmeter. If the direction of the magnetic field is not known, the magnetron must be rotated until the voltmeter reading is a maximum. The direction of the field will then be given by the direction of the axis of the magnetron, and its magnitude by the voltmeter reading, by means of Eq. (1).

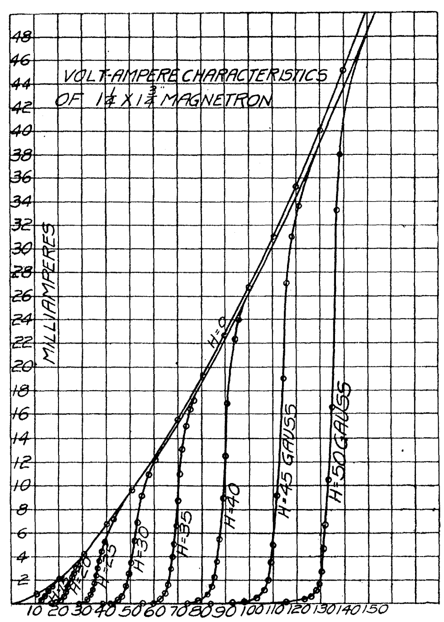

Fig. 5. Volt-ampere characteristics of magnetron for a series of values of magnetic field.

This relation is a theoretical one, and has been found to be reliable within the limit of accuracy of construction of the magnetron. The constant of the instrument, $6.72/R$, depends only on the radius R of the anode, assumed to be of circular section and concentric with the cathode. With ordinary care in construction, using the value of R measured either before or after assembly, this constant may be relied on within 5 per cent. The $V^{\frac{1}{2}}$ relation is exceedingly accurate over a narrow working range; so that for relative measurements, such as testing the uniformity of a field, the limit of accuracy is that of reading the voltmeter. An ordinary voltmeter may easily be read to $\frac{1}{2}$ per cent, making an accuracy of $\frac{1}{4}$

per cent in H; and with a precision voltmeter and constant voltage an accuracy of 1/20 per cent is attainable.

For a wide range of values of H there is a small systematic deviation from the $V^{\frac{1}{2}}$ relation when a fixed value of resistance r is used. This is due to the finite slope of the volt-ampere curves, as shown in Figs. 5 and 8. It may be avoided by varying r, as explained below, or may be allowed for by calibration.

The operation of the circuit shown in Fig. 4 is most easily described by reference to the volt-ampere diagrams shown in Figs. 5 to 8.

Fig. 5 shows a family of volt-ampere characteristics, for a series of values of magnetic field, of a magnetron with anode $1\frac{1}{4}$ inches in diameter and $1\frac{3}{4}$ inches long. The upper curve, marked $H=0$, gives the current when there is no magnetic field. This is the maximum current that can be obtained at the corresponding voltage. Its value is limited only by the space charge of the electrons, and is given accurately (except for end corrections) by the equation[3]

$$i(\text{amp.}) = 14.7 \times 10^{-6}(L/R)E^{3/2} \text{ (volts)}$$

where L and R are the length and radius of the cylinder. The family of curves marked 15, 20, 25 etc. represent the observed volt-ampere characteristics with constant magnetic fields of 15, 20, 25, etc., gauss respectively. It will be observed in each case that for voltages below a critical value the current is zero, and above the critical value it is practically the same as when there is no magnetic field. The critical voltages are less abrupt at low than at high voltage, because of the voltage drop along the filament. They are proportional to the square roots of the corresponding magnetic field strengths (cf. Eq. 1).

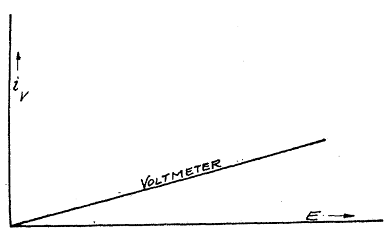

Fig. 6. Volt-ampere characteristic of voltmeter in Fig. 4.

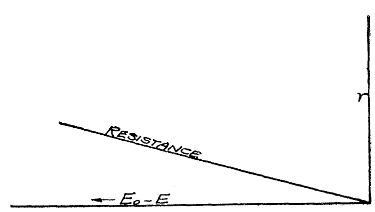

Fig. 7. Volt-ampere characteristic of series resistance in Fig. 4.

Figs. 6 and 7 show the volt-ampere characteristics of the voltmeter and the resistance r respectively. The abscissas in Fig. 7 are plotted to the left, in order that the graph may be inserted without change in Fig. 8.

Fig. 8 gives the complete volt-ampere representation of the circuit of Fig. 4. The family of curves marked "magnetron+voltmeter" are ob-

[3] Langmuir, Phys. Rev. **2**, 450, Dec. 1913.

tained by adding the ordinates of Figs. 5 and 6, and represent the volt-ampere characteristics of the multiple arc combination of magnetron and voltmeter. The curve marked "resistance" is the same as Fig. 7 with its origin placed at the point $E = E_0$ (E_0 = generator voltage). The actual current and voltage for any magnetic field H is given by the intersection of the "resistance" curve with the "magnetron+voltmeter" curve for that value of H. This follows from the facts that the current i_r through the resistance r is equal to the sum of the currents i_p and i_V through the magnetron and voltmeter (Fig. 4), and the total voltage E_0 is the sum of the voltages across "magnetron+voltmeter" and "resistance."

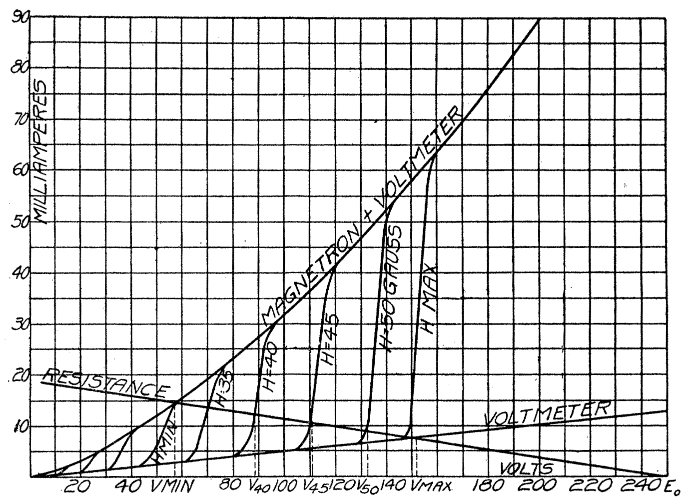

Fig. 8. Combined volt-ampere characteristic of circuit shown in Fig. 4. The voltmeter reading is given by the intersection of the curve marked "resistance" with that one of the "magnetron+voltmeter" curves which corresponds to the actual magnetic field.

It is evident from Fig. 8 that the voltages V_{40}, V_{45}, V_{50} etc. read by the voltmeter are the "critical" voltages for the corresponding magnetic fields,[4] and may therefore be used as a measure of the magnetic field, according to Eq. (1). This applies, of course, only to the range of magnetic field between H_{min} and H_{max}. For fields weaker than H_{min} the voltmeter reading is determined by space charge, and is independent of magnetic field, while for fields stronger than H_{max} the voltmeter reading is determined by the resistances of the voltmeter and series resistance r,

[4] The error due to the slope of the "critical" portions of these curves is discussed below. This error may be avoided by adjusting the resistance for each reading so that it intersects the vertical portion of the volt ampere curve at its middle point.

and is entirely independent of the magnetron. In practice it is easy to find these limits by observing once for all (for a given r and E_0) the voltmeter readings below which and above which the magnetic field has no effect, or by observing, in the absence of magnetic field, the values of voltmeter reading with magnetron filament lighted (lower limit, V_{min}) and not lighted (upper limit, V_{max}) respectively.

The range H_{min} to H_{max} may be adjusted to include any desired values of H, within wide limits, by changing the resistance r or the voltage E_0, as is evident from Fig. 8. The weakest field that can be measured is determined principally by the voltage drop in the filament, which makes the critical voltage indefinite at low anode voltages. For ordinary tubes this limit is about 20 gauss. The strongest field that can be measured is determined by the ability of the tube to operate on high voltage without evolution of gas. With ordinary tubes this upper limit is about 200 gauss, but a well evacuated tube of small anode diameter may measure as high as 500 gauss. It is to be noted that the magnetic field increases as the square root of the voltage, so that a 10 to 1 range of magnetic field requires a 100 to 1 range of voltage.

When the magnetron is operated as shown in Figs. 4 and 8 with fixed values of r and E_0, there is a small systematic variation from Eq. (1), due to the fact that the resistance curve does not cut the different magnetron curves at corresponding points, but near the top of the steep portion for weak fields, and near the bottom for strong fields (See Fig. 8). This error may be corrected by changing r or E_0 so as to make the tube current i_p increase uniformly with increasing V; or so that V is always in the middle of the range between V_{min} and V_{max}. The magnitude of this error and the degree of correction may be seen from Tables I, II and III.

The values in Table I are those obtained with constant voltage supply E_0 and constant series resistance r, using ordinary shop voltage for both

TABLE I

E_0 and r constant; $E_0=825$ volts; $r=3600$ ohms

V (volts)	i_p (milliamp.)	H (gauss)	H (calc.) (gauss)
149	20.5	84	87
198	19.2	98	100
255		112	114
304	16.5	123	125
354		134	135
405	13.6	136	144
453		158	152
501	11.	176	160

V and H. From 85 to 150 volts, which is the operating range, there is a slow systematic deviation of the experimental values from about 2 per cent below the true value to 2 per cent above. The tube used in this test had an anode $\frac{3}{4}$ inch in diameter by $1\frac{1}{4}$ inches long.

Table II gives the values obtained with the same tube when the generator voltage is varied so that i_p is proportional to V. The values of $H=(6.72V^{\frac{1}{2}}/R)$ given by the magnetron agree with those of the calibrating coil (H, column 3) within the limit of constancy of the calibrating field, which was operated from shop voltages.

TABLE II

E_0 varied to make $i_p \propto V$

V (*volts*)	i_p (*milliamp.*)	H (*gauss*)	H (*calc.*) (*gauss*
25	1.25	38	36
50	2.5	51	51
100	5	72	72
200	10	101	101
400	20	143	143
600	30	176	175

Table III shows the degree of reliability of the readings obtained when an ordinary 250 volt shop supply is used for plate voltage, and the series resistance r is adjusted for each reading so that the reading is approximately in the middle of the "range" ($V_{min}-V_{max}$). The range was determined, for each value of r, by observing the voltmeter reading with filament on and off respectively, with $H=0$. In this test the calibrating field was furnished by a long single layer solenoid 4 inches in diameter by 4 feet long, operated from storage batteries. The magnetron anode had

TABLE III

r varied to keep V in middle of "range"

V_{min} (*volts*)	V_{max} (*volts*)	V (*volts*)	H (*calibrating coil*) (*gauss*)	H (*magnetron*) (*gauss*)
11.0	18.0	14.0	15.7	15.8
13.3	21.2	16.5	16.9	17.1
14.5	24.0	19.7	18.83	18.75
16.3	27.8	22.0	19.75	19.81
18.5	32.2	25.3	21.25	21.26
21.3	38.6	29.4	22.80	22.90
25.5	47.5	34.6	24.40	24.81
32.0	61.0	45.7	28.43	28.6
44.5	89.0	65.5	33.95	34.2
58.0	114.	84.4	38.77	38.8
73	140	102.4	42.7	42.8
89	160	115.5	45.4	45.4
96	165	129.7	48.2	48.2
100	170	135.6	49.4	49.2

a diameter of $1\frac{1}{4}$ inches and a length of $1\frac{3}{4}$ inches. The voltmeter was a standard G. E. type DP_2 voltmeter, reading 150 volts full scale. Voltage was measured from the negative end of the filament. The series resistance r varied from 200,000 ohms for the lowest reading to 4000 ohms for the highest. It is seen that the values of H given by the voltmeter readings are very accurate over the whole range, the maximum error being only about 1 per cent.

This last method of operating the magnetron is very convenient. In measuring an unknown field the procedure is as follows: The magnetron is placed in the field, with its axis parallel to the estimated direction of the field, with filament lighted and maximum series resistance. The voltmeter should read about 10 volts.

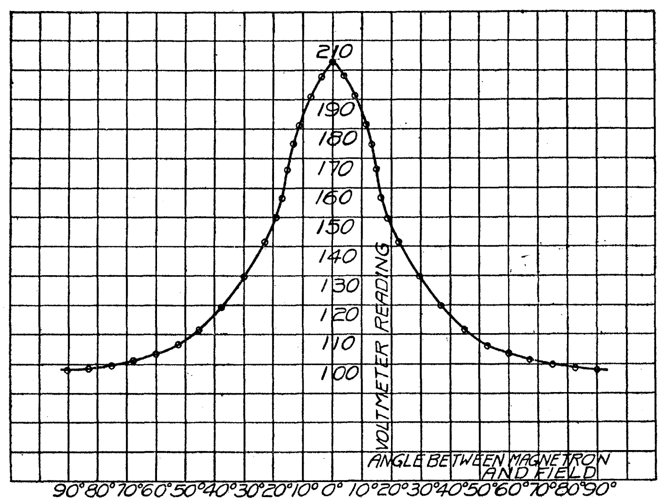

Fig. 9. Effect of the direction of magnetic field upon the reading of the voltmeter in method 1, using circuit shown in Fig. 4.

As the resistance is gradually decreased, the voltmeter reading at first increases, then remains nearly constant over a certain range of resistance change, and then increases again. The range of voltage over which the voltmeter reading is approximately constant is the working range. The resistance is set at the approximate middle of this range, the tube is rotated about axes at right angles to its length until the voltmeter reading is a maximum, and this maximum reading is taken as a measure of the magnetic field. As a check the tube may be removed from the field (or the field annulled) and the resistance adjusted so that the voltmeter readings with filament lighted and not lighted respectively lie at equal distances below and above the observed reading in the magnetic field. The tube should then be again placed in the field, rotated until the voltmeter reading is a maximum, and the final accurate reading observed.

The sensitiveness of the magnetron to direction of the resultant magnetic field may be seen from Fig. 9, which shows the effect of rotating the magnetron in a uniform magnetic field. The circuit is that represented in Fig. 4, with $E_0=250$ volts, $r=5400$ ohms, and a constant value of $H=102$ gauss. The ordinates are voltmeter readings of a 300 volt type DP_2 voltmeter in multiple with a $\frac{3}{4}$ inch diameter by $1\frac{1}{2}$ inch long magnetron. The abscissas are the angles between the axis of the magnetron and the magnetic field. It is seen that the voltmeter reading is a maximum when the axis of the magnetron is parallel to the field, and falls off rapidly as the axis is rotated, so that the direction of the field can easily be determined within one or two degrees. It will be noted that the decrease in voltmeter reading with angle is more rapid than the cosine of the angle at small angles and less rapid at large angles, i.e. the magnetron does not measure the *component* of magnetic field parallel to the axis, but only the *resultant* field when parallelism has been established.

Method 2. Weak fields. For measuring weak fields it is desirable to supply separately the ineffective part of the magnetic field (H_0, Fig. 3), by means of a constant current in a solenoid wound around the magnetron. The variation of plate current with magnetic field is then that corresponding to the steep portion ($c-d$, Fig. 3) of the $I-H$ curve, and it is possible to measure fields as weak as 1/10 gauss with an ordinary milliammeter, with an accuracy of about 1 per cent.

Since the polarizing field H_0 is large compared with the fields to be measured, it must be very constant if the *zero reading* is to be steady, and the anode voltage must likewise be constant. This is not difficult with a good storage battery for the filament, and either storage battery or dry cells for the anode. But there is a much simpler method of maintaining a steady zero, based on the fact that the equation of the zero point, which is the middle of the steep portion of the $(I-H)$ curve, is by Eq. (1)

$$H_0=k\ V^{\frac{1}{2}}$$

Hence, if the field H_0 is furnished by a current i_0, which is derived from the anode generator voltage V by means of a device that makes i_0 proportional to $V^{\frac{1}{2}}$, then $H_0 \propto i_0 \propto V^{\frac{1}{2}}$, which is the required relation; and the zero reading will be independent of fluctuations of V.

A satisfactory device for producing a current through the polarizing coil proportional to $V^{\frac{1}{2}}$ is a standard gas filled tungsten lamp connected in series with the coil. It happens that in these lamps the variation of resistance with temperature and the cooling effect of the gas cooperate in such a way that the current through the lamp is accurately proportional to the square root of the voltage at its terminals, over a wide range of voltage. Fig. 10 A shows a logarithmic plot of the volt-ampere characteristic, between 50 and 130 volts, of a 125 volt, 75 watt Mazda C (gas-

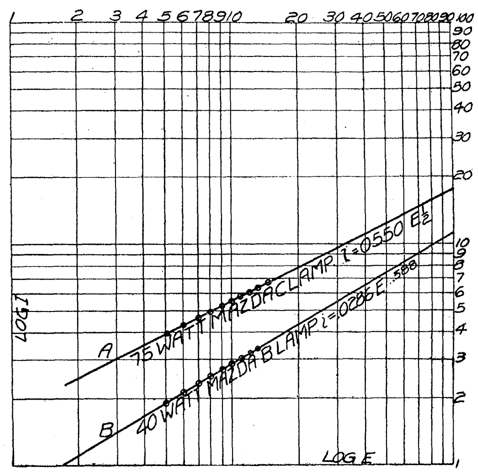

Fig. 10. A and B. Logarithmic plots of volt-ampere characteristic of Mazda C and Mazda B lamps, respectively.

filled) lamp. The points lie on a straight line whose slope is exactly $\frac{1}{2}$, representing the equation $i = .0550\ V^{\frac{1}{2}}$. Fig. 10 B shows a similar characteristic for a Mazda B lamp (vacuum type). The points again lie on a straight line, but its slope is .59, representing the equation $i = .0286\ V^{0.59}$. Hence the gas-filled lamp is a more perfect "regulator" for the

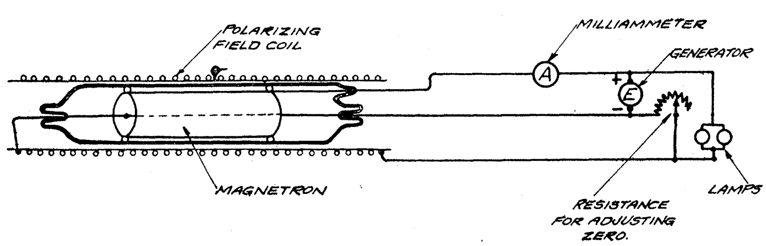

Fig. 11. Circuit for measuring weak magnetic fields with a magnetron.

current i_0 through the solenoid than the vacuum lamp. It is obvious that in order to utilize this regulating property of the tungsten lamp, the resistance of the field coil must be small compared with that of the lamp.

Fig. 11 shows the circuit for measuring weak fields, utilizing this resistance characteristic of a tungsten lamp in series with the polarizing solenoid. For convenience, the solenoid is wound with such size of wire that it can be operated in series with the filament, furnishing the required heating current. The value of the current is fixed, once for all, to give the correct filament temperature, by the use of one or more standard 110 volt lamps in multiple. The number of turns of the solenoid is then adjusted so that this current produces approximately the required polarizing field (H_0, Fig. 3); and the final adjustment to exactly the right value is made by shunting the solenoid with a small variable resistance. The solenoid should have a length several times its diameter, so that the field inside the magnetron may be very nearly uniform.

A milliammeter, reading from 10 to 25 milliamperes at full scale, is connected in series with the anode. This milliammeter may conveniently by the same meter that is used as voltmeter in method 1, fitted with a "no resistance" binding post.

Curve I, Fig. 12, shows the calibration of a magnetron with anode $1\frac{1}{4}$ inches in diameter and $1\frac{3}{4}$ inches long, operated as shown in Fig. 11, from a 132 volt shop supply. The sensitiveness is approximately 15

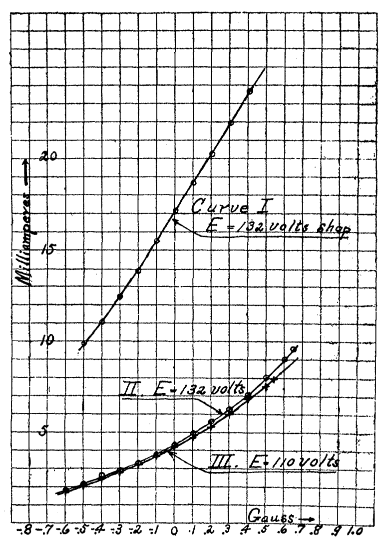

Fig. 12. Calibration curves of magnetron magnetometer for weak fields.

milliamperes per gauss. Curves II and III, Fig. 12, show two calibrations, of a magnetron, also $1\frac{1}{4}$ by $1\frac{3}{4}$ inches, at 110 and 132 volts respectively, with no change in the circuit constants. It is to be noted that the increase of voltage from 110 to 132 volts has increased the plate current uniformly by just 4 per cent over the whole range of magnetic field. This example is given to show the degree to which this method of operation is independent of fluctuations in supply voltage. The voltage fluctuations which occur on standard supply lines are of the order of 5 volts or less, and it is seen that fluctuations of this magnitude would cause a variation of only 1 per cent in plate current.

Extension of range. The sensitiveness of method 2 may be reduced to any desired extent by inserting resistance in series with the anode. In this way the range may be adjusted to include any desired field from 1/100 gauss to 100 gauss. A convenient method of accomplishing this

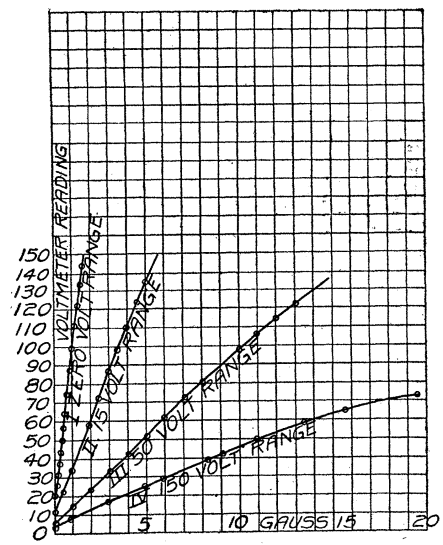

Fig. 13. Calibration curves of magnetron magnetometer, using circuit shown in Fig. 11, with different resistances in series with milliammeter to increase the range.

adjustment is to use as milliammeter a multiple scale voltmeter, which may be the same voltmeter used in method 1. Fig. 13 shows a series of calibrations taken with a standard type DP-2 voltmeter fitted with 4 scales, reading respectively 1/100, 15, 50, and 150 volts at full scale. It is seen that they cover satisfactorily the whole range from 1/100 to 20 gauss. In order to extend the range to 100 gauss with the same anode voltage, a magnetron with smaller anode diameter viz. $\frac{1}{4}$ inch, should be used. This would decrease the sensitiveness for weak fields. Conversely, the sensitiveness can be increased by the use of larger anode diameter.

Research Laboratory,
General Electric Co.,
Schenectady, New York,
March 9, 1923.

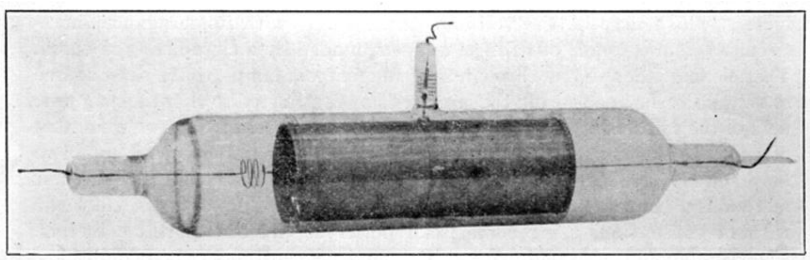

Fig. 1. Photograph of typical magnetron.