

THE SCATTERING OF LOW SPEED ELECTRONS BY PLATINUM AND MAGNESIUM

BY C. DAVISSON AND C. H. KUNSMAN

ABSTRACT

Scattering of electrons by platinum and magnesium.—Electrons from a tungsten filament were accelerated with voltages up to 1000, and a restricted beam was directed against a platinum target at 45° incidence. The number of scattered electrons with over 9/10 the energy of the primaries was determined for various directions by measuring the current to a Faraday box collector which could be revolved so as to explore the range from within 15° of the primary beam ($\Psi = 15^\circ$) to the plane of the target ($\Psi = 135^\circ$). For bombarding potentials up to 200 volts the intensity of scattering decreases more or less regularly from low to high values of Ψ , indicating a maximum for $\Psi = 0$. At higher potentials the scattering pattern develops two well marked lobes besides the one for $\Psi = 0$. When the platinum target was covered with a deposit of magnesium the patterns found are simple with the exception of a small lobe on the curves for potentials less than 150 volts. Up to 500 volts the maximum intensity is apparently in the direction $\Psi = 0$, but for higher voltages the direction shifts to around $\Psi = 90^\circ$. *Theoretical interpretation.* Since low speed electrons are more easily deflected than α -particles, their scattering patterns depend not only on the field immediately about the nucleus but also upon its nature in regions beyond various or all of the structural electrons. The complicated patterns for platinum are taken to indicate that the field out from the nucleus in the platinum atom is characterized by several approximate discontinuities resulting from more or less definite concentrations (shells) of electrons at certain distances from the center. The magnesium atom having a single strong concentration of electrons (the L group) gives rise to simple patterns.

Arrangement of electrons in the magnesium atom.—The simple patterns for Mg agree in the main features with theoretical patterns calculated for the scattering of electrons by a positive nucleus of limited field (Davisson, Phys. Rev. **21**, 637, 1923). On the basis of this theory the results indicate a concentration of 7 or 8 electrons in a shell with a radius of about 1.5×10^{-9} cm. The theory has not been sufficiently developed to permit an analysis of the patterns for Pt.

I. INTRODUCTION

WHEN a stream of electrons is directed against a metal target the area under bombardment becomes temporarily a source of electron emission. The results of numerous investigations¹⁻¹² have established that a part at least of this emission is made up of electrons, originally in the target, that have been expelled in some way by the incident or primary electrons. This is certain from the fact that ordinarily there can be found a range of bombarding potentials over which the number of electrons pro-

ceeding from the target exceeds the number incident upon it. Whether or not the whole of the emission is made up of such secondary electrons is a question that in the nature of the case has been less definitely answered. Baeyer,¹ Gehrts² and Campbell³ observed that for bombarding potentials up to about 12 volts the emission is comprised in part of electrons whose speeds are not appreciably less than that of the primaries, and inferred that these were primary electrons that had been turned back from the target. At higher potentials, however, this class of electrons was not detected. As the bombarding potential was increased beyond 12 volts the maximum speed of the emitted electrons was found to fall below that of the primaries and finally to reach a limiting value which was independent of further increase in the speed of the incident particles. In observations extending to 3000 volts this particular result has been obtained repeatedly. The limiting speed has been fixed by different observers as corresponding to potential differences of from 5 volts to 40 volts. Quite recently, however, experiments have been described by Farnsworth¹² in which electrons of all speeds up to that of the primaries were observed proceeding from a nickel target for bombarding potentials up to 110 volts. Results similar to those of Farnsworth were also obtained several years ago in this laboratory by W. Wilson and J. R. Weeks for a number of metals and for bombarding potentials up to 250 volts. Unfortunately the results of these measurements have not been published.

Experiments are described in this paper from which it is evident that electrons having speeds not appreciably less than that of the primaries proceed from platinum and magnesium for all bombarding potentials up to the highest investigated in each case, 1000 volts and 1500 volts, respectively. In preliminary notes¹³ we have also described similar results obtained in experiments with targets of nickel and aluminium.

These relative high speed electrons make up only a small fraction of the total number leaving the target, particularly at high bombarding

¹ Baeyer, *Verh. Deut. Phys. Ges.* 96 and 953, 1908; *Phys. Zeit.* 10, 176, 1909

² Gehrts, *Ann. der Phys.* 36, 995, 1911

³ Campbell, *Phil. Mag.* 25, 803, 1913; 28, 286, 1914; 29, 369, 1915

⁴ Horton and Davies, *Proc. Roy. Soc. A* 95, 408, 1919

⁵ Hull, *Phys. Rev.* 7, 1, 1916

⁶ Dadourian, *Phys. Rev.* 14, 434, 1919

⁷ Millikan, *Proc. Nat. Acad. Sci.* 7, 13, 1921

⁸ Barber, *Phys. Rev.* 17, 323, 1921

⁹ Tate, *Phys. Rev.* 17, 395, 1921

¹⁰ McAllister, *Phys. Rev.* 19, 247, 1922

¹¹ Baltruschat and Starke, *Phys. Zeit.* 23, 403, 1922

¹² Farnsworth, *Phys. Rev.* 20, 358, 1922

¹³ Davison and Kunsman, *Science*, 64, 522, Nov. 25, 1921; *Phys. Rev.* 19, 253 and 534, 1922.

potentials. In a tube exhausted to 10^{-7} mm of mercury they can be observed without difficulty, but at higher pressures their detection becomes uncertain or impossible on account of the masking effect of positive ions. It can hardly be regarded as surprising, therefore, that they have avoided detection rather consistently, particularly as much of the experimenting in this field was done prior to the development of present day vacuum technique.

II. THEORETICAL CONSIDERATIONS

The view taken in this paper in regard to these electrons that proceed from the target with very nearly the speed of the primaries is that they are themselves primary electrons that have been scattered without appreciable loss of energy in encounters with single atoms in or near the surface of the target. The mechanism is assumed to be similar to that of α -ray scattering but to differ from it in certain respects. An α -ray, on account of its large ratio of kinetic energy to charge, is unappreciably deflected except at distances from a nucleus that are less, in general, than the distance to the nearest structural electrons. The angle distribution of the scattered particles is dependent only upon the nature of the field at these small distances. A low speed electron, however, is much more easily deflected. It may pass quite outside various of the groups of structural electrons and still be turned through a large angle. It is to be expected, therefore, that the angle distribution of these particles will depend upon the arrangement of electrons in the atom as well as upon the nature of the field near the nucleus. If the electrons are arranged in a number of more or less definite shells the scattering pattern will exhibit irregularities determined by the constants of these shells and the speed of the bombarding electrons. If the electrons are arranged in a single strong shell correspondingly simple patterns are to be expected.

A calculation has been given recently by one of us¹⁴ of the angle distribution of electrons scattered by a positive nucleus whose field ends abruptly at a distance ρ from the center. It was shown that the intensity of scattering in a direction making an angle Ψ with the primary beam will be given by

$$I_{\Psi} = \text{const} \left(\frac{2\mu - 1}{(2\mu - 1)^2 (1 + \cos \Psi) + (1 - \cos \Psi)} \right)^2$$

where $\mu = V\rho/E$. V is the potential drop through which the incident electrons acquire their speed, and E the nuclear charge.

¹⁴ Davisson, Phys. Rev. **21**, 637, 1923

One object of this investigation has been to find if this formula is at all capable of representing the scattering patterns of magnesium (atomic number 12). The magnesium surface was formed by sublimation on a target of platinum, so that it has been possible by making observation before and after forming the deposit to determine the patterns for both metals.

III. THE EXPERIMENTAL METHOD

The experimental tube is shown diagrammatically in Fig. 1. The principal parts are the tungsten filament F_1 , for supplying electrons, the nickel cylinder and chute C , for limiting the cross-section of the bombarding stream, the platinum target T and the double Faraday box collector B_1 and B_2 . The target system, comprising the target proper, a

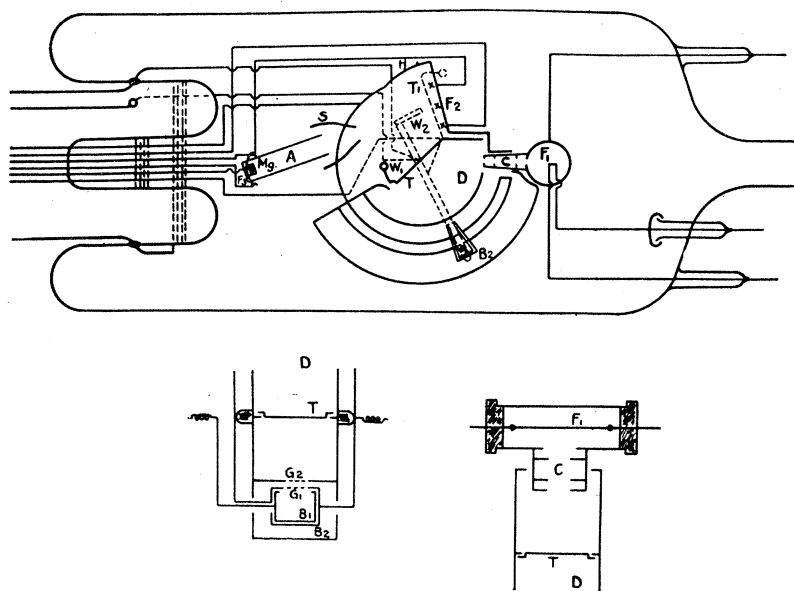


Fig. 1.

counterpoise W_1 and a shielding plate which is shown making an angle with the target, is arranged to rotate about an axis coincident with the junction of the target and shielding plate. By tilting the tube the target can be made to take either of two extreme positions. The normal position is that indicated in section by solid lines and designated as T . The second position is that indicated in section by dotted lines and designated as T_1 . In this position the target is brought in range of a tungsten filament F_2 , from which it can be heated by electron bombardment for the purpose of cleaning its surface. In this position the target is also in range

of the "magnesium gun" shown at *A*. This gun is a platinum cylinder, closed at one end and charged with a small piece of magnesium held in place in the closed end of the tube. A tungsten filament F_2 serves as source of electrons for heating the cylinders by bombardment, and a glass shield *S* protects various parts of the apparatus from magnesium vapor. The shielding plate which has already been referred to as a part of the target system also serves this purpose.

The filament F_1 is located accurately in the axis of the nickel cylinder, and is held taut by a molybdenum spring not shown in the diagram. A series of apertures in the cylinder and chute for limiting the stream of bombarding electrons from the filament, is clearly indicated in the figure. The three apertures in the chute are each 6 by 1.5 mm, and that in the cylinder is somewhat larger. From the filament to the furthest aperture is a distance of 19 mm, and this also is the distance from this aperture to the center of the target.

The target in its normal position forms with the chute and other metal parts a complete enclosure. These other metal parts suggest a section of a drum and will be referred to collectively as the drum. The axis is parallel to F_1 and passes through the center point of the target face. In making measurements the cylinder and chute, the target, and the drum, which are insulated from one another, are ordinarily held at the same potential positive to F_1 . The bombarding electrons thus acquire their speed within the cylinder, and proceed through the apertures to the target, on which they are incident at 45° , without further acceleration. It will be seen from the geometry of the system that the area under bombardment is approximately 6 mm (parallel to F_1) by 4 mm.

The double Faraday box collector is swung on a pair of arms pivoted in the axis of the drum so that the collector revolves in an arc of a circle about the source of the scattered electrons. Two cylindrical flanges fixed to the inside of the drum are interposed between the target and the collector as shown at G_2 in one of the detail drawings. The opening between their inner edges (6 mm wide) is covered by a grid formed of two sets of fine wires stretched diagonally across the opening at right angles to each other.

The opening in the outer Faraday box is 6 by 1.5 mm and is covered by grid wires. The opening in the inner box is somewhat larger and is not so covered. The two parts of the box are insulated from one another by thin quartz plates. The lead from the inner box is especially protected from stray electrons that might reach it inside the tube. From its point of emergency from the outer box to the point at which it passes through the wall of the tube it is completely encased in small quartz tubing.

The various parts of the apparatus are supported from three glass rods fixed in the base of the tube. These are not shown in the figure. The numerous leads are brought out through the base of the tube and through ears in the top. Those to the target, drum and collector are separated as far as possible from the others and guard rings are interposed to protect them from leakage currents.

All metal parts were thoroughly pre-glowed in a vacuum oven immediately before assembly. During baking and pumping such of the parts as could be bombarded by electrons were raised to high temperatures for considerable periods. The complete unit sealed to the pumps comprised, besides the experimental tube, an ionization manometer and a narrow side tube containing coconut charcoal, which was held at 460° C throughout the pumping period. The unit was sealed from the pumps with the pressure at about 10^{-5} mm of mercury and with the charcoal tube at 450° C. On cooling the charcoal with liquid air the pressure became less than could be measured, i.e. less than 10^{-8} mm of mercury. Liquid air was maintained on the charcoal during the entire period of the measurements.

The tube was mounted with the axis of the drum parallel to the horizontal component of the earth's magnetic field. The angular position of the collector could then be varied by rotating the whole tube about this axis, the collector system hanging as a pendulum. The angular range thus covered was from within 15° of the primary beam to somewhat beyond the plane of the target. The earth's magnetic field could be compensated, when this was desirable, by means of appropriate Helmholtz coils.

IV. EXPERIMENTAL RESULTS

Platinum. In making observations we have ordinarily worked with what we have termed "ten per cent" electrons, that is, electrons that leave the target with losses in energy that do not exceed ten per cent of their incident energy. If, for example, the distribution of this class of electrons was to be determined for a bombarding potential of 100 volts, the cylinder and chute, the target, and the drum were held at +100 volts relative to the mid-point of the filament, and both boxes of the Faraday collector at +10 volts. There was thus a retarding potential of 90 volts between G_1 and G_2 (Fig. 1), and only electrons that had left the target with 90 volts energy could enter the collector. With this arrangement of potentials the currents to the drum, target, and collector were measured, and the ratio of the collector current I_c to the total current received by drum and target I_t , was evaluated as a measure of the

intensity of scattering for the particular angular setting of the collector. The sum of the currents to the target and drum measures, to a close approximation, the total number of electrons incident per second on the target, and I_c measures the corresponding number entering the inner box. The total current I_t ranged from 3×10^{-6} to 9×10^{-5} amperes, and was measured with an ordinary wall type galvanometer. The current I_c varied in different measurements from 10^{-5} to 10^{-3} of the total current and was measured with a Leeds and Northrup galvanometer of high sensitivity, $1.5 (10)^{-10}$ amp. per mm deflection.

Four sets of distribution curves for platinum are shown in Fig. 2, covering the range of bombarding potentials from 10 to 1000 volts.

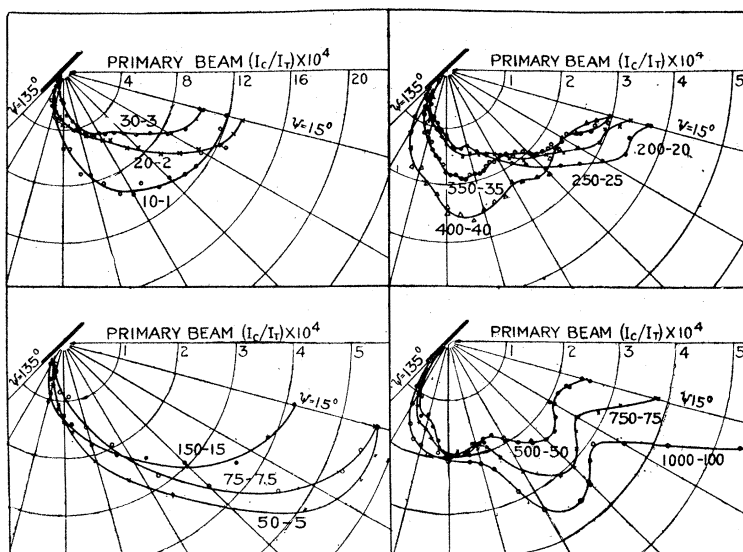


Fig. 2. Angular distribution of electrons scattered by platinum.

The notation "30-3" is used to indicate that the bombarding potential was 30 volts and the collector potential 3 volts. All curves in this figure are for "ten per cent electrons." It will be noted that for bombarding potentials up to 200 volts the distribution of the scattered electrons is comparatively simple, much more simple than might reasonably be expected for the scattering by an atom as complex as that of platinum. Above 200 volts, however, the expected irregularities are much in evidence. The development of various lobes and their variation in angular positions with increase in bombarding potential can be easily traced. All of these curves are quite reproducible and are independent of the total bombarding current over a wide range.

In Fig. 3 additional curves are shown for bombarding potentials 500 and 1000 volts. The three curves for 500 volts corresponding to three different values of the retarding potential. With the collector at 10 volts, the electrons entering the collector left the target with at least 98 per cent of their incident energy, or 99 per cent of their incident speed. The 1000 volt curves are likewise for a series of three different collector potentials as indicated in the figure. It will be noted that the measured intensity of the scattering in a given direction is about proportional to the collector potential. This is at least an indication that the two vanish together, and that, therefore, some among the scattered electrons have speeds not appreciably less than that of the primaries. As the collector potential is decreased the various features of the pattern stand out more sharply; otherwise the form of the curve is independent of the retarding potential over the range explored.

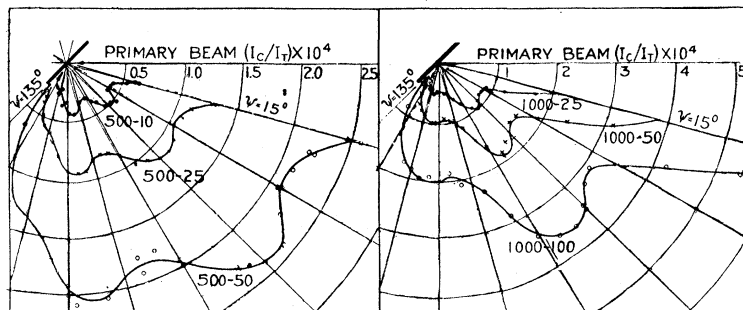


Fig. 3. Angular distribution of electrons scattered by platinum.

The means of analyzing distribution curves in terms of the present theory are not sufficiently developed to permit of a quantitative consideration of patterns as complex as these exhibited by platinum. The most that can be said at present is that they indicate several distinct concentrations of electrons in the platinum atom at various distances from its center.

Magnesium. A light deposit of magnesium was formed on the platinum target by the method already described, and a series of distribution curves obtained for bombarding potentials to 1500 volts. No means were available for estimating the thickness of this deposit. The vaporization was continued very gently until a barely visible spot had been formed. A second deposit was then formed on top of the first, and the series of observations repeated.

A set of these curves for bombarding potentials from 24 to 1500 volts is shown in Fig. 4. With the exception of a small lobe or spur which is noticeable near $\Psi = 90^\circ$ on curves in the range 24 to 125 volts, the dis-

tributions are strikingly simple and conform in a general way to the requirements of the theory for the scattering by single shell atoms. Starting from $V=24$ volts the elongation of the pattern in the direction $\Psi=0$ first increases with V , and then decreases, the intensity in the direction $\Psi=0^\circ$ finally becoming less than in the direction $\Psi=90^\circ$. This behavior is sufficiently like that calculated for the idealized system to warrant our neglecting the small spur for the present and attempting a calculation of the radius of the shell from the characteristics of the principal feature of the patterns.

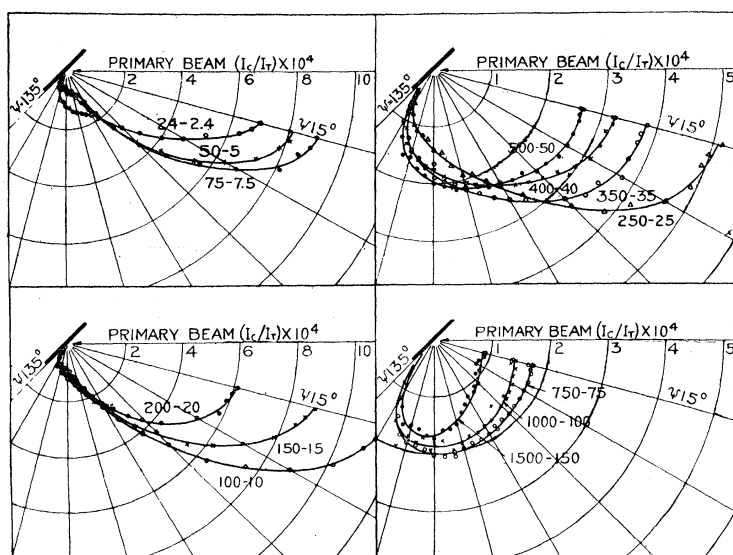


Fig. 4. Angular distribution of electrons scattered by magnesium.

It is important to recognize, in dealing with the curves quantitatively, that the form of each is determined in part by the absorption of scattered electrons in the metal. Experiments with a tube in which the target can be rotated, indicate that many of the scattered electrons receive their large deflections at considerable depths in the metal and are subjected to absorption on their way to the surface. The effect of this absorption is to reduce the observed intensities differently in different directions, least in the direction normal to the surface and most in the directions in its plane. In the plane of the target the observed intensity is always zero. This results from absorption and partly perhaps from the microscopic roughness of the surface.

The form of the observed curves must also be affected to some extent by the variation in angular width of the bombarded area as viewed from the collector. This width is greatest when the collector stands in the

normal to the target, and in this position fewer electrons enter the collector than would if the bombardment were more concentrated. As the collector moves out of this position the bombarded area is viewed more nearly edge on, and the apparent intensities are somewhat increased. Fortunately, the effects both of the absorption and of the variability of angular width of the source should be symmetrical with respect to the normal to the target. We may reasonably expect, therefore, that the ratio of observed intensities in directions making equal angles on opposite sides with the normal will be the same as if the distribution curves were ideal.

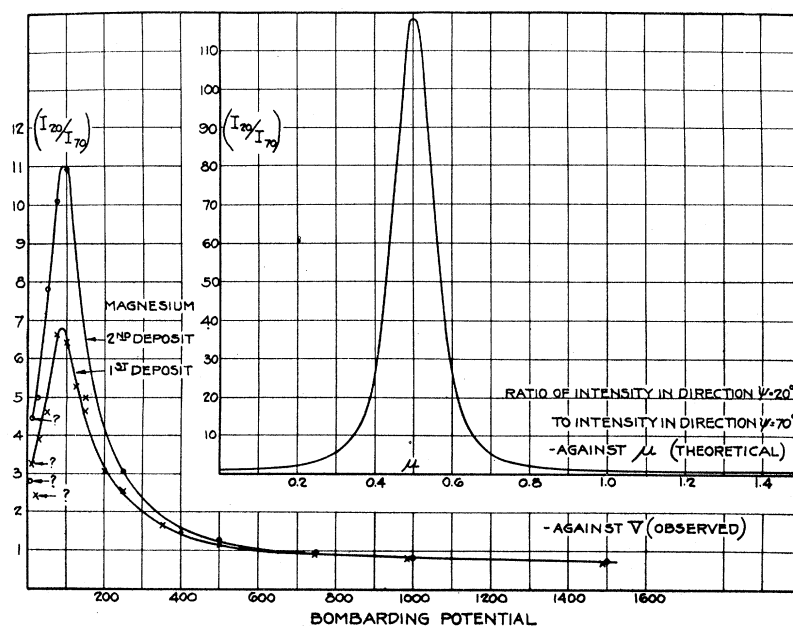


Fig. 5.

In Fig. 5 we have plotted the ratios of observed intensities for directions $\Psi = 20^\circ$, ($45 - 25$), and $\Psi = 70^\circ$, ($45 + 25$), against bombarding potentials. It should be mentioned that in the distribution curve for 24 volts the lobe is a definite feature extending to a value of Ψ less than 70° . In this case the ratio of intensities plotted in Fig. 5 is that between the actual intensity at 20° and the intensity at 70° as read from an extrapolation of the main distribution. At potentials less than 20 volts the main distribution and the lobe are apparently merged and indistinguishable. The ratios for these voltages have been taken directly from the distribution patterns and plotted in Fig. 5. They have been disregarded, however,

in drawing in the curves in this figure. We have no explanation for the fact that the maximum of the curve for the second deposit is higher than that for the first.

V. COMPARISON WITH THEORY

The relation to be expected between this ratio and μ (proportional to V) can, of course, be calculated from the formula for I_ψ . It is shown graphically in the same figure to one tenth the vertical scale of the observed curves. The outstanding differences between the theoretical and the observed curves are (1) the difference in height of their maxima, and (2) the difference in position of their maxima relative to other features of the curve. It seems not unlikely that the former of these differences results merely from the fact that the field about the nucleus is not as sharply limited as assumed in arriving at the theoretical curve. The position of the observed maximum at 90 volts instead of much further to the right has the appearance of being a more serious discrepancy. It is significant, however, that each experimental curve, so far as it has equal ordinates on the two sides of its maximum, is reasonably symmetrical. The experimental curve can be most simply described as one resembling the theoretical curve in form that has been shifted several hundred volts to the left. That is, the scattering is reasonably in accord with what might be expected if the bombarding potential were in each case several hundred volts greater than that actually used. This suggests that the nucleus is surrounded by a shell of electrons which does not completely compensate its charge, and that the several hundred additional volts are acquired by the incident electrons in the field external to this shell. The incident electrons would, of course, be deflected to some extent in such a field. The principal effect, however, should be to increase their velocity toward the limited field. This is particularly true as we are dealing only with those electrons that suffer large total deflections, that is, with those that pass nearest the nucleus, and, therefore, approach most nearly head-on. For the present we shall assume that this is the cause of the shift, and disregard the deflections that are involved.

Next to the position of the maximum at $V=90$ volts, the most dependable result from the experimental curve would appear to be the potential at which the ratio of intensities falls to unity. This occurs at $V=650$ volts. Approximately the same value of this potential is obtained when curves are plotted for other pairs of values of Ψ satisfying the condition $\Psi_1 + \Psi_2 = 90^\circ$. $650 - 90 = 560$ volts should then correspond to the value of V in the equation $\mu = \frac{1}{2} = V\rho/E$, which leads to

$$E/\rho = 2 \times 560/300 = 3.73 \text{ e.s.u./cm}$$

where E is the central charge and ρ is the radius of the shell.

On this basis the amount of the voltage shift is $560 - 90 = 470$ volts. Using E' to represent the residual charge of the atom out to and including the principal shell of electrons, and regarding the field outside this shell as essentially unlimited, we also have

$$E'/\rho = 470/300 = 1.57 \text{ e.s.u./cm.}$$

Combining these results, $E'/E = 0.42$. This is the ratio of 5 to 12, so that, if the central charge is $12e$, the residual charge of nucleus and shell should be $5e$, indicating a shell of seven electrons with the remaining five considerably outside this shell. On the other hand if two K-electrons are assumed so near the nucleus that the central charge is effectually $10e$, then the ratio of E' to E would indicate six electrons in the L-shell and four others outside these, while the assumption of three K-electrons would lead to the arrangement three-five-four. If the central charge is $12e$ then

$$\rho = (12 \times 4.78 / 3.73) (10)^{-10} = 1.54 \times 10^{-9} \text{ cm.}$$

For central charges $10e$ and $9e$ the values of ρ are respectively $1.28 (10)^{-9}$ cm and $1.15 (10)^{-9}$ cm. These values are smaller but reasonably in accord with the radius of the two quantum circular orbit of the magnesium atom as calculated on Bohr's theory. If the radius of the one quantum orbit of the hydrogen atom is $a_0 = 5.3 \times 10^{-9}$ cm, then for nuclear defect zero the radius of the two quantum orbit in the magnesium atom should be $a = 4a_0/12 = 1.8 \times 10^{-9}$ cm. For nuclear defect $3e$ we would have $a = 4a_0/(12 - 3) = 2.6 \times 10^{-9}$ cm.

We return now to the small lobe which is a feature of the distribution curves for bombarding potentials up to 125 volts. It is conceivable that this lobe is due to electrons which have been deflected only in the field outside the principal shell. For such a class of electrons the principal shell would function as an obstacle. Only those electrons would appear in the distribution whose trajectories lay entirely outside its boundary. If the external field due to the residual charge E' be regarded as essentially unlimited, we have, from the ordinary theory of conic orbits, that the nearest approach of an electron to the center of the system

$$b = \frac{E'}{2V} \left[\left(\frac{2}{1 + \cos \Psi} \right)^{\frac{1}{2}} - 1 \right].$$

The distribution we are considering would then include only electrons for which b is as great or greater than ρ . It would begin abruptly at the critical value of Ψ which satisfies the above equation with b set equal to ρ , and would be altogether lacking in the angular range below this value. It will be seen that this critical value of Ψ should increase with increase in V . In Fig. 6 we have reproduced the portions of distribution curves for bombarding potentials up to 125 volts which include the small lobe.

The radial scales of these curves are arbitrary and have been adjusted to avoid overlapping. It will be seen that the value of Ψ at which the added distribution begins is fairly definite, and that it does, in fact, increase with increase in V .

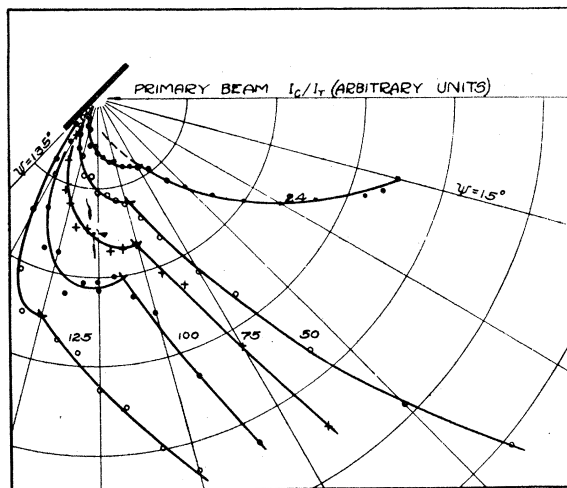


Fig. 6. Portions of distribution curves of magnesium showing lobe.

Setting b equal to ρ and rewriting the equation in the form

$$\left[\left(\frac{2}{1 + \cos \Psi} \right)^{\frac{1}{2}} - 1 \right] = \frac{2\rho}{E'} V$$

we see that the function of the critical value of Ψ on the left should be proportional to V , the factor of proportionality being $2\rho/E'$. The values of this function for all of the curves in which Ψ can be located are plotted against V in Fig. 7. The line through the origin has been drawn in without reference to the plotted points. Its slope is $2\rho/E'$ as computed from the value of E'/ρ found in the calculation based on the main feature of the patterns. The observed points fall on this line as well perhaps as might be expected.

This explanation of the lobe would be untenable, of course, if it should turn out that the lines along which the electrons must be assumed to approach are as distant from the scattering center as the distance between adjacent atoms in the magnesium crystal. That this is not the case is clear from the following considerations. If d represents the distance from the center of force to the line along which an electron approaches, ρ the distance from the force center to the apse of its orbit, and Ψ the direction in which the electron finally proceeds from the system, it can be shown

that in the case of an unlimited field these quantities are related through the equation

$$d = \rho \left[\frac{(1 - \cos \Psi)^{\frac{1}{2}}}{\sqrt{2} - (1 + \cos \Psi)^{\frac{1}{2}}} \right]$$

The largest values of d correspond to the smallest values of Ψ . The minimum observed value of the critical angle in any of the curves is 55° . For this case $d = 4.1\rho$. Or, if we use 1.5×10^{-9} cm as the value of ρ , $d = 6.1 \times 10^{-9}$ cm. The electrons forming the added distribution in this extreme case would then approach along lines not less than 6×10^{-9} cm

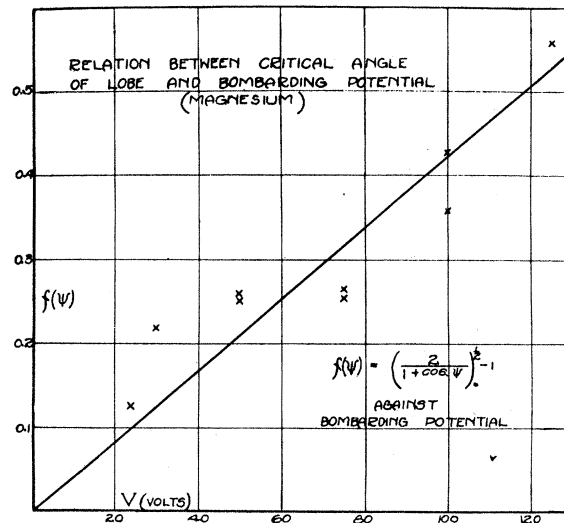


Fig. 7.

from the center of the atom. This distance is about one-fifth the shortest distance between atoms in the magnesium crystal. It appears, therefore, that it would be possible under favorable circumstances for electrons to have approximately ideal encounters of the sort considered with single atoms, even in the extreme case corresponding to $\Psi = 55^\circ$.

We now inquire within what distances, on the present theory, the scattered electrons forming the main distribution have approached to the center of the system. It may be shown in the case of the limited field that if the distance of nearest approach b is expressed as a fraction a of the radius of the field ($b = a\rho$), then a will be related to the other quantities involved through the equation

$$a = \delta^2 \left[\frac{2\mu}{1 + [1 + 4\mu(\mu - 1)\delta^2]^{\frac{1}{2}}} \right]$$

where

$$\delta = \left(\frac{d}{\rho}\right)^2 = \left[\frac{(1 - \cos \Psi)}{(1 - \cos \Psi) + (2\mu - 1)^2 (1 + \cos \Psi)} \right]$$

d being the distance from the nucleus to the line of approach. A table of values of α calculated from this equation and applicable to the present results is given below.

TABLE I

μ	V (Volts)	α ($\Psi = 15^\circ$)	α ($\Psi = 45^\circ$)	α ($\Psi = 90^\circ$)
0.45	34	0.35	0.68	.78
0.50	90	1.00	1.00	1.00
0.55	146	0.43	0.83	0.95
0.75	370	0.049	0.33	0.73
1.00	650	0.017	0.15	0.50
1.50	1210	0.0065	0.060	0.26
2.00	1770	0.0038	0.037	0.17

Of the electrons forming the observed patterns those scattered in the direction $\Psi = 15^\circ$ when V had its maximum value (1500 volts) passed nearest the nucleus. For these $\alpha = .005$. If $\rho = 1.5 \times 10^{-9}$ cm then the nearest approach was 7×10^{-12} cm. This is about double the distance of nearest approach estimated in the experiments on alpha ray scattering. The distance is much less, however, than the radius of the orbit of the K electrons as calculated on Bohr's theory. If $a_0 = 5.3 \times 10^{-9}$ cm then the K-electrons in the magnesium atom should be at a distance from the nucleus given approximately by $a_0 / (12 - \frac{1}{4}) = 0.45 \times 10^{-9}$ cm. This distance corresponds to $\alpha = \frac{1}{3}$. It is clear then from the figures in Table I that over a wide range of V certain electrons pass tangent to a sphere of this radius and emerge in the angular range $\Psi = 15^\circ$ to $\Psi = 90^\circ$. In Table II we have written down the angles corresponding to $\alpha = 0.33$ for values of V for which curves are shown in Fig. 4.

TABLE II

V :	24	50	75	(90)	100	150	200	250	500	750	1000
μ :	0.44	0.46	0.485	(0.5)	0.51	.55	0.60	0.64	0.87	1.09	1.31
Ψ :	15°	10°	3°	(0°)	3°	12°	21°	28°	59°	78°	90°

If there are irregularities in the distribution curves caused by the K-electrons at or near these angles they are too slight to be detected with the present experimental arrangement. In particular it seems impossible to associate the small lobe at low voltage in any way with the K-electrons.

We must conclude then that on the basis of the present interpretation and within the accuracy of the measurements there is no evidence of

any orbital electrons in the range 7×10^{-12} cm to 1.5×10^{-9} cm from the nucleus. The spherical shell system of central charge $12e$ that would yield patterns most nearly in accord with those observed is one in which the central charge is surrounded by a single shell of charge $-7e$ and of radius 1.5×10^{-9} cm.

VI. DISCUSSION OF RESULTS

In attempting to infer the nature of a scattering system from the angular distribution of scattered particles the only possible method of procedure seems to be to calculate distribution patterns for systems of assumed characteristics and to compare these with the observed distribution. If the calculated distribution can be made to represent the experimental results by an appropriate choice of constants the assumed system is similar to the actual system, at least in the way it scatters particles. It is not necessarily similar to it in other respects, nor does it follow that other models might not be found that would also be similar to the actual system in this particular respect. This limitation to the use that can be made of the data of scattering experiments has been stated very clearly by C. G. Darwin.¹⁵

In the present case we have compared the observed distribution curves with those calculated for an assumed system which is admittedly inadequate to represent any actual atom, except in the most hazy manner. Recognizing this deficiency in the model it has seemed legitimate to see in any agreement between the two sets of curves, however general, an equally general agreement between the atomic system and the model. It has also seemed legitimate to take advantage of this general agreement to calculate the constants of the model. It should be recognized, however, that the constants are primarily those of the model and not those of the atom. In the present case the constants are those of the spherical shell model that most nearly resembles the actual system in the way that it scatters electrons. These are not necessarily related in any very simple way to the constants of the actual system. It has not been shown, for example, that the spherical shell that most nearly represents a cubical arrangement of eight electrons when appropriately integrated over all orientations is a shell of charge $8e$ and of radius equal to the distance from the center to a corner of the cube. The octet might very well be most nearly equivalent to a more contracted shell of fewer than eight charges plus a certain external field. If the electrons are arranged as in the Bohr model with half of the L-electrons moving in elliptical orbits it is difficult

¹⁵ Darwin, *Phil. Mag.* **12**, 486, 1921.

to conjecture the probable relation between the actual system and its most nearly equivalent spherical shell model. This is particularly true as the system not only has directional properties, but in a particular orientation also presents a variety of aspects to incident electrons depending upon the phases of the structural electrons in their orbits. At all except the lowest bombarding potentials the time of transit of the electron through the atom is small compared with the periods of the orbital electrons.

No attempt has been made to give a separate interpretation of the results in terms of the scattering patterns which are found when it is assumed that the electrons in passing through the atom execute orbits of the type calculated by Darwin. So far as we can judge from a careful examination of the curves there is no evidence that the electrons have executed these orbits. On the simple theory, or on H. A. Wilson's view in regard to encounters between an electron and a positive nucleus,¹⁶ identical patterns are to be expected for bombarding potentials V_1 and V_2 satisfying the condition $V_1 + V_2 = 2V_m$, where V_m is the potential for which the pattern has its maximum elongation. On the basis of Darwin's orbits, however, these curves would be expected to exhibit differences in form of the type shown in Fig. 7 of the theoretical article.¹⁴ A comparison of the magnesium curves for potentials 24 and 150 volts, and for 50 and 125 volts shows that the curves of each of these pairs have closely the same form. It should be pointed out, however, that the differences in form to be expected between the curves of these pairs on the basis of Darwin's calculations is much less than shown in the figure referred to. The curves in the figure are for $\mu = \frac{1}{4}$ and $\mu = \frac{3}{4}$, while the values of μ for the curves which have been compared are, as we suppose, much closer to the value one-half. In any event the calculations which have been based on the critical voltages read from the curve in Fig. 5 would not be appreciably affected by assuming that the electrons had executed Darwin orbits.

In conclusion we take pleasure in expressing our thanks to Dr. H. D. Arnold and Dr. W. Wilson for encouragement to carry out this investigation and to G. E. Reitter for the care with which he constructed the special apparatus required.

RESEARCH LABORATORIES OF THE
AMERICAN TELEPHONE AND TELEGRAPH COMPANY
AND THE WESTERN ELECTRIC COMPANY, INCORPORATED,
April 11, 1923.

¹⁶ Wilson, Proc. Roy. Soc. A **102**, 9, Oct. 2, 1922.