THE SENSIBILITY OF THE EAR TO SMALL DIFFERENCES OF INTENSITY AND FREQUENCY.

By Vern O. Knudsen.

Abstract.

Intensity and pitch sensibilities of the ear as functions of loudness and frequency.—Intensity sensibility is defined, as usual, as the ratio of the least perceptible difference of energy to the total energy of that tone, $\Delta E/E$, while pitch sensibility is the ratio of the least perceptible difference of frequency to the frequency of the tone, $\Delta N/N$. There are considerable discrepancies among the results of previous investigators. The author used as a source of sound a telephone receiver actuated by a current from a vacuum tube oscillator. The intensity or the pitch could be changed periodically, once a second or so, by automatically changing the resistance or capacity in the oscillating circuit. The method of observation, then, was to change ΔE or ΔN continuously until the threshold of perception of fluctuation was reached. Separate observations usually checked within 10 per cent. As auxiliary experiments, in which the intensity was varied by changing the distance, showed that the acoustical energy of the source was a linear function of the electrical energy input, the latter was used as a convenient measure of the intensity of the sound. The frequency scale was determined by calibration. While the curves for the ears tested show individual differences, the results are in general the same for all. The intensity sensibility was found to be about 0.10 for moderate and high intensities but to increase to the limiting value 1 as the intensity decreases to the threshold. The curves are very similar to those obtained for the eye, and the *modification of the Weber-Fechner law* proposed by Nutting for light sensation also fits the results for audition satisfactorily: $\Delta E/E = F + (1 - F)(E_0/E)^n$, where E_0 is the threshold intensity and F is about 0.10. The exponent n varies somewhat with the frequency, being 1.65 for 200 d.v. and 1.05 for 1,000 d.v.; nevertheless at the same loudness level, for instance 10,000 E_0, $\Delta E/E$ is nearly independent of frequency, showing only a 10 per cent. variation from 100 to 3,200 d.v. The results were the same whether harmonics were present or not. It is concluded that for 1,000 d.v. under favorable circumstances the normal ear can distinguish about 400 gradations of loudness between the threshold and a painful intensity 10^{12} times as loud. The pitch sensibility, $\Delta N/N$, was found to depend on relative loudness in nearly the same way as $\Delta E/E$. For the same loudness level, $\Delta N/N$ decreased from 0.01 at 50 d.v. to 0.003 at 600 d.v. and then remained constant up to 3,200 d.v. The limit of perception of variation of pitch was about the same or trained as for other ears, but training helped in distinguishing which note was higher. Two ears were found more sensitive than one ear to small differences of pitch but not to small differences of loudness.

I. Introduction.

We can arrive at a restricted arbitrary solution of the sensibility of the ear by determining the smallest perceptible increments of loudness, pitch, and quality for tones of all gradations of loudness, pitch , and quality. Thus the ratio of the smallest discernible difference of any arbitrary tonal stimulus to the whole stimulus will serve as a means of expressing the sensibility of the ear at that loudness, pitch, and quality. Hence if we can determine the ratios of the least discernible differences of loudness and pitch and quality of a tonal stimulus to the whole stimulus for tones of all gradations of these three properties, we should obtain a relative measure of how the sensibility of the ear depends upon these characteristic properties of sound. This leads us to the purpose of the present investigation, namely:

I. To determine how the sensibility of the ear to small differences of intensity varies with loudness and pitch. This "intensity sensibility" is expressed by the ratio of the smallest perceptible difference of the energy of a tone to the total energy of that tone, $\Delta E/E$.

II. To determine how the sensibility of the ear to small differences of frequency varies with loudness and pitch. This "pitch sensibility" is expressed by the ratio of the smallest perceptible difference of the frequency of a tone to the whole frequency of that tone, $\Delta N/N$.[1]

II. Historical Survey.

1. *Sensibility of the Ear to Small Differences of the Intensity.*

The history of this phase of the sensibility of the ear begins with the Weber-Fechner law, which states that for each of our senses the increase of a stimulus necessary to produce a just discernible increase of sensation bears a constant ratio to the total stimulus. This constant ratio is called the Fechner constant.[2] In standard works on psychology, such as Wundt's, the value given to this ratio for the perception of sound intensity is one third.[3] This value is about the average of a number of early measurements obtained for noises produced by the impacts of balls against wood or metal plates, or by varying the distance from a ticking watch to the head of an observer.

Thus, in 1880, Fischer and Wundt[4] conducted a large number of experiments in which the smallest perceptible increment of sound intensity was determined by comparing the heights of fall of two similar lead balls which made successive impacts upon the fall-plate of a Hepp fall appa-

[1] This measure of sensibility has been employed by previous investigators and for that reason is used in this article. It is a little misleading since the smaller the values of $\Delta E/E$ and $\Delta N/N$ the greater the sensitivity of the ear. It would be better to use the inverse ratio or the logarithm of the inverse ratio to express these sensibilities.

[2] In the present paper we shall call this ratio the "Fechner ratio" and not the "Fechner constant," since, as we shall see, it is not a constant.

[3] Wundt's Human and Animal Psychology, p. 30. Wundt gives this same value to the Fechner ratio for the sensations of "warmth" and pressure.

[4] Wundt, Phil. Stud., p. 495.

ratus. By using balls of different weights and various heights of fall, they could produce various intensities of sound. Their results indicate that the Fechner ratio is independent of the intensity of the noise. They give for the value of the ratio .4055.

Max Wien[1] may be properly called the pioneer worker in determining the Fechner ratio for musical tones. Wien used as a source of tone a telephone receiver near the mouth of a Helmholtz resonator. He measured the intensity of the sound by optically projecting and amplifying the motion of a diaphragm placed within the resonator. A rubber tube connected the resonator to the observer's ears. Wien used chiefly the method of minimal change. His average values of $\Delta E/E$ for the three tones with which he worked are:

Pitch	a(220 d.v.)	e^1(337 d.v.)	a^1(440 d.v.)
$\Delta E/E$	.224	.176	.131

He therefore concludes that $\Delta E/E$ is a function of the frequency and uses the above data to determine the relative sensibility of the ear at the three different frequencies. His value of $\Delta E/E$ also varies with the intensity, increasing in general for very feeble and very loud tones, but rather irregularly.

A. Deenik[2] working with tuning forks and organ pipes records the following values:

(*a*) With Tuning Forks,

Pitch	c^1(256 d.v.)	c^2(512 d.v.)	c^3(1024 d.v.)
$\Delta E/E$	.332	.295	.195

(*b*) With Organ Pipes,

Pitch	C	c	c_1	c_2	g_2	c_3	g_3	c_4	c_5	c_6
$\Delta E/E$	.232	.218	.163	.131	.105	.125	.112	.085	.107	.192

2. Sensibility of the Ear to Small Differences of Frequency.

Vance[3] has given a good historical survey of the problem of the smallest perceptible difference of pitch and has also contributed valuable data to the subject. One has only to examine the existing data on the pitch-discriminating power of the ear given by Preyer,[4] Luft,[5] Meyer,[6] Stucker,[7]

[1] Über die Messung der Tonstärke; Max Wien, Wied. Ann. der Phys., 36, 834 (1888).

[2] Über das Unterschiedungsvermögen fur Tonintensitaten. A. Deenik, Versl. K. Ak. van Wet. afd. Natuutk., 14, 396.

[3] Variation in Pitch Discrimination Within the Tonal Range, T. F. Vance, Psychol. Monograph, No. 69, 16, p. 115. See also Nagel's Physiologie des Menchen, 3, p. 483.

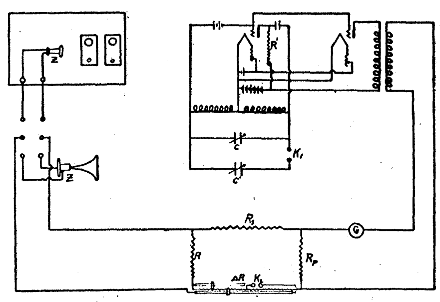
[4] Über die Unterschiedsempfindlichkeit fur Tonhöhen, W. Preyer, Jena, 1876, p. 24.

[5] Die Unterschiedsempfindlichkeit fur Tonhöhen, E. Luft, Phil. Stud., 4, *511* (1888).

and Vance[8] to learn that there are notable discrepancies among the results of these principal investigators.

III. Apparatus and Method.

1. *Apparatus*.—Fig. 1 shows a schematic circuit diagram of the essential features of the apparatus used for the experiments described in this paper. The source of sound used is a telephone receiver actuated by energy from a vacuum tube oscillator. The oscillator can produce in the receiver tones of any desired frequency between 30 d.v. and 20,000 d.v. By means of a divided and balanced resistance circuit in the output of the oscillator the intensity of the tones can be varied by any desirable and measurable intervals from barely audible tones up to tones sufficiently loud to excite the sensation of pain. The wave form of the oscillator current is maintained practically sinusoidal by making R', the plate resistance in the oscillating tube, sufficiently large to limit the amplitude of the oscillations to the extent that the tube amplifies linearly. In many of the tests the possible error from overtones was further eliminated by associating the receiver with appropriate resonators.



SCHEMATIC CIRCUIT DIAGRAM

Fig. 1.

The circuit is so designed that a motor-controlled key periodically changes, by any desired intervals, the resistance R across which the receiver is shunted. The tone emitted by the receiver will therefore periodically and abruptly fluctuate from a tone of one intensity to a tone of greater or lesser intensity. The duration of each tone and the rate of fluctuation are easily controlled—the former by the amount of mercury in a cup into which the tongue of the key dips, the latter by the

[6] Über die Unterschiedsempfindlichkeit fur Tonhöhen, M. Meyer, Zsch. Psy. u. Physiol. d. Sinn., *16*, 352 (1889).

[7] Über die Unterschiedsempfindlichkeit fur Tonhöhen in Verschiedenen Tonregionen, N. Stücker, Setz. d. & ath. Kl. d. Kaiserl. Ak. d. Wiss., Wien, 1907, 116, 367.

[8] Loc. cit.

speed of the motor which operates the key. For best working conditions the two tones are of equal duration and alternate at a rate of about 50 per minute. This period is often changed and interrupted, and sometimes the key is operated manually, in order to eliminate the possibility of the observer possessing any after image or persistence of the rhythm of the changes after his liminal difference has been reached.

The means employed for changing the frequency is as simple as that used for changing the intensity. The same motor-controlled key is used to successively open and close K_1. This periodically increases the capacitance of the oscillator circuit by an amount c'. c' is made up of a combined glass and air variable condenser whose capacitance can be varied continuously from zero upwards. The frequency of the tone emitted by the receiver will therefore alternately wax and wane by an amount which is determined by the values of c and c'.

2. *Method of Making Measurements.*—The procedure for determining the smallest perceptible difference of intensity is as follows: The observer holds the receiver snugly to his ear, or, when the resonator is used as the source of the tone, he holds to his ear a rubber tube which is connected to the resonator. The resistance R and ΔR (Fig. 1) are adjusted to such values that when K_2 is opened or closed (either manually or by the motor control) the difference of intensity of the two emitted tones is plainly perceptible. Then ΔR, a portion of a slide wire resistance, which is added to R when K_2 is closed, is gradually decreased until the observer no longer recognizes a difference of intensity. The observer signals his judgments to the operator by means of two signal lamps. The judgment of the disappearance of the difference of intensity of the tones is a rather simple judgment since it consists of deciding when a flutter tone merges into a steady tone. Then a second judgment is made to determine when the difference is just barely perceptible as the difference of intensity is increased from a non-perceptible amount; that is, ΔR is increased from zero until the observer recognizes the first appearance of the flutter. With a little practice and careful attention one can easily reproduce decisions of appearance or disappearance of the flutter within limits of about 10 per cent. A number of observations, four or more, are taken for each tone and the average of these is used to compute the Fechner ratio $\Delta E/E$. The method of this computation depends upon a linear relation between the electrical energy which actuates the receiver and the acoustical energy developed by the vibrating diaphragm.

3. *Relation between the Acoustical and Electrical Energies in a Telephone Receiver.*—The following experiments indicate a linear relation between the acoustical and electrical energies in a telephone receiver. The ex-

periments were performed over the campus lawn away from reflecting surfaces and where, as Wien has shown,[1] the intensity of sound radiation obeys the inverse square law. A telephone receiver, actuated by the vacuum tube oscillator already described, was supported overhead by a pulley, cord, and reel arrangement so that the receiver could be suspended at any distance, up to 10 meters, above the head of the observer. The current in the receiver could be adjusted by the attenuating circuit in the output of the oscillator.

In one experiment the telephone receiver was adjusted to such a distance above the ears of the observer that the tone just ceased to be audible. Then the distance from the ears to the receiver was altered and the receiver current adjusted until the threshold intensity was again obtained. Curve I., in Fig. 2, shows the relation between the current in the receiver, I_R, and the distance from the ears of the observer to the receiver to produce the threshold intensity for a tone whose frequency was 700 d.v. Similar curves were obtained for tones of 250 d.v. and 2,000 d.v. Since the presence of noise seriously interferes with the accuracy of locating the threshold intensity, these data were taken during the summer recess, 1921, and most of the observations were made at night under conditions particularly free from disturbing noises.

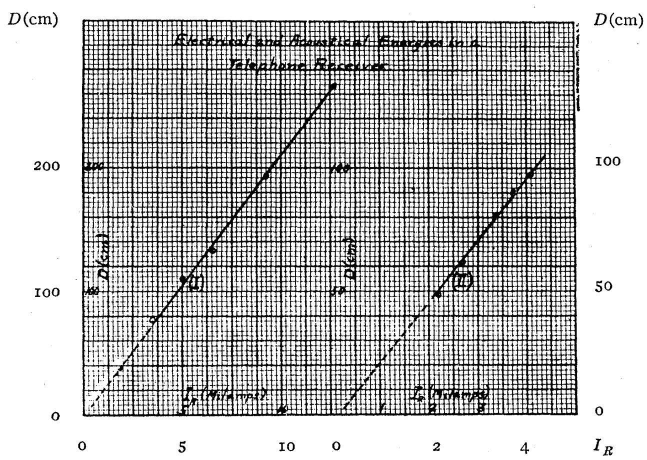


Fig. 2. Electrical and Acoustical Energies in a Telephone Receiver

In a second experiment the tone in a movable receiver was compared and made equal in intensity to the intensity of a tone in a receiver maintained at a constant distance above the observer's ears and actuated by a constant current of the same frequency as the current in the movable

[1] Loc. cit.

receiver. Ten separate judgments were averaged for each comparison, five with the compared tone just louder than the standard tone and five with the compared tone just feebler than the standard tone. Curve II, Fig. 2, shows the relation between the distance from the movable receiver to the observer's ears and the current required in the movable receiver to produce in it a tone of the same intensity as that of the constant standard tone in the fixed receiver. The frequency of the tone for Curve II was 700 d.v. Similar curves were obtained for tones of 250 d.v. and 2,000 d.v.

Assuming the inverse square law, the linear character of Curves I and II shows that for a wide range of receiver currents the acoustical energy developed by the vibrating diaphragm is a linear function of the electrical energy which actuates the receiver. For any fixed frequency, therefore, the electrical energy in the receiver is a convenient and reliable relative measure of the acoustical energy developed by the receiver diaphragm.

The change of the current in the receiver can therefore be used as a measure of the smallest perceptible difference of intensity of musical tones. Thus the ratio of the smallest discernible change of amplitude of a tone to the total amplitude will be given by the ratio of the change of current in the receiver to the total current. This latter ratio is conveniently computed from a simple equation which we shall now derive.

4. *Equation for Computing the Fechner Ratio* $\Delta E/E$.—The attenuating and measuring circuit by which the current in the receiver can be controlled and computed is shown in Fig. 1. R, R_s, and R_p, are non-inductive resistances. R is a rather low resistance, about 20 ohms or less. ΔR is usually of the order of one ohm and is added to R when K_2 is open. R_s—divided and distributed in two equal parts to keep the line balanced—is large, 5,000 ohms. R_p may have any value between zero and infinity. Z, the impedance of the receiver when held against the ear, varies from 105 ohms at 100 d.v. to 2,080 ohms at 9,000 d.v. G is a Duddell Thermo Galvanometer which measures the current I flowing into the attenuating network.

With K_2 closed the current in the receiver, I_R, is

$$I_R = \frac{R_p R I}{\left(R_p + R_s + \dfrac{RZ}{R + Z}\right)(R + Z)}. \tag{1}$$

With K_2 open R is given the increment ΔR and I_R is given the increment ΔI_R. Since R and ΔR are small compared with R_s, I will not be sensibly changed. Therefore,

$$I_R + \Delta I_R = \frac{R_p(R + \Delta R)I}{\left(R + R_s + \dfrac{(R + \Delta R)Z}{R+\Delta R+Z}\right)(R + \Delta R + Z)} \cdot \qquad (2)$$

Hence,

$$\frac{\Delta I_R}{I_R + \Delta I_R} = \frac{\Delta RZ}{(Z + R)\,(R + \Delta R)} \cdot \qquad (3)$$

Equation (3) therefore gives the ratio of the smallest perceptible increment of the amplitude of vibration of the tone to the total amplitude. Then the Fechner ratio, that is, the ratio of the smallest perceptible increment of energy of the tone to the total energy of the tone, $\Delta E/E$, is given by

$$\frac{\Delta E}{E} = \left(1 + \frac{\Delta I_R}{I_R + \Delta I_R}\right)^2 - 1. \qquad (4)$$

5. *Method for Determining* $\Delta N/N$.—In paragraph 2 was described the procedure for making measurements on the smallest perceptible difference of loudness. The same procedure is followed for making measurements on the smallest perceptible difference of pitch, except that the frequency instead of the intensity of the tone emitted by the receiver alternately waxes and wanes. The $\Delta N/N$ could be determined from the average value of c', c, and the equation for the frequency of the oscillator; but since the latter is rather complicated it is simpler and more accurate to calibrate the frequency of the oscillator experimentally for small changes of c. This was done by comparing the frequency of the oscillator with the frequency of standard König tuning forks which were maintained at a constant temperature. By counting the number of beats for slightly different values of c the rate of change of frequency for small changes of c, $\Delta N/\Delta c$, was determined for tones covering the entire range of pitch from 50 d.v. to 4,800 d.v. From these values of $\Delta N/\Delta c$, curves were prepared from which one can read directly the percentage change of frequency corresponding to any possible change of c.

IV. Experimental Results.

1. *Sensibility to Small Differences of Intensity as a Function of Loudness.*—Fig. 3 shows how the sensibility of the ear to small differences of intensity depends upon the loudness of sound. The curves show the relation between the Fechner ratio, $\Delta E/E$, and the relative amplitude of vibration of tones of 400 d.v., 1,000 d.v., 2,000 d.v., and 4,000 d.v. The relative amplitude of vibration of the tones is expressed by the ratio of the current

in the receiver, I_R, to the current in the receiver, I_0, required to produce the threshold intensity. Each curve shown is an average of four separate curves taken on different days. Each point on the separate curves, not shown in the figure, is the average of six independent judgments. Therefore each point indicated on the curves in the figure is an average of 24 separate judgments.

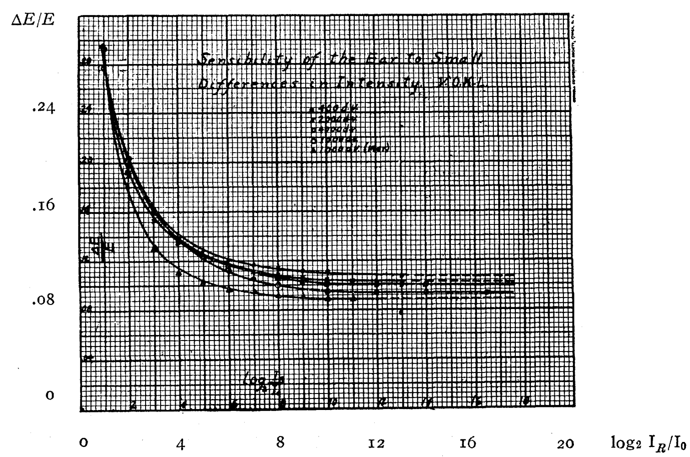


Fig. 3. Intensity Sensibility of the Ear

All but one of the curves in Fig. 3 are for the writer's left ear. Similar curves were obtained for the ears of three other persons, one of which is the lowest curve in Fig. 3. None of the other curves differed appreciably from those shown. The character of all of these curves shows that the sensibility of the ear to small differences of intensity is some continuous function of the intensity, but not as simple as the Weber-Fechner law requires. For feeble intensities the Fechner ratio is almost inversely proportional to some logarithmic function of the intensity. For moderate and high intensities the ratio is nearly constant, and hence for moderate and loud tones the original form of the Weber-Fechner law is quite valid. The value of the ratio, however, at least for musical tones, is not one third but more nearly one tenth.

From the values of the smallest perceptible difference of intensity shown by the curve for 1,000 d.v., it follows that the normal ear, under the most favorable conditions, can distinguish about 400 gradations of loudness between the limits of intensity to which the ear responds. The energy of the loudest audible tone at 1,000 d.v., that is, the energy of a tone which is sufficiently loud to just excite the sensation of pain, is about 10^{12} times as great as the energy of the threshold tone at that frequency. (It was observed that for tones appreciably lower or higher in pitch than 1,000 d.v., the ratio of the energy of the tone which just excited the sensation of pain to the energy of the threshold tone became much less than 10^{12}. This may have an important bearing upon the interpretation of threshold curves which are used for determining the sensitiveness of the ear.)

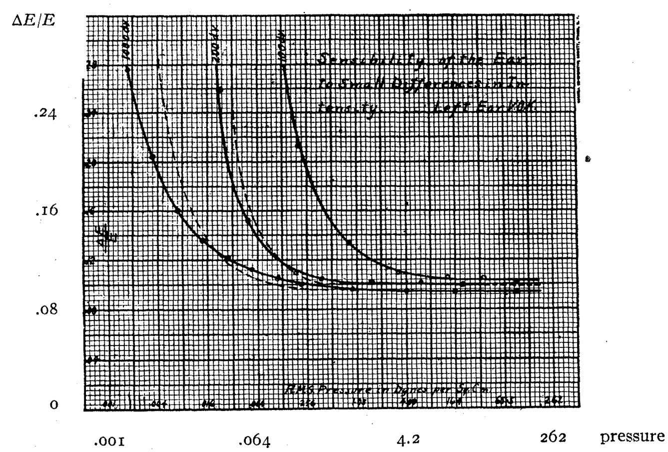


Fig. 4. Intensity Sensibility of the Ear

Fig. 4 shows three of the curves of Fig. 3 with the intensities plotted to an absolute rather than a relative scale. The absolute values of intensity for these curves were computed by giving to the threshold intensities the absolute average values obtained by Fletcher and Wegel[1] from measurements on 41 normal ears. This transformation simply shifts the curves in Fig. 3 along the horizontal axis. As the frequency increases the threshold energy decreases. Consequently of the three curves shown in Fig. 4 the one for 1,000 d.v. is to the extreme left. For decreasing frequency the curves shift to the right.

This group of curves for the ear is strikingly like a similar group of curves for the eye obtained by König and Brodhun and recalculated by Nutting.[2] A reproduction of these curves for the eye, giving the relation between the Fechner ratio for the eye and the intensity of the light, is shown in Fig. 5. It is interesting to note the fundamental agreement of form and position of these two families of curves for the ear and for the eye. The curves for the eye, like those for the ear, shift to the right as the frequency decreases. There is one essential difference between the

[1] The Frequency-Sensitivity Characteristic of Normal Ears, H. Fletcher and R. L. Wegel, Phys. Rev., Vol. 19, p. 553, 1922.

[2] The Luminous Equivalent of Radiation, P. C. Nutting, Bur. of Stds., 5, 261.

Fechner ratios for the ear and eye; namely, the Fechner ratio for the ear approaches the constant value 0.10 at high intensities of sound, whereas the Fechner ratio for the eye approaches the constant value 0.015 at high intensities of light.

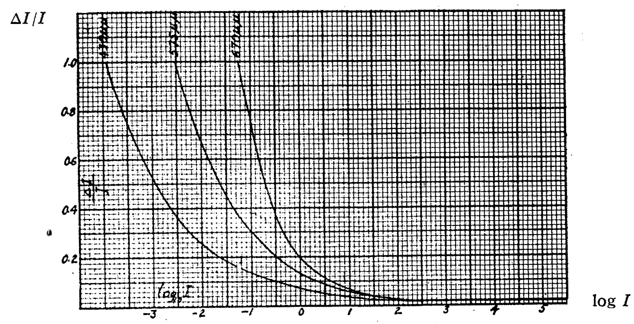


Fig. 5. Fechner Ratio for the Eye as a function of Intensity of Light

2. *Sensibility to Small Differences of Intensity as a Function of Frequency.*—Fig. 6 shows how the sensibility to small differences of intensity depends upon the pitch of tones within the chief portion of the tonal range used in speech and music. The curves are for 19 individual ears from 16 persons. For three persons the curves for both right and left ears are

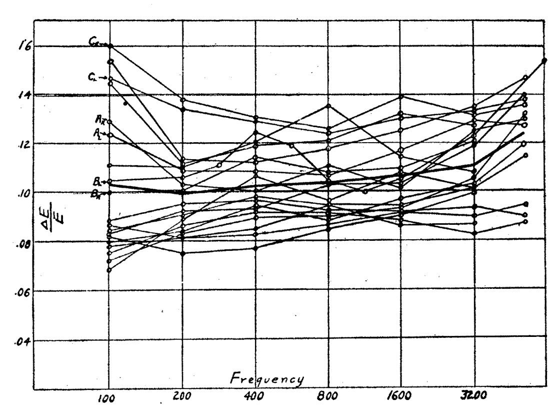


Fig. 6. Intensity Sensibility as a function of Frequency

shown. All of the curves are for people who heard normally well and were experienced in making careful observations. All of the observations were made upon tones of the "same" loudness. The "same" loudness, as here employed, means that the amplitude of the tones used for these tests all bore a constant ratio to their respective threshold amplitudes.

The value of this constant ratio was 100. Such tones are of suitable loudness for making reliable judgments and yet not loud enough to produce serious fatigue effects.

The average curve for the 19 individual ears is indicated by the heavy line. Although the individual curves differ appreciably and in many cases show that the Fechner ratio depends upon the pitch of the tones, the average curve indicates that from 100 d.v. to 3,200 d.v. the Fechner ratio is almost constant, provided that the tones have the same loudness. The value of $\Delta E/E$ over the above-named range is roughly 0.10.

A few measurements were made upon tones of higher pitch than those shown in Fig. 6, but their reliability was questionable because of contact noises which were not entirely eliminated for tones whose frequency exceeded 4,800 d.v. Such measurements as were made, however, indicate that for tones above 5,000 d.v. the Fechner ratio increases appreciably, perhaps to about .20 at 12,000 d.v.

Curves numbered A_R, A_L, B_R, B_L, and C_L are for the right and left ears of observers A, B, and C respectively. These curves indicate that in general the intensity-differentiating mechanisms in a person's two ears are more nearly alike than those in two ears of different persons, although a person's two ears show noticeable differences in this function. See for example the curves for B's right and left ears.

Tests made with binaural hearing showed that within the limits of perceptional error the single ear could distinguish the same percentage change of intensity as could both ears operating together, provided as much energy was fed into the single ear as was fed into both ears. The Fechner ratio for tones of the same loudness was also found to be independent of the mode of generating and receiving the tones. Thus the same value of $\Delta E/E$ was obtained whether the observer listened to a tone produced by the loud speaker receiver, by the double-phone head receivers, or by the König resonators.

The curves shown in Fig. 6 were computed from data which were obtained by the method of minimal change, already described. The observers were required to perceive *only a difference* of intensity and in general were not required to determine the *direction* of the difference. But with a little training it was found that one can nearly always determine the direction of the difference whenever one can perceive that a difference, however small, exists.

To verify this, and also to check the correctness of the data from which the curves in Fig. 6 were obtained, the method of right and wrong guesses was employed for determining the $\Delta E/E$ for five different ears. The two tones whose intensities were to be compared were separated by a time interval of 1/3 second. Each pair of tones to be compared was presented twice, in the same order, the order of presentation having been determined by previously tossing up a coin.[1] The observer was required to decide or guess whether the first or the second tone sounded the louder.

With the telephone receiver as the source of sound, the method of right and wrong guesses applied to tones whose amplitudes were 100 times their respective threshold amplitudes, gave the following values:

Pitch	200 d.v.	400 d.v.	800 d.v.	1600 d.v.
$\Delta E/E$	.118	.115	.113	.106

The same method, but with the receiver in front of a König resonator and the ear coupled to the resonator by means of a rubber tube, gave the following values;

Pitch	220 d.v.	400 d.v.
$\Delta E/E$	.117	.108

These values agree very well with those obtained by the method of minimal change.

3. *Sensibility to Small Differences of Intensity as a Function of Quality.*— To determine whether the purity of the tone affected the value of the Fechner ratio, measurements were taken on two ears for tones of the same intensity and pitch, but of different quality. The quality was varied by changing the plate resistance of the oscillator circuit. As the plate resistance becomes small the wave-form of the generated a.c. becomes flattened and consequently harmonic overtones are effectively introduced. With a plate resistance of 200,000 ohms, the oscillator tube amplifies almost linearly and therefore the wave-form is nearly sinusoidal. With a plate resistance of only 3,000 ohms, the oscillations are not restricted to the linear portion of the characteristic curve of the oscillator tube, and as a result the wave-form of the generated current differs greatly from a pure sine curve.

The following average values of $\Delta E/E$ for a tone of 200 d.v. were obtained for different values of the plate resistance:

Plate resistance	200,000 ohms (pure tone)	50,000 ohms (fairly pure tone)	3,000 ohms (impure tone)
$\Delta E/E$	.114	.112	.112

Similar data were obtained for tones of 400 d.v. The data indicate that the Fechner ratio is almost independent of quality,

[1] The method of right and wrong guesses used in these experiments is essentially the same as that used by C. E. Seashore at the University of Iowa.

4. *Dependence of $\Delta E/E$ upon the Time Interval.*—The method of right and wrong guesses was further used to ascertain how the Fechner ratio varied with the interval of time between the two tones whose intensities were to be compared. Fig. 7 shows how $\Delta E/E$ varied with the time interval, up to 10 seconds, for five persons. The average curve is indicated by the heavy line. Insufficient individual curves were obtained to give reliably the exact form of the average curve, but even as it is—an average of only five ears—it closely resembles well-known parabolic memory curves.

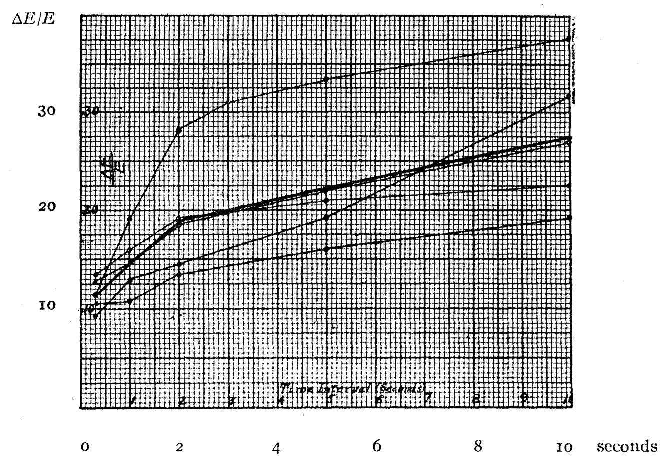


Fig. 7. Variation of $\Delta E/E$ with the Time Interval.

5. *Sensibility to Small Differences of Frequency for Tones of Different Intensities.*—Fig. 8 shows how the sensibility of the ear to small differences of frequency depends upon the loudness of musical tones. The ratio of the smallest perceptible difference of frequency of the tone to the total frequency of the tone, $\Delta N/N$, is plotted against the relative loudness, I_R/I_0. I_R/I_0, as already defined, means the ratio of the test current in the receiver to the current in the receiver required to produce the threshold intensity. The curves in Fig. 8 show that the $\Delta N/N$ depends upon the loudness in nearly the same way that $\Delta E/E$ depends upon loudness.

The error for separate judgments in determining the liminal difference of pitch is about the same as for discriminations of loudness, namely, about 10 per cent. The uncertainty for either pitch or loudness discriminations is greater for very feeble tones. The curves shown are for the writer's left ear only, but data on other ears exhibit the same general properties. The curves show plainly that the resolving power is also different for tones of different pitch. This will be treated more fully in paragraph 7.

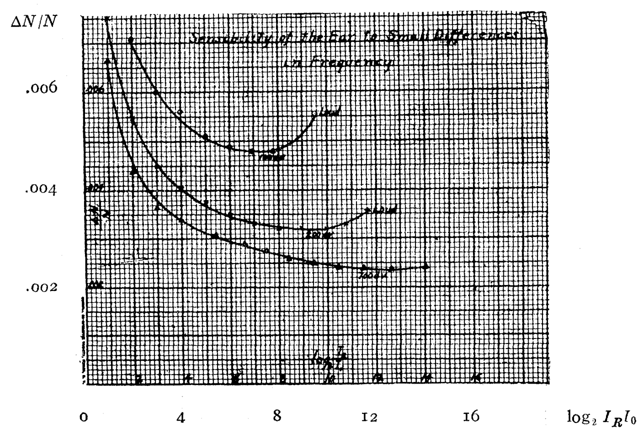


Fig. 8. Pitch Sensibility of the Ear.

6. *Influence of the Mode of Reception upon the Sensibility to Small Frequency Differences.*—The data on pitch discrimination given in the last paragraph are for monaural reception in which the observer holds the single telephone receiver to one of his ears. Tests made on binaural reception indicate that a person's two ears are better than a single ear for appreciating small differences of pitch. The results of some measurements comparing different modes of reception are shown in Fig. 9. Curves I. and II. show that the resolving power for the observer's left ear is nearly the same as for his right ear. Curve III., obtained with

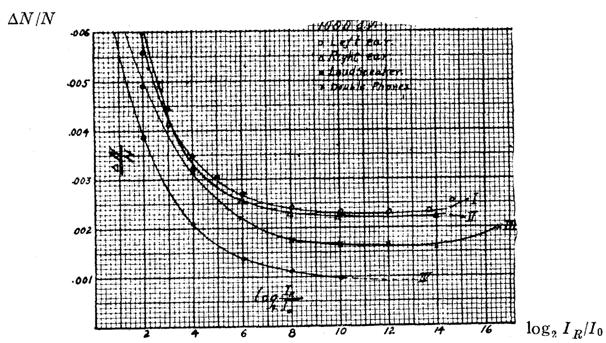


Fig. 9. Influence of Mode of Reception on Pitch Sensibility

double-phone receivers held to the two ears of the observer, indicates that binaural reception enables one to resolve smaller differences of pitch than monaural reception does. Curve IV., obtained with the loud speaker receiver as the source of sound, apparently indicates that this mode of reception yields a higher resolving power than either monaural or binaural reception with the receivers held to the ears. However, near the barely perceptible change of pitch, it is difficult to determine whether the difference is one of pitch or of loudness. One is prone to judge a slightly louder tone as a tone of slightly higher pitch, and vice versa. It was suspected therefore that the extraordinary results indicated by Curve IV. might be partially attributed to the interference pattern surrounding the ears. Such an interference pattern, caused by reflections from obstacles in the room, would produce at the ears of the observer slight differences of intensity of the two tones compared which at the receiver differed only in frequency. Tests showed that the interference pattern actually did affect the judgment of liminal differences of pitch. Thus at 1,000 d.v. the smallest perceptible difference was first found for binaural reception, using the double-phone head receivers as a source of tone. Then that same difference was produced with the loud speaker receiver. The difference, and in general the direction of the difference, were then clearly perceptible. However, if the observer moved his head to different positions in the room there were definite places where the difference of pitch was no longer perceptible, and still other positions where the direction of the difference was judged reversed, that is, where the graver tone seemed the higher. Hence, where stationary waves are produced, the method of liminal change does not give the true resolving power of the ear, but gives a hybrid resolving power which exceeds the true value because the difference of pitch as heard by the ears of the observer is accompanied by a difference of intensity—the magnitude of the latter depending upon the nature of the interference pattern surrounding the head.

The writer was unable to find any reason for explaining the departure of Curve III. from I. and II. unless it be assumed that binaural reception actually yields a higher resolving power than monaural reception. If such a difference as that shown in Fig. 9 does exist, a careful study of the problem may be helpful in determining the nature of the pitch-differentiating mechanism in the ear.

7. *Sensibility to Small Differences of Frequency for Tones of the Same Loudness but of Different Pitch.*—Fig. 10 shows how $\Delta N/N$ varies with the frequency for 15 different ears from 12 persons. The amplitude of vibration for each test tone was 100 times its threshold amplitude. The tones were produced in the single telephone receiver which was held against the observer's ear. The average curve indicates that $\Delta N/N$ decreases from about .01 at 50 d.v. to about .003 at 600 d.v. From 600

d.v. to 3,200 d.v. it remains at the constant value of about .003. Thus for the intensity of tones employed for these tests the average single ear can just appreciate a difference of 0.5 vib. at 50 d.v., 0.66 vib. at

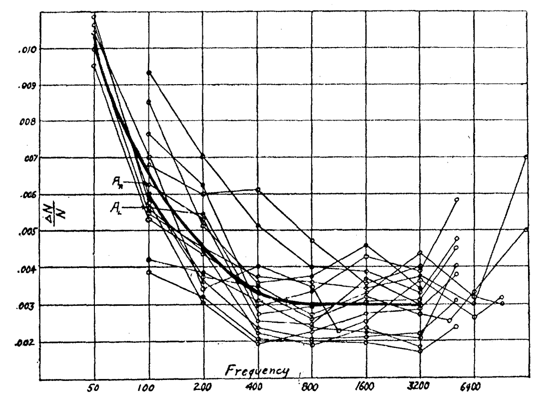


Fig. 10. Variation of Pitch Sensibility with the Frequency

100 d.v., 1.6 vibs. at 500 d.v., 3.0 vibs. at 1,000 d.v., and 9 vibs. at 3,000 d.v. This does not mean that the average untrained ear can distinguish which of the two compared tones is the graver for such small differences. The observers were required to determine *only* a *difference* of pitch. The trained ear can determine the correct direction of the difference for much smaller intervals than the untrained ear. Thus, tests made upon two well-trained musicians showed that as long as they could perceive a difference of pitch of two tones they could also determine which one of the two was the graver. Other people with but little musical ability or training could distinguish differences nearly as small as the trained musicians but these people were not at all certain about the directions of the differences near their just perceptible values.

V. Discussion of Results.

1. *Comparison of Data with the Results of Previous Investigators.*—The numerical values of $\Delta E/E$ presented in the last section agree fairly well with Wien's value at 440 d.v. and with Deenik's organ pipe values between c_2(261 d.v.) and c_5(2,088 d.v.). Deenik's higher values of $\Delta E/E$ for tones below c_2 may be partially attributed to his working with relatively feeble intensities for the graver tones. The results given in the present paper indicate that $\Delta E/E$ is more nearly independent of the frequency than is indicated by the results of Wien or Deenik.

The data for the resolving power of the ear given in this paper more nearly accord with Stücker's average values than with those of other investigators. They are in good agreement with Vance's data for tones above 256 d.v. The discrepancies among other investigators, it seems to the writer, can be reasonably attributed to inaccuracies in measuring the frequency, to the presence of higher partial tones, to the differences of loudness of the tested tones, to the presence of sound interference patterns, and to the different time intervals between the two tones whose frequencies are compared.

2. *Modification of the Weber-Fechner Law.*—About sixty years ago G. T. Fechner expressed Weber's law in the mathematical form $\Delta E/E =$ constant. He assumed that the constant $\Delta E/E$ was proportional to the corresponding increment in sensation, ΔS, that is, $\Delta E/E = C\Delta S$. He further assumed that ΔE and ΔS were true differentials, and integrating the above expression he obtained the important law of psycho-physics, namely, that the sensation is proportional to the logarithm of the exciting stimulus. Nutting[1] has shown that this is subject to serious defects, and in the case of light sensation has proposed the following form for expressing the Weber-Fechner law,

$$\Delta I/I = P_m + (1 - P_m)\,(I_0/I)^n. \tag{5}$$

This applies quite satisfactorily to low as well as high intensities. $\Delta I/I$ is the Fechner ratio, P_m is the minimum value the $\Delta I/I$ approaches at high intensities, I_0/I is the ratio of the threshold intensity to the total intensity of the stimulus, and n is an arbitrary number depending upon the wave-length.

The $\Delta E/E$ for sound perception can be expressed by a similar equation,

$$\Delta E/E = F + (1 - F)\,(E_0/E)^n, \tag{6}$$

where F is in the constant minimum value the $\Delta E/E$ approaches at high intensities, and E_0/E is the ratio of the energy of the threshold tone to the energy of the tone producing the sensation. This equation satisfies the terminal conditions, for at the threshold $E_0/E = 1$ and therefore $\Delta E/E = 1$, and at very high intensities E_0/E is very small and therefore $\Delta E/E = F$. The dotted curves in Fig. 4 were obtained by plotting the graphs of (6), in which n was the average value resulting from the substitutions of observed values of $\Delta E/E$, F and E_0/E in equation (6). For the 1,000 d.v. curve $n = 1.05$ and for the 200 d.v. curve $n = 1.63$.

The similarity of the sensibility curves for the eye and the ear shown in Figs. 4 and 5 suggests the need of determining the exact form of the

[1] The Luminous Equivalent of Radiation, P. C. Nutting, Bur. of Stds., 5, 261. See also: Sur L'interpretation de la Loi de Weber-Fechner, V. Henri and J. L. des Bancels, C. R. de la Soc. de Biol., 72, 1075–78.

corresponding curves for other sensations, as taste, smell, pressure, warmth, and so forth. If all of these should prove to be similar to the curves for the sensation of light and sound, it might be possible to formulate a general relation between stimulus and sensation for stimuli of all intensities which affect our senses.

The writer is indebted to many of his colleagues at the Ryerson Physical Laboratory who assisted in contributing the data. He wishes to acknowledge the helpful suggestions received from Prof. H. G. Gale, Prof. A. C. Lunn, and Dr. John P. Minton. Finally, he wishes to thank Mr. Walter H. Merrymon, whose coöperation greatly facilitated the experimental work.

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