THE EFFECT OF TEMPERATURE ON GRAVITATIVE ATTRACTION

By P. E. Shaw and N. Davy

Abstract

Variation of gravitational attraction with the temperature of the larger mass, 20° to 250° C.—It is invariably assumed in computing the masses of the heavenly bodies that the temperature effect is zero; but except for the work of the authors, we have no direct evidence for this assumption, since it must be allowed that the results of balance experiments in which the temperature of the smaller mass is varied up to 100° only are inconclusive. In 1915 a laborious series of experiments was performed with a Cavendish torsion balance, which indicated a positive temperature coefficient for the gravitational constant, of 10^{-5} per degree. Recently the apparatus has been changed by substituting a more rigid support, iron girders for the former wooden scaffolding, and by providing better thermal insulation between the heated outside large masses and the vacuum chamber containing the smaller masses. The result of eleven series of readings is a mean difference (hot minus cold deflection) of $-$ .05 mm in 208 mm, or 1/40 per cent on a range of 200°. While most of the differences are negative, two positive results were obtained; and since the observational error is of the same order as the result, the conclusion is that the effect, if it exists, is less than 2×10^{-6} per degree, and may well be zero. The previous result is explained as probably due to a slight change in the relative positions of the large and small masses, reversible with the temperature change.

Introduction

Some years ago one of us published an account of experiments on this subject.[1] The results shown in that paper seemed to indicate that the attraction between masses depends, under certain conditions, on their temperature. The ultimate result attained in that research was that, on raising the temperature of the large external masses M, M, of a torsion balance system in which the small internal masses m, m remain at constant temperature, the attraction of the large on the small masses increases by about $1/10^5$ per 1° C.

In a later paper [2] it was shown as the result of further work that the above numerical result is increased about 8 per cent when allowance is made for expansion of the beam carrying the large masses. The above

[1] Shaw, Phil. Trans. (May, 1916). The text of this paper unfortunately contains a few errors of an important nature. For these we append the following errata: p. 356 Equation III should read: $\theta = CM^{1/3} \cdot D^{2/3} T^2/a$; p. 357 (line 9) should read: Sensitiveness $\propto$ R.D.; p. 357 (line 10) should read: Radius and Density are of equal importance; pp. 373, 375, 380: The oscillation periods should be 252, 280, 280 seconds (not minutes).

[2] Shaw and Hayes, Proc. Roy. Soc. (April, 1917).

temperature effect was obtained after eight years of work, mostly spent on overcoming the technical difficulties peculiar to the experiment.

This result is certainly important if true. Thus, the masses and densities of the earth, sun and planets and other heavenly bodies are all derived by supposing these bodies, though at very high mean temperatures, to have the same gravitative pull as if their temperatures were those of the masses used, say, in a laboratory in a Cavendish torsion balance at 20° C. And although in the experiments about to be described the temperature only rises to about 250° C at the most, whereas the mean temperature of the sun and stars may be reckoned in four or five figures on the Centigrade scale, yet any evidence of a temperature effect found at low scale temperatures would at once invalidate the commonly quoted figures for mass and density, and we should have to regard them at best as "effective" and not true values. In that case, since we can never deal experimentally with these very high temperatures, we could never hope to know the true masses of the heavenly bodies.

The *nil* effect obtained by Poynting and Phillips[3] and that of Southerns[4] do not directly affect the issue. They are both "weight" experiments made with a gravity balance and therefore directly deal with the small mass only, whereas the present research deals with the large mass. But even if it be claimed that these weight experiments provide an *a priori* proof that there is no temperature effect, this claim can only be made for temperatures up to 100° C, which was the upper limit used in these researches. Therefore we have no experimental warrant apart from the present series of researches for considering the Newtonian constant of gravitation to be unaffected by rise of temperatures above 100° C; and the present work only carries us up to 270° C.

Method and Apparatus

The general methods of the present research are identical with those employed before and the materials also are unchanged. But the supports and the suspending systems of the large masses M, M' and of the small masses m, m' are entirely different; they are stronger and more massive. It is clear from a study of the old experiments that even minute movements occurring during an experiment of the small masses relative to the large ones might cause spurious effects of the order of the small temperature effect found. So it was felt that in repeating the work special care should be taken to provide rigidity and immobility in these particulars. The changes introduced have greatly improved the apparatus and enabled a higher order of accuracy to be reached, and

[3] Roy. Soc. Proc. A, **76** (1905).
[4] Roy. Soc. Proc. A, **78** (1906).

it is now found that the temperature effect is less than 2×10^{-6} and may well be zero.

Of the apparatus and processes used only a brief account is required as they are similar to those given in detail in the former paper,[1] to which the reader is referred for a further account of methods and precautions employed. Stated in general terms, the object of the research is to measure the change in attraction, if any, occurring between two solid masses, one large and one small, when the temperature of the large one is raised. A torsion balance of the Boys-Cavendish type is employed. The torsion system carries the small masses in a vacuum, outside which are the large masses. Between the large and small masses screens are arranged to stop the flow of heat to the inner system from the large masses when these are raised in temperature.

The Inner System.—Figs. 1 and 2 are side and front views of the main part of the apparatus. In the center of Fig. 1 is the vacuum tube *VV* which is 120 cm long and 5 cm in diameter, and contains the torsion balance with its small suspended masses *m*, *m′*, hanging by separate wires, 32 mm apart. *VV* is suspended inside the sheath *S* by a cranked iron bar *D* supported on and swivelling in sleeves on a board *T*. This board is heavily loaded with lead *L* and is supported by 16 steel springs whose upper ends are attached to an iron frame *F* which is bolted in four places to the pair of stout angle irons *AG*. These irons stretch across the width of the vault and are recessed into the holes in the brick walls. They are heavily loaded and rest on steel balls which are imbedded in rubber pads *P*. Thus the angle irons and their attachments form a "floating" system free in part from the vibrations of the walls. The board *T* and the vacuum tube suspended from it in cantilever fashion form an *inner* floating system whose lateral movements are checked by rubber pads *R* resting on the angle irons while the tube *S* resting on the tripod *Tr* constitutes an *outer* system.

The lower end of the vacuum tube is firmly socketed into a brass tube which penetrates a brass ring *K* in which are four set-screws to adjust the tube to a plumb position and hold it there. The ring *K* is clamped to two piers of cemented bricks *Br* resting on concrete slabs *C*. The whole masonry base weighs 500 lb and rests on rubber blocks *Ru*. Thus the supports of the vacuum tube and its contents both above and below consist of heavy floating systems. In this way the torsion balance is well guarded from the tremors of the building and ground. These special anti-tremor devices are required by reason of the location of the apparatus in the University College in the center of Nottingham and near a trunk railway. In using an asymmetrical system such as this torsion balance, which has the small masses at different heights, any tremors when once received are converted into pendular movements of two frequencies and thence into torsional vibrations. These vibrations are long retained but die away gradually by the viscous drag of the residual air in the vacuum.

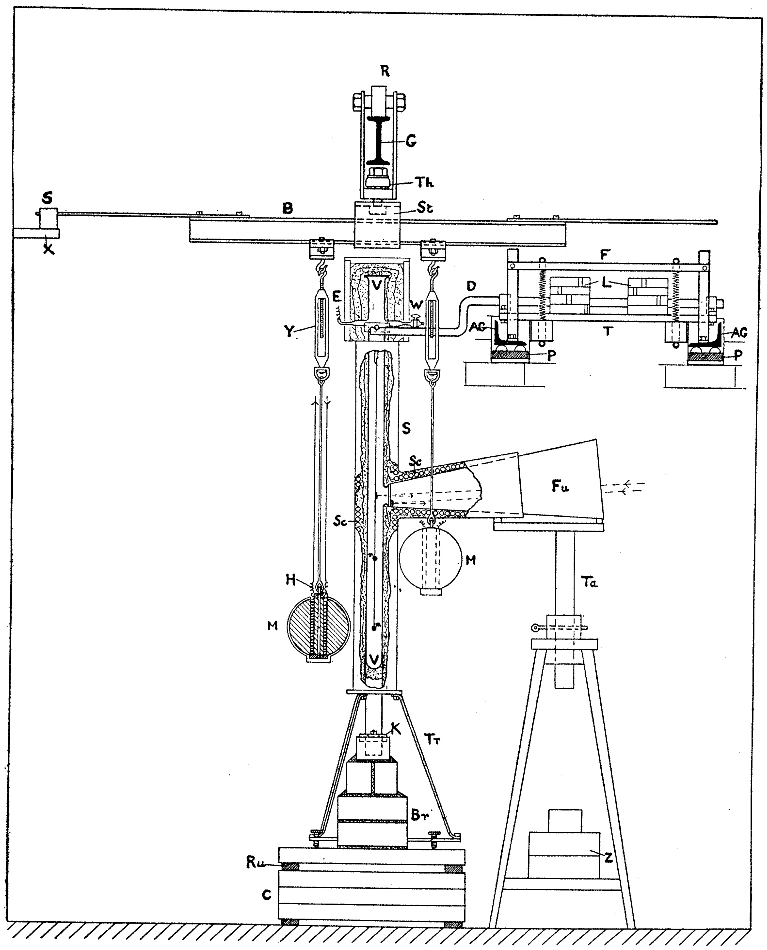

Fig. 1

At the top of the vacuum tube VV are seen the wire E, for earthing the inside of the tube, and the tap W, which is connected by tube VT, Fig. 2, to the vacuum pump and manometer. The vacuum tube is surrounded by cotton wool lagging, helical rubber water tubes, and polished copper sheath, as shown in Fig. 1, the whole forming a wall all round the vacuum tube, 4 cm thick, designed to maintain the temperature in the vacuum constant even when the large external masses M, M' are at 270° C. The water tube system *Sc* surrounding the vacuum is shown; its action was explained in the former paper; the triple inlet *IW* and outlet *OW* are shown in Fig. 2. The funnel *Fu* which guards the window of the vacuum tube from heat given off by the large masses M, M' is also thoroughly protected from heating, by lagging, water tubes, and copper sheath. In order to prevent passage of air into the funnel its mouth is covered by a card with a rectangular hole V through which the two mirrors can be seen from the telescope. The funnel is supported on the table *Ta* which for stability is loaded at Z; and the vacuum sheath rests on a brass tripod *Tr* which stands on the cement slabs C.

The Outer System.—The large masses M, M, of lead, each of 47 kg, are carried with their heating coils H by rods and adjustable turn-screws Y attached to a girder beam B (3″ × 3″). This beam rests in a stirrup *St* and is supported from the wheel R which travels along the girder G (3″ × 6″). G stretches across the vault and its ends rest in recesses in the side walls and are there bricked in solidly. These girders and other supports are designed to carry a load of 300 kg; and in order that the rotation at *Th* and the to and fro movement of R on the girder G may be easy and smooth throughout an experiment, the former is provided with a thrust ball-bearing *Th* and the latter with a journal ball-bearing surrounding its axle. The wheel R can be brought to any desired position on G and rigidly clamped there by girder clips Q_1, Q_2, one on each side of R. Suppose that the rotating center *Th* has been brought accurately over the torsion head in the vacuum tube; then the masses M, M can be swung on B coaxially round the inner masses m, m, until the A, B positions of maximum couple are attained (see Fig. 3). Stops S are provided on a bracket X which is fixed on the end wall of the vault, and extension bars shown on the beam B are brought to these stops and are clamped there.

This system has mobility. It is possible during an experiment to unclamp either Q_1 or Q_2, rotate the beam B and run wheel R and all depending from it to one end of girder G, and then bring up an alternative system R' into position over the vacuum tube.

For further elucidation of the apparatus two photographs are given. Fig. 3, Front view as seen from one side of the telescope; Fig. 4, Back view, the telescope and scale being at the far end of the vault. The angle girders which carry the inner vacuum systems are seen prominently high up, Fig. 3. The Gaede pump and manometer, as also water pipes passing to and from the sink, are seen in both photos. In order to obtain clearer views, some outer wrappings have been removed from the

Physical Review, Vol. 21, Second Series,
March, 1923.

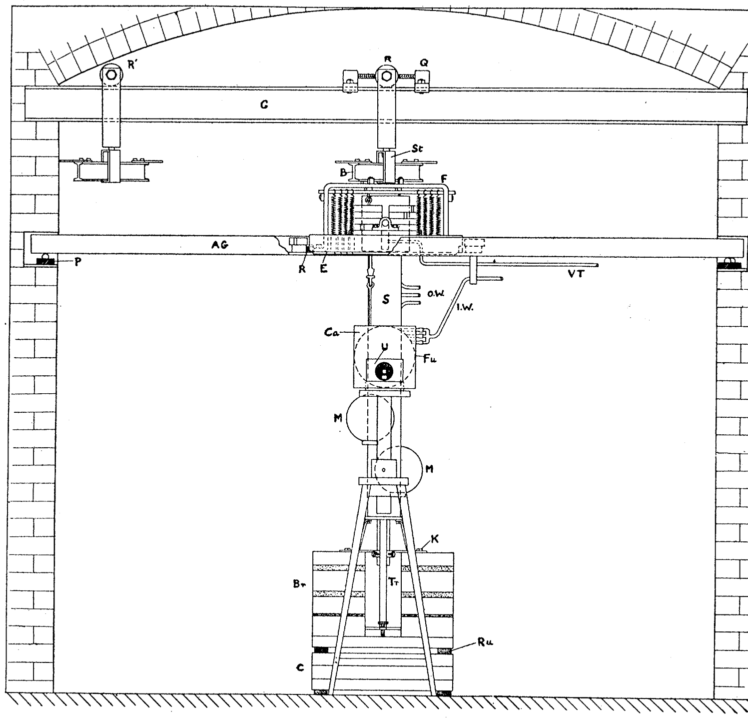

Fig. 2.

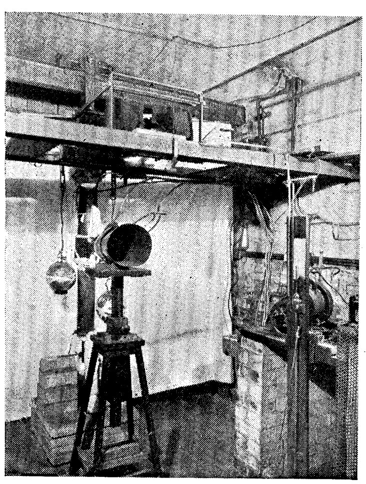

Fig. 3

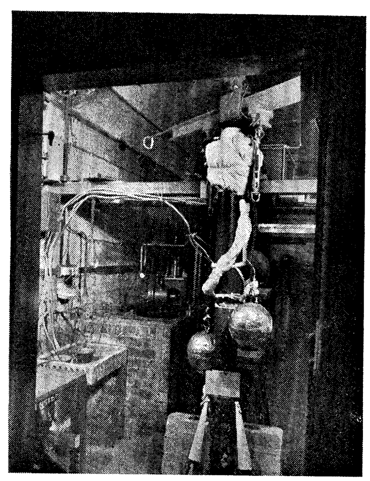

Fig. 4

P. E. SHAW AND N. DAVY.

funnel and its supporting table. The two white surfaces seen so prominently in Fig. 4 are cotton wool lagging introduced for protection, respectively, of the top of the vacuum tube and of an exposed water pipe which passes round the sheath of the inner system. The scale used for the telescopic readings is 6 meters from the inner system. It is seen between the two bright horizontal surfaces at the far end of the vault, with the telescope field lens showing bright below it. The height and width of the room, as seen in Fig. 2 are each 10 ft.

To the right of, and 6 meters from the torsion balance, but not seen in Fig. 1, are the telescope and the scale graduated in mm. As explained in the former paper it is desirable to have two mirrors; one fixed to the outside of the vacuum tube, another in the usual way on the beam of the torsion balance. The former enables us to make correction for rotation, if any occur, of the vacuum tube. By deducting any rotation of the former mirror from that of the latter we obtain the true net rotation of the torsion beam. In the present arrangement both mirrors reflect light from *the same* scale whereas formerly the two mirrors reflected light from different scales into the telescope. Thus in the present arrangement we eliminate one link in the optical arrangement and avoid any error due to possible displacement of the two scales relative to one another.

In Fig. 5 we have a plan showing the vacuum tube with beam and

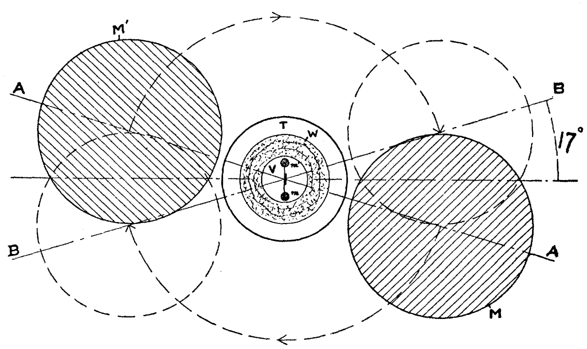

Fig. 5.

small masses m, m' surrounded by cotton wool W and the water tube space T. The large masses M, M' are shown set at the A position of maximum influence, 17° from the normal to the torsion beam. When the B position is required, the masses M, M' are swung round coaxially with V, as shown by the arrows. The shading of the masses is meant to show that M and m are in one horizontal plane whereas M' and m' are in another.

Experimental Results

The experiments were conducted and tabulated just as described in the old paper. The readings, in full, of one typical experiment are given in Table I. Columns 2 and 3 show the azimuth mirror readings of the vacuum tube and the beam respectively. In column 3 are shown the extreme scale readings as the beam swings to and fro. These are in sets of three, each set containing A and B positions alternately (see Fig. 5). The error of a single reading is generally only 0.1 mm but may be on occasions as much as 0.2 mm. Most of the latter error is due to the fact that the scale as seen in the telescope is generally quivering. In column 4 are entered the equilibrium positions Z, as found from column 3 by the expressions used in the previous research:

$$Z = C - \frac{(c - b)^2}{(a - b) + (c - b)} \qquad \text{or} \qquad Z = \frac{ad + b}{1 + d},$$

where Z = equilibrium position; a, b, c are successive scale readings, and d = decrement. The results in brackets, column 4, are obtained from those immediately above by correcting for the tube readings, column 2. The last column gives the temperatures of the lead spheres M, M' as the experiment proceeds.

A glance at column 2 will show how steady is the reading of the tube azimuth, there being only 0.1 mm variation during the cold readings and 0.2 in the hot. In the old experiments this variation amounts to 0.5 mm or more. This rotation of the vacuum tube is as we have seen not in itself a source of error but it suggests that the vacuum tube has too great a degree of freedom relative to the large masses M, M'. And the movement of the outer system relative to the inner, brought about by heat, may have introduced errors in the results of the former experiment. This will be discussed further below.

Summarized results of the experiments are shown in Table II. In column 2 for dates 8/2 and 8/4 are shown two experiments *C–H* in which readings were taken with the coils in the lead spheres heated so that in those cases the temperature is rising during the experiment. It is more satisfactory to have temperature falling and no heating current passing, and this is the usual method; but it will be observed that the results for these two *C–H* experiments are not materially different from the other nine given in column 6.

To explain Table II., take as an example of cold and hot experiments those of date 7/30. For the first the temperature is 18°; for the second it falls from 230° to 140°. Seven results are found for the cold equilibrium position whose mean is 207.06 mm on the scale. Twelve results are found for the hot with mean 206.98. The mean difference in range is − 0.08 mm. The minus sign indicates that the gravitative attraction between the masses is less when the masses M, M' are hot than when they are cold. In column 7 we see that the differences in equilibrium positions are not more than 0.3 mm cold and 0.3 mm hot. These are a measure of the accuracy with which scale readings can be taken in cold and hot experiments respectively, as the image of the scale moves to and fro in the field of the telescope.

TABLE I

A Typical Hot and Cold Experiment

Cold Readings					
1	2	3	4	5	6
Time	Tube readings mm	Torsion beam readings mm	Z Equilibrium position mm	Range mm	Temperature of M, M'
12.10 p.m.	208.5	280.5 508.5 355.5	416.95		18° C
				206.95	
		788.7? 513.4 698.0	623.9		
				207.0	
		277.1 510.8 353.8	416.9		
				207.2	
		788.4 513.9 698.0	624.1		
				207.2	
	208.6	252.3 527.6 342.9	417.0 (416.9)		
More water run in the tubes than this morning					
3.12 p.m.	208.6	253.5 527.1 343.4	417.2 (417.1)		18°
				207.0	
		792.1 511.7 699.5	624.2 (624.1)		
				207.0	
		265.0 519.1 349.1	417.2 (417.1)		
				207.0	
		510.4 701.1 572.3	624.2 (624.1)		
				207.06	Mean

Hot Readings

Same water flow as this afternoon

1	2	3	4	5	6
Time	Tube readings mm	Torsion beam readings mm	Z Equilibrium position mm	Range mm	Temperature of M, M'
8.25 p.m.	208.7	330.9			
		475.5	417.3		
		378.1	(417.1)		
	208.6	777.3			
		521.7	624.2	206.9	220°
		692.9	(624.1)		
8.40 p.m.	208.75	254.5?	417.2		
		526.2		207.1	213°
	208.8	344.2	417.3		
		466.5	(417.0)		
				206.95	200°
	208.7	749.1			
		540.0	624.1		
	208.6	680.7	(623.95)		
				207.15	194°
	208.8	264.5			
		519.5	417.1		
		348.4	(416.8)		
				206.9	188°
	208.7	781.8			
		518.1	623.9		
		694.8	(623.7)		
				206.85	180°
	208.8	266.3			
		518.6	417.25		
		349.2	(416.95)		
				206.85	172°
	208.7	517.5			
		695.4	623.9		
		575.9	(623.7)		
				206.9	165°
	208.8	291.0?			
		502.0	417.1		
		359.0	417.1		
		455.7	(416.8)		
				207.1	152°
	208.75	757.8?			
		534.3	624.1		
	208.7	684.5	(623.9)		
				207.1	147°
9.55 p.m.	208.8	263.7			
		520.0	417.1		
		348.0	(416.8)		
				206.95	140°
	208.75	775.6			
		522.5	624.0		
		692.1	(623.75)		
				206.98	Mean

The oscillation period between consecutive readings, column 3, is 290 sec.

TABLE II

Summarized Results of Experiments

1	2	3	4	5	6	7	8
Date 1921	Nature of experiment *C*=cold *H–C*= from hot to cold	Temperature of large spheres *MM'*	Number of ranges taken	Mean range mm	Difference in mean ranges hot–cold mm	Extreme variation in equilibrium position mm	Extreme variation in tube readings mm
7/21	*C*	18°	12	208.43		.2	.45
7/22	*C*	18°	12	208.40	−.04	.2	.4
7/23	*H–C*	200°–130°	11	208.37	−.07	.4	.4
7/24	*C*	17°	7	208.46		.3	.4
7/25	*H–C*	210°–120°	11	208.38	−.07	.6	.2
7/26	*H–C*	240°–160°	8	208.40		.6	.2
7/30	*C*	18°	7	207.06	−.08	.3	
	H–C	230°–140°	12	206.98		.3	.2
8/1	*C*	18°.5	6	207.09	−.01	.3	
	H–C	250°–150°	12	207.08		.2	.0
8/2	*C*	18°	11	207.19		.4	
					−.10	.25	.1
	C–H	75°–160°	12	207.09		.6	
8/4	*C*	20°	7	207.11	−.08	.3	
	C–H	20°–220°	8	207.03		.3	.15
	H–C	230°–130°	13	207.19	+.09	.5	
8/6	*C*	19°	6	207.38		.3	
	H–C	215°–120°	10	207.40	+.02	.4	.0
8/7	*C*	19°	6	207.72	−.12	.1	
	H–C	220°–130°	11	207.60		.4	.2
8/10	*H–C*	230°–120°	7	207.53	−.10	.4	
	C	19°	6	207.63		.3	.15
				Mean	−.05		

Notes: (1) The vacuum for the first six dates was 10 mm. It was let down to 20 mm on date 7/30, and further to 25 mm on date 8/7. Apparently any vacuum between these limits is suitable.

(2) The water flows through the rubber tubes round the vacuum at the rate of about 1 litre per minute. The flow must be commenced at least two hours before the commencement of an experiment and is maintained constant until the experiment is over. The temperature of the water flow will then remain constant to 0.1°.

Now consider Table II. as a whole. The last column in Table II. shows that except in the first few experiments the scale, telescope, and vacuum tube are very nearly at rest relative to one another, since the extreme variation is not much greater than the smallest perceptible movement, 0.1 mm. Number 6 is the most important column as it shows the magnitude of the temperature effect, if any exist. These eleven results have values of 0.12 mm or less. Two have positive values, nine have negative, the algebraical mean result of all eleven experiments being − .05 mm. The temperature ranges are not the same in all the experiments but this need not be considered in taking the final mean. The result − .05 mm seems to be an indication of a small temperature effect.

Taking the figures of this experiment we find that the probable error of the cold experiment is ± .03 mm and that of the hot experiment ± .02 mm, and the probable error of the mean of the whole series of experiments is less than .01 mm of scale.

TABLE III

Observed differences in ranges, hot minus cold, in mm.

Old series: + .58, .51, .48, .45, .42, .40, .38, .36, .32, .30, .28, .26; mean + .40.
New series: − .12, .10, .10, .08, .08, .07, .07, .04, .01, + .02, .09; mean −.05.

For comparison, the old and the new series of results for difference in range, hot–cold, are given in Table III. This shows clearly that the new results are more consistent than the old; and that the temperature coefficient, instead of being 1.2×10^{-5}, as given by the previous work, is certainly less than 2×10^{-6} and, in view of the observational errors, may well be zero.

It can be readily calculated that a small displacement of M, M' relative to m, m' of only 1 mm in a horizontal direction perpendicular to the line between them, or of 4 mm in a vertical or radial direction, if reversible with temperature, might give rise to an apparent gravitative temperature effect of the amount found in the earlier experiments. The old apparatus was carried by a stout wooden scaffolding firmly secured to the cement floor and vaulted roof and was crossed-braced with stout scantling. The scaffolding surrounded the apparatus, and though cards were used to screen heat from all wooden parts, the latter probably yielded during the heat experiments.

The apparatus has now been left intact as recently used; so that the work can be extended at any future time. Small modifications in it might be applied with advantage, chiefly with the view of confining the freedom of the outer and inner systems each to one degree, viz., rotation on a vertical axis. A serious difficulty which will be encountered in any attempt to attain a higher order of accuracy is the vibration of the torsion system. As this cannot be overcome by providing more freedom to its semi-floating supports and as the vacuum cannot be let down so as to damp out vibrations as they arise, there only remains the (unpleasant) course of conducting all standard experiments in the least vibrational period of the day, *i.e.*, between midnight and 4 a.m. One of us has already burnt much after-midnight oil on this arduous work. The removal of the apparatus to some spot in the country remote from town vibrations is a luxury denied to those whose research must be done, if at all, in the town in which they work and in the College in which there exists suitable equipment, including work-shops.

These experiments and those published in 1915 are the only attempts to conduct the Cavendish gravitation experiment with hot bodies. They have shown conclusively that such gravitation experiments can be performed with high accuracy provided the torsion balance is in a vacuum and is surrounded by a protective screen of flowing water and non-conducting material. Contrasting the methods of the recent experiments with those of seven years ago, we see that there is no difference in the heating or the heat-screening arrangement or in the vacuum or its contents. This proves that the old effect which now appears spurious was not caused by heat trouble but probably by mechanical defects which we have now removed, and that our heating arrangements have all along been satisfactory.

We wish to record our great obligation to the Royal Society for a grant in aid of the research.

University College,
Nottingham, England,
November 10, 1922

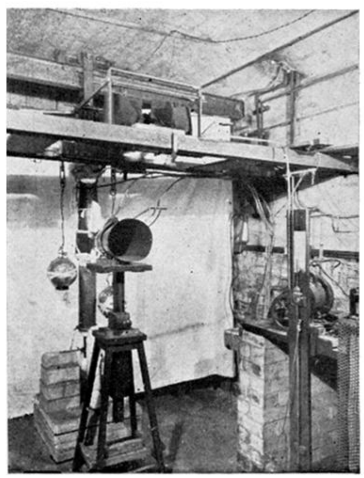

Fig. 3

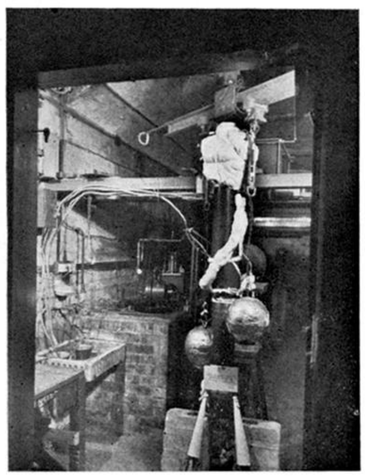

Fig. 4