MINIMUM INTENSITY FOR AUDITION

By Frederick W. Kranz

Abstract

Minimum intensity of sound for audition for frequencies of 128 to 4,096 p.p.s.—(1) *Thermophone as source.* The receiver was held to the ear like a telephone receiver and was actuated by current from an oscillator, the threshold intensity being calculated from the corresponding current by Wente's formula. Tests of 14 ears for frequencies an octave apait show logarithmic sensitivities lying between the results of Wien and those of Fletcher and Wegel, the mean values increasing from 5 for 128 p.p.s. to a practicallv constant sensitivity of about 8.5 for the range 512 to 4,096 p.p.s. (2) *Telephone receiver as source* was then tried to see whether results nearer Wien's would be obtained. Two methods of calibration of diaphragm amplitude as a function of current were used, one involving the use of a microscope to magnify the motion of a quartz fiber attached to the diaphragm, the other amplifying the motion by means of a rocker which rotated a light mirror, both, however, involving extrapolation from relatively large amplitudes to those of the order of 10^{-9} cm. The results for two listeners agreed in showing a somewhat lower sensitivity with the telephone than with the thermophone as source, the energy ratio increasing from about 2 for higher frequencies to 7 for 128 p.p.s. This difference may be due to the extrapolation used in reducing the readings. The results of the two methods show a much better agreement with each other than with the results of Wien.

Suggested number scale for auditory sensitivity. Logarithmic sensitivity is defined as the $\log_{10}(1/J_0)$, where J_0 is the threshold intensity in ergs per cm^2 per second. The advantages of such a scale are pointed out.

IN a previous paper by the author [1] a method was described by which determinations were made of the minimum sound intensity necessary for audition. The present paper is to give further results by this method and to present check measurements by a second method.

I. Thermophone as Source of Sound

The thermophone used consists of a thin strip of platinum foil about 0.0002 cm in thickness, mounted in a small telephone receiver case which was held tightly to the ear of the observer, thus forming a closed cavity containing the platinum strip and having the ear drum as one of the boundary walls. The alternating current flowing through the strip causes periodic variations in the temperature of the strip, thus heating the adjacent air, which periodically expands and produces pressure changes in the cavity. Thus there are no moving mechanical parts. The foil was also heated by a direct current of one ampere, the alternating

[1] Phys. Rev. **17**, p. 384 (1921).

current and direct current being kept separate by condensers and inductance coils, the latter being about one henry each.

Because of the inefficiency of the thermophone, it was possible to measure directly the alternating current going into the thermophone by means of a thermo-couple and micro-ammeter, thus avoiding any uncertainties in reduction calculations. A resistance equal to that of the thermo-couple was placed in the line to keep the two sides of the electrical circuit balanced, and similarly the condensers and inductances were symmetrically placed as were the resistances used in the control of the current. The circuit arrangements are shown in Fig. 1.

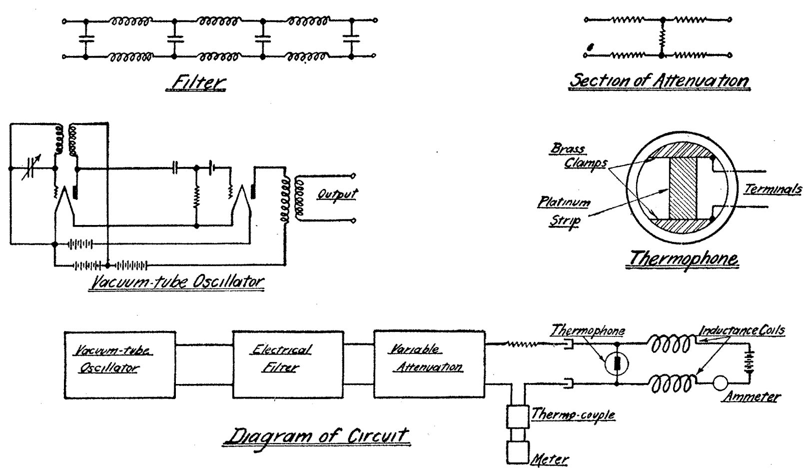

Fig. 1

The thermophone as used was held tightly against the ear so it can be regarded as producing pressure changes in a closed cavity, and the energy may be calculated from these pressure changes if the wave-length of the sound used is large compared to the dimensions of the enclosure. The formula for the amplitude of the pressure change in terms of the constants of the thermophone, as given by Wente,[1] was used. For the temperature and area of foil, volume of cavity, and the frequencies used in this experiment, this formula may be considerably simplified.

It becomes

$$\delta p = \frac{0.478 R I_0 i}{\sqrt{2}\,\dfrac{V_0 \alpha T_a}{2 p_0}\left(1 - \dfrac{k-1}{k}\dfrac{\theta_0}{T_a}\right)\left[(\gamma\omega)^2 + 8(\alpha\kappa)^2 + 4(\gamma\omega)(\alpha\kappa)\right]^{1/2}}$$

[1] Phys. Rev. **19**, p. 336 (1922).

and substituting the constants of the air and the platinum,

$$\delta p = \frac{6.6 \times 10^5}{(273/\theta_0)^{3/4}(1 - .001\theta_0)}\left(\frac{RI_0}{V_0}\right)\left[1 + \frac{1.1}{\sqrt{f}}\left(\frac{273}{\theta_0}\right)^{1/4} + \frac{0.613}{f}\left(\frac{273}{\theta_0}\right)^{1/2}\right]^{-1/2}\left(\frac{i}{f^{3/2}}\right).$$

The quantities R, V_0, I_0, and θ_0 depend on the particular thermophone used and the amount of direct current used in it. The resistance R is the resistance of the foil when heated with the direct current I_0. In all of the measurements made, the value of I_0 was kept at one ampere. By means of a potentiometer arrangement in which the voltage drop across the thermophone was compared with that in another circuit, it was possible to determine the resistance of the foil with a current of a few milliamperes flowing through it in which case the heating is inappreciable, and also when the current was one ampere. This latter value was taken for R, while the difference between the two values allowed the temperature θ_0 of the foil to be calculated. The enclosed cavity V_0 includes the thermophone cavity and also the volume of the ear. The volume of the former was obtained from the dimensions while the ear volume was determined by placing the receiver cap of the thermophone tightly against the ear and filling the ear with water through the hole in the receiver cap. Thus the portion of the outer ear which actually contributed to the enclosed cavity as used was correctly taken into account. Substituting the values of R, V_0, I_0, and θ_0 gives δp as a function of f and i, the frequency and the alternating current.

In determining the limit of audibility, the thermophone was held tightly to the ear under test by means of a telephone head band, the other ear being covered by a dummy head receiver. The experiments were conducted in an isolated room, the walls and ceiling of which were lined with felt, so that disturbing external noises were eliminated. A second person opened and closed the alternating current circuit by means of a knife switch, and this current was reduced in small steps until the observer became uncertain in his judgments as to when the switch was opened and closed. The value of the alternating current at this point being read and the frequency being known, the corresponding pressure change in the cavity was determined by means of the formula given.

It is conceivable that the pressure change in the cavity might be diminished by the motion of the ear drum in response to the pressure on it. It can be shown that the pressure change without taking the movement of the drum into account should be multiplied by a factor which varies from unity to 0.75 as the elasticity of the ear drum is considered to vary from ∞ to zero. It may be safely assumed, however, that the elasticity is sufficiently large to make the correction negligible, especially in view of the limits of accuracy of the determinations being made.

Although the determinations as made were of the values of the pressure changes exerted on the ear drum necessary for audition, calculations were made from these pressure changes of the intensities of the corresponding free progressive waves for the sake of comparison. If such a wave is reflected from a surface, the pressure changes exerted on the surface are twice the pressure changes in the wave. However the ear drum is at the end of a somewhat twisted canal, and the case is not that of simple reflection, and the above factor of two will be reduced. The irregularity of the canal does not allow an accurate calculation of the pressure on the drum, but considering the fact that it has approximately a uniform cross-section expanding but slightly just on reaching the drum and that its length is considerably shorter than the wave-length of the sound over the frequency range used, it seems that the intensities calculated from the observed pressure changes on the drum are not far from those of such free progressive waves as would be just audible.

The intensity J of a free progressive wave having a given value of pressure change may then be calculated from the formula $J = (\delta p)^2/2c\rho$, where c is the velocity of sound and ρ is the density of the air. In this case, the expression will be of the form $J = Ci^2/Lf^3$, where C is a constant depending on the thermophone and other factors mentioned, and L is the quantity given in brackets in the expression for δp. It may be noted that L is of the form $1 + A/\sqrt{f} + B/f$, where A and B are constants depending on θ_0 and, as used here, A is of the order of unity and B is of the order of 0.6, so that L is only slightly a function of the frequency f, varying by less than 10 per cent over the range of frequencies used. Thus the thermophone has an efficiency which is practically inversely proportional to the cube of the frequency and this serves to minimize the effect of harmonics in the alternating current supplied to the thermophone.

The results are given in terms of "logarithmic sensitivity," which is the logarithm to the base 10 of the reciprocal of the necessary intensity at the limit of audition, the intensity being expressed in absolute units, ergs per square centimeter per second. Thus the high parts of the curves represent the greater sensitivities and on the logarithmic scale equal differences in height have much more nearly equal importance in audition than if plotted on a linear scale, also small values are not obscured at the bottom of the paper. This logarithmic sensitivity seems to be the most logical way to express sensitivity. The numbers involved are small, equal differences between numbers are of approximately equal importance, the large ranges of sensitivity encountered in normal and deaf ears are easily expressed, and there is a very close connection with our absolute system of units as used in physical measurements. It is not necessary to give a name to the unit of sensitivity on this scale, as a "logarithmic sensitivity of 6" or even "a sensitivity of 6 units" has a clear meaning, once the system of calculation is understood.

The two ears even of the same individual have in general sufficient differences to be easily distinguished. Fig. 2 shows the results of tests

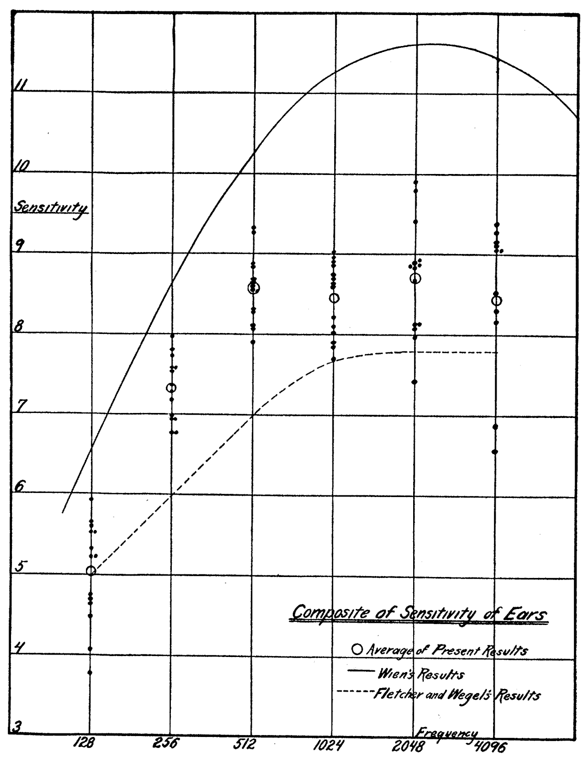

Fig. 2

made at intervals of an octave from 128 to 4,096 p.p.s. for 14 ears, and from this the variations between ears may be easily seen.[1] A sensitivity difference of 3 corresponds to an energy ratio of 1000, and variations of this magnitude are found between ears of people who would be classed as having normal hearing. The circles on this figure give the average

[1] The single figure of absolute sensitivity given in the author's previous paper does not agree with the highest values shown in the present data because of the use of the formula of Arnold and Crandall without the correction of the obvious error of a factor of θ_2, the absolute temperature. Wente's formula also includes another slight correction.

values of sensitivity for the ears shown. For the sake of comparison, Wien's[1] curve obtained by the use of telephone receivers is shown in the solid line. It is seen that the present results show a less sensitivity, that is a greater intensity required for audition, than found by Wien. The two sets of data agree in indicating a diminished sensitivity at the lower frequencies, but the present data show a less variation of sensitivity with frequency above 512 p.p.s. than indicated by Wien. The difference in sensitivity between the two sets of data varies from one and one half units to three units, the corresponding energy ratios varying from 30 to 1000. The dotted curve shows the averaged results of Fletcher and Wegel,[2] obtained fundamentally principally by the calibration of a telephone receiver by means of a thermophone. This curve falls below the average of the present results by a sensitivity difference of about one unit in general, except at the lowest frequencies, the two curves coinciding at 128 p.p.s.

II. Telephone Receiver as Source of Sound

Because of the difference between the results obtained by use of the thermophone and those obtained by Wien by use of a telephone receiver, it was considered advisable to make determinations of minimum audibility by means of a telephone receiver to parallel those of the thermophone as a check.

For absolute determinations with a telephone receiver, the amplitude of motion of the diaphragm must be found. At minimum audibility, the motion is so small that it cannot be measured directly. The method here used was to determine the relation between the diaphragm motion and the voltage across the receiver for larger amplitudes and extrapolate this relation downwards to small values.

Two methods of measurement of the amplitude of the diaphragm motion were used. In the first one, a light thin rod of wood about the size of a toothpick was waxed to the center of the diaphragm, the other end being split into a fork across which was fastened a fine quartz fiber. The telephone receiver was so mounted that the motion of the fiber could be viewed by means of a microscope having a micrometer eye-piece, and having such a magnification that one division of the scale in the eye-piece corresponded to a distance of 0.00087 cm. The telephone was thus calibrated for amplitude of motion of the center of the diaphragm per volt across the receiver for five frequencies from 128 to 1,536 p.p.s. The arrangement used in this calibration is shown in Fig. 3. The voltage across the receiver was calculated from the resistance and current in the

[1] Wien, Archiv fur die gesamte Physiologie **97**, pp. 1–57 (1903).
[2] Fletcher and Wegel, Phys. Rev. **19**, p. 553 (1922).

circuit across which the receiver is shunted. After calibration, the wooden stick with its fiber was removed from the diaphragm and an equivalent weight in the form of a small piece of brass was put on, so that the vibration characteristics would be maintained.

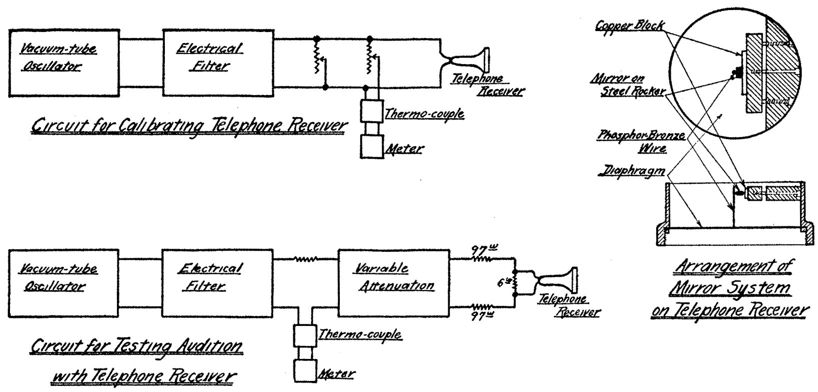

Fig. 3.

The circuit with which the receiver was used in determining the limit of audibility is also shown in Fig. 3. The attenuator is an arrangement of electrical resistances so adjusted that the different sections which can be introduced into the electrical circuit attenuate the current passing through by a definite known amount. A necessary condition is that the attenuator be terminated in a constant resistance, this being 200 ohms for this attenuator. As the impedance of the telephone receiver is neither pure resistance nor constant with frequency, a special resistance unit was made, consisting of a resistance of 6 ohms having connected to it on either side, resistances of 97 ohms. The telephone receiver was connected across the 6-ohm resistance, and since its impedance was large compared to the 6 ohms, the whole combination had a resistance of essentially 200 ohms, and this was connected to the output of the attenuator. The impedance offered by the attenuator to the circuit into which it is connected is 200 ohms regardless of the number of sections of the attenuator being used. The current into the attenuator was measured with a thermo-couple and a micro-ammeter, and the current output was easily calculable from the sections of attenuation thrown into circuit.

A unit of attenuation may probably best be defined in terms of the ratio of the electrical energy input to the electrical energy output, and for many purposes the relation between energy ratio and attenuation units may best be taken as a logarithmic one. So a unit of attenuation will be defined thus: $n = \log_{10} m$, where n is the number of attenuation units corresponding to an energy ratio of m. Thus one attenuation unit corresponds to an energy ratio of 10, two units to a ratio of 100, four units to a ratio of 10,000, etc. The attenuator used has steps of 0.025, 0.05, 0.1, 0.2, 0.4, 0.8, 1.6, 3.2, and 6.4, a total of 12.775 attenuation units, and thus energy ratios could be obtained and accurately known from a change of 6 per cent to a ratio of about six trillion in steps of 6 per cent changes. Electrical energy being proportional to the square of the current, current reduction ratios may be easily calculated from the number of attenuation units used. Thus $n/2 = \log_{10} m'$, where n is the number of attenuation units corresponding to a current ratio of m'. The voltage across the receiver was calculated on the assumption that all of the current output of the attenuator goes through the 6 ohms which shunts the receiver. This assumption introduces an error of not over 6 per cent at low frequencies, decreasing to about 1 per cent at 1,000 p.p.s.

After calibration of the telephone receiver, measurements were made on the ears of two individuals and the necessary voltage across the receiver at the limit of audition was determined. The amplitude of motion of the telephone diaphragm was found from the calibration described above, and the pressure changes produced on the ear drum were then calculated from the relation $\delta p = p_0 \gamma \delta V / V_0$, where V_0 is the volume of the cavity formed by the diaphragm and case of the receiver and the ear, δV is the volume change produced by the motion of the diaphragm, and γ is the ratio of the specific heats of air.

The second method of measuring the amplitude of motion of the diaphragm was by means of a mirror so mounted as to rotate by movements of the diaphragm. In the first system of this type tried, a small mirror was fastened to a length of a hair spring from a watch and mounted in such a way that motions of the diaphragm acted on a projecting point on the back of the mirror, giving it a rotary motion. A line of light reflected from the mirror and on to a scale of ground glass was then broadened into a band of light corresponding to the amplitude of motion of the diaphragm. The objection to this system was that there was an uncertainty as to whether all of the motion imparted to the brass point by the diaphragm was taken up in producing rotation of the spring and mirror system or whether part of it was taken up in producing vibrations which of course would have no effect on the width of the reflected line of light and so would not be measured.

Because of this objection, another mirror system was used, for the suggestion of which the author's thanks are due to Dr. Max Mason. A sketch of this system is shown in Fig. 3. A phosphor bronze wire of some stiffness was mounted to project from the center of the diaphragm and this rested against one sharp edge of a small rectangle of steel, another sharp edge being held against a copper block by the force exerted by a slight bending of the phosphor bronze wire. This force was regulated by adjustments of the position of the copper block which was fastened to the receiver case. Thus movements of the diaphragm caused the wire to rock the steel piece about the edge which rested on the copper as an axis. A small mirror waxed on to the steel piece gave a broadening of a reflected line of light just as in the other mirror system. The steel piece was about 1 × 1.5 mm in size. The great advantage of this system is that it has no natural period of its own since it has no elastic force separate from the diaphragm. The mirror and steel piece merely add a little mass to the moving system, and should follow the motions of the diaphragm faithfully. The sharpened edges of the steel piece make grooves in the wire and in the copper block, and rock in these.

The relation between diaphragm motion and the width of the band of light was found by use of a micrometer screw. The telephone receiver used was so made that the portion back of the diaphragm, including the pole-pieces, could be removed without disturbing the clamping of the diaphragm, so the micrometer screw was mounted on a frame with the clamped diaphragm, and the center of the diaphragm was deflected by pushing it with the end of the micrometer screw and the resulting movement of the line of light reflected from the mirror was noted. An amplification of 1,600 was obtained, that is the motion of the line of light as indicated by its broadening was 1,600 times the amplitude of the diaphragm motion, the amplitude of motion of the telephone diaphragm being determined in terms of the voltage across the receiver. With this system, measurements were made on the sensitivity of the ear with the receiver in the same condition as when calibrated, the mirror attachment remaining in place. Tests were made at five frequencies between 128 and 1,536 p.p.s.

A determination of sensitivity was made by the thermophone method in each case just before making a measurement with the telephone receiver, thus giving quite a direct comparison of the results by the two methods. The results are tabulated below, S and K refer to two different observers and R and L to the right and left ears. The first method is that in which the diaphragm amplitudes were measured by means of a microscope, and the second that in which the vibrating mirror was used.

METHOD 1

Frequency	Ear		Sensitivity		Difference in Sensitivity (1)−(2)	Energy Ratio
			Thermophone (1)	Telephone (2)		
128........	S	R	5.44	5.12	0.32	2.1
		L	5.57	4.62	0.95	8.9
	K	R	4.96	4.33	0.63	4.2
		L	5.18	4.53	0.65	4.5
256........	S	R	7.63	7.01	0.62	4.2
		L	7.86	6.72	1.14	14
	K	R	7.47	6.41	1.06	11.5
		L	7.37	6.51	0.86	7.2
512........	S	R	7.87	7.59	0.28	1.9
		L	8.69	8.39	0.30	2.0
	K	R	8.62	8.09	0.53	3.4
		L	8.32	8.39	−0.07	0.83
768........	S	R	7.65	7.71	−0.06	0.87
		L	7.63	7.61	0.02	1.05
	K	R	9.33	9.05	0.28	1.9
		L	9.43	9.15	0.28	1.9
1,536	S	R	8.78	8.95	−0.17	0.67
		L	7.86	7.95	−0.09	0.83
	K	R	8.78	8.71	0.07	1.2
		L	8.78	8.41	0.37	2.4

METHOD 2

Frequency	Ear (K)	Sensitivity		Difference in Sensitivity	Energy Ratio
		Thermophone	Telephone		
128........	R	5.45	4.34	1.11	13
	L	5.28	4.34	0.94	8.8
256........	R	7.53	6.89	0.64	4.4
	L	7.32	6.61	0.71	5.2
512........	R	8.58	8.3	0.28	1.9
	L	8.48	8.1	0.38	2.4
640........	R	9.24	8.60	0.64	4.4
	L	9.24	8.64	0.60	4.0
1,024.......	R	9.21	8.94	0.27	1.86
	L	8.82	8.44	0.38	2.4
1,536.......	R	8.84	8.74	0.10	1.26
	L	8.35	8.24	0.11	1.28

It is seen that the sensitivities obtained with the thermophone are in general somewhat greater than those obtained with the telephone; that is, the necessary energy for audition is less as measured by means of the thermophone than as measured by means of the telephone. The two telephone methods agree well in this. The ratio of the two measured values of this energy is given in the last column. The averaged values of these energy ratios follow.

Frequency:	128	256	512	640	768	1,024	1,536
Energy ratio:	6.9	7.7	2.1	4.2	1.4	2.2	1.3

The uncertainty in determining the limit of audition is greater at the lower frequencies than at the higher, but mere uncertainty should not make the ratios all larger, as is found to be the case.

It is seen that the telephone measurements agree with those obtained with the thermophone in giving considerably lower values of sensitivity than found by Wien.

Discussion

As to the difference found between thermophone and telephone results, in the latter measurements, the relation between diaphragm amplitude and voltage across the telephone receiver was determined for amplitudes down to about 6×10^{-4} cm, and this relation was used at the limit of audibility where the amplitude of the diaphragm motion varied from about 10^{-7} cm for the lowest frequency to 10^{-9} cm for the higher frequencies. In the range where observations were possible, this relation was found to vary somewhat with the amplitude, the variation being such that the amplitude per unit volt was greater for the larger amplitudes than for the smaller. This variation was not large but was found consistently with both methods of measurement of the diaphragm amplitude, so it may be possible that the extrapolation to the small amplitudes should not be made linearly. This variation is in the right direction to explain the difference in results between the telephone and the thermophone, for if the telephone became less and less efficient at lower amplitudes, it would take a larger amount of electrical energy to produce a given motion of the diaphragm than a linear extrapolation would indicate, and this corresponds to a decreased sensitivity of the ear as measured with the telephone, which is what is found if the thermophone be taken as a standard. Also it may be that the statistical theory of condensations and rarefactions in the propagation of sound impulses may not hold accurately when the amplitudes of motion are as small as are here considered, these being, for the higher frequencies, of the order of one thirtieth of the diameter of a nitrogen molecule and about one ten-thousandth of the mean free path of the molecules.

The agreement between the values for the necessary energy at minimum audibility obtained by the use of the thermophone and the telephone is sufficiently close, however, to leave no uncertainty about the order of magnitude of the sensitivity of the human ear.

It is recognized that tests of minimum audibility at a rather few separated frequencies do not justify any conclusions being drawn as to the form of the curve of audition in the intervals between these frequencies. A method has been developed which eliminates this uncertainty and allows a complete curve of audition to be made. Many such curves have been obtained and will be published soon.

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