THE VARIATION WITH FREQUENCY OF THE POWER LOSS IN DIELECTRICS.

By Hector J. MacLeod.

Abstract.

Variation of the power loss in dielectrics with frequency, 500 to 1,000,000 cycles.—(1) *A new bridge method of measurement* is described. This bridge has two resistance ratio arms, and two similar variable condenser arms which are made to act as pure capacities by connecting their dielectrics in parallel with the resistance arms. The condenser to be studied is connected in parallel with one of the bridge condensers set at zero. With this bridge the equivalent series resistance can be obtained directly without the assumption that the loss in the standard condenser is negligible. The theory of this bridge is given. For frequencies above 3,000 a resonance substitution method was used, correction being made for the losses in the standard precision condenser. (2) *Results for glass, Pyrex, paraffin, ceresin, mica and Murdock composition* are given. The power loss for unit voltage is equal to $2\pi fCF$, where f is the frequency, C the capacity and F the power factor, which depends only on the material. Since the phase difference is small, F is taken equal to $2\pi fCR$, where R is the equivalent series resistance. It was found that, approximately, $R = A/f^k$, $P = Bf^n$; hence $F = D/f^{(k-n)2}$. For paraffin, mica, and Pyrex, the values found for D are .00174, .0132, and .0264; the values of $(k - n)$ are .30, .23, and .215, respectively; and the values of $(n + k)$ are all close to 2, the difference being due to the small change of capacity with frequency. (3) *Relation to phenomenon of residual charge.* The constant n is the same as in the equation of E. v. Schweidler for the residual charge current: $i = EC_0\beta t^{-n}$, where E is the harmonic impressed electromotive force and β is a constant. This equation leads to the above equation for power loss and also to the equation for the capacity: $C = C_0(1 + Mf^{n-1})$, where M is a constant. Values of n determined from the variation of capacity with frequency agree closely with those given above. This agreement both confirms the theory and indicates the accuracy of the measurements.

Introduction.

In a perfect condenser the conductors or plates would have no resistance, the dielectric would show no absorption and its resistance would be infinite. In such a condenser the power loss would be zero and the current and impressed electromotive force 90° out of phase. Actual condensers only approximate to this ideal case, for the resistance of the conductors, though generally negligible, is never zero and the dielectric absorption is always accompanied by a loss of power which appears as heat in the condenser. This power loss signifies that there is a component of electromotive force in phase with the current. The effect of absorption is therefore equivalent to that of a resistance either in series or in parallel with the condenser.

The equivalent series circuit and vector diagram for a condenser with dielectric loss or conductor resistance is shown in Fig. 1. The resistance r is called the equivalent series resistance. θ is the phase angle between the current and voltage and cos θ is the power factor. The difference between 90° and the actual phase angle θ is called the phase difference ψ.

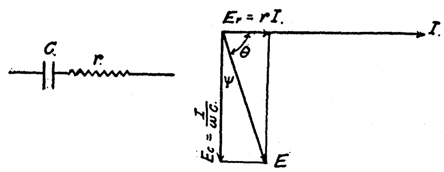

Fig. 1.

The power loss is given as for any part of a circuit by

$$P = EI \cos \theta = EI \sin \psi.$$

In all except extremely poor condensers, ψ is very small. The angle ψ, its sine, and its tangent may, therefore, be taken as equal. When this is true, the power factor and phase difference are synonymous.

The power factor of a condenser may be determined by measuring the energy loss or the equivalent resistance. Since the energy loss is small it is difficult to obtain accurate results by the first method. However, by using the condenser as part of a resonance circuit, Rosa and Smith[1] were able to measure the loss by means of a wattmeter.

It is however less difficult to measure the equivalent resistance and this method has been used in a large number of experiments including those of Hanauer,[2] Monasch[3] and Lawther.[4] Lawther used a bridge method up to 1,360 cycles and a resonance method for higher frequencies. The ratio arms of the bridge were two similar condensers. The condenser to be investigated formed the third arm while the fourth contained a non-inductive resistance and an air condenser whose losses were considered negligible. He found that the series resistance R may be represented by the equation $R = Af^k$ and the power loss by $P = Bf^n$ for condensers of glass and mica.

Method and Apparatus.

Previous determinations of the energy losses in condensers have been made by comparison with an air condenser whose losses were considered negligible. In many cases, at least, this assumption has not been justi-

[1] E. B. Rosa and A. W. Smith, Phys. Rev., 8, 1899.
[2] J. Hanauer, Wid. Annalen, 65, 1898.
[3] B. Monasch, Ann. d. Physik, 22, 5, 1907.
[4] H. P. Lawther, Thesis, Harvard Univ. 1916.

fiable. In the present investigation, a bridge method was used for frequencies up to 3,000 and a special type of bridge was designed, which is believed to be new. The ratio arms are two equal fixed resistances and the other two arms of the bridge are variable condensers. The dielectric of the condensers is placed in parallel with the resistance arms and consequently the variable capacity arms of the bridge show no dielectric loss. Hence, neglecting the plate resistance, the capacity arms act as perfect condensers, assuming, of course, that air is a perfect dielectric.

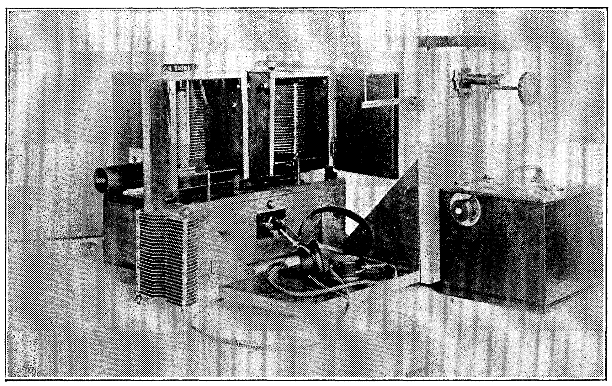

Fig. 2. The bridge.

Figure 2 is a photograph of the bridge. The diagram of connections is shown in Fig. 3 and the general arrangement of the four arms is illustrated by Fig. 4. The letters A, B, C, and D indicate corresponding points in the two figures.

Referring to Fig. 3: The middle points of the primary and secondary windings of the input transformer are grounded. Since the resistance arms are equal, the point D will remain at approximately earth potential. A and C will be high potential points and when the bridge is balanced, B will remain at the same low potential as D.

Referring to Fig. 4: The copper lining and cover of the case containing the two resistance arms are connected to the point D. They are, therefore, at low potential. The high potential plate of the condenser C_1 above A is supported by the block of hard rubber E_1 resting on the copper lining which is connected to the point D. This dielectric is therefore in parallel with the resistance R_3 between the points A and D. The low potential plate of the same condenser and also the copper lining of its case are connected to the low potential point B. The condenser case is supported by four small blocks of hard rubber, which are fastened to the case of resistance arms. When the bridge is balanced, these two cases are at the same low potential, consequently there is no loss in the hard rubber supports. The arrangement of the second condenser is similar in every way.

Special care was taken to have the bridge as symmetrical as possible. The blocks of hard rubber E_1 and E_2 (Fig. 4) were cut from the same piece. The condenser plates are separated by special 0.5 cm. spacers and the maximum capacity of each condenser is about 1,000 $\mu.\mu.$f. The ratio arms are made of equal resistances, each being approximately 11,500 ohms.

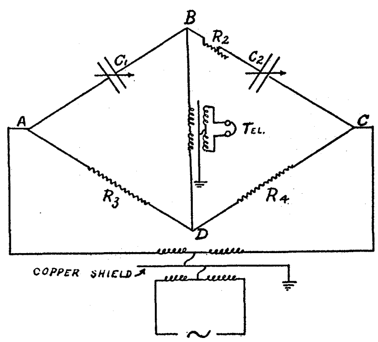

Fig. 3. Diagram of bridge circuit.

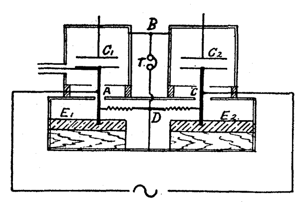

Fig. 4. Arrangement of bridge arms.

A balance in the bridge is indicated by means of a set of Baldwin phones, which are connected to the points B and D (Fig. 3) through a transformer completely enclosed by a grounded copper shield. The telephone and input transformers are similar to those used in the General Radio Company's capacity bridge. Each is made up of two sections symmetrically wound on a small closed laminated iron core. The primary and secondary windings are separated by a grounded copper shield. The middle point of the secondary winding of the telephone transformer is grounded, while in the input transformer both middle points are grounded. Thus there is no actual ground within the bridge itself. The fine adjustment of the condenser necessary for balance is obtained by a 17-inch lever arm fastened to the shaft of the condenser C_2 and controlled at its outer end by a rack and pinion.

When the bridge is balanced, the condenser C_1 is reduced to its zero value and the condenser, whose loss is to be determined, is connected in parallel with it by means of the terminals indicated in Fig. 4. The high potential terminal is a brass rod connected to the supports of the high potential plates. It is surrounded by a cylinder connected to the lining of the case and forming the low potential terminal. There is no direct dielectric support between the terminals.

It is important to have the resistance R_2, by which the bridge is balanced, practically a pure resistance. For resistances above 1,000 ohms a special resistance box was built in which six 1,000-ohm coils were placed three inches apart. The coils not in use were entirely disconnected from the circuit by means of double-throw switches.

The inductive effect of these coils was determined at 1,000 cycles by the substitution method with an impedance bridge. The standard was a 1,000-ohm single loop, whose dimensions were known and whose inductance could be calculated. 1,000 ohms in the resistance box was found to be equivalent to 1,000 ohms pure resistance shunted by 2.7 μ.μ.f., while five coils were equivalent to 5,000 ohms shunted by 4.7 μ.μ.f. The effect of these values on the power factor and capacity was negligible.

The standard of capacity used was a carefully calibrated 1500 μ.μ.f. variable precision condenser made by the General Radio Company. In comparison with ordinary condensers its loss is small.

The Oscillator.

A diagram of connections of the source of oscillations is shown in Fig. 5. It is a well-known arrangement and need not be described in

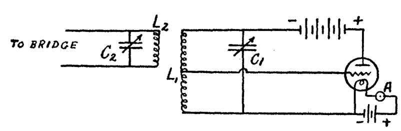

Fig. 5. Diagram of oscillator circuit.

detail. Resonance was obtained in the secondary circuit by means of the decade mica condenser C_2. The circuits were coupled as loosely as possible. The resonance e.m.f. impressed on the bridge was about 90 volts.

The Resonance Circuit.

At high frequencies the ordinary resonance substitution method was used but the losses in the precision condenser had previously been determined at low frequencies and could be taken into consideration. The double-pole double-throw switch, by which either condenser could be thrown into the circuit, was made of paraffin containing small holes filled with mercury. For frequencies above 100,000 a Leeds and Northrup galvanometer, having a resistance of 7.23 ohms and a period of 1.8 seconds, was used with a thermocouple. For lower frequencies this instrument was replaced by a sensitive Weston Electric galvanometer.

Theory of Bridge Circuit.

Let z_1, z_2, z_3 and z_4 represent the impedances of the corresponding arms of the bridge as indicated in Fig. 3.

$$z_1 = R_1 - J/\omega C_1, \qquad z_3 = R_3,$$
$$z_2 = R_2 - J/\omega C_2, \qquad z_4 = R_4.$$

The condition for zero current in the telephone circuit is that

$$z_1/z_2 = z_3/z_4$$

or, assuming an impressed sinusoidal electromotive force,

$$R_1/R_2 = C_2/C_1 = R_3/R_4.$$

$R_3 = R_4$ and therefore when the bridge is balanced $C_1 = C_2$ and $R_1 = R_2$.

Let C_x be the capacity of the condenser to be investigated and R_x its equivalent series resistance. This condenser is connected in parallel with C_1. Then by neglecting R_x^2 in the expression

$$R_x^2 + \frac{(C_x + C_1)^2}{C_x^2\, C_1^2 \omega^2}$$

it may easily be shown that

$$R_0 = \frac{R_x C_x^2}{(C_x + C_1)^2} \tag{1}$$

and

$$C_0 = (C_x + C_1), \tag{2}$$

where R_0 is the equivalent series resistance of this parallel circuit and C_0 its series capacity. In the condensers studied, R_x was of the order of 1,000 ohms at 1,000 cycles, C_x was 900 $\mu.\mu.$f. and C_1, 30 $\mu.\mu.$f. R_x^2 is, therefore, of the order of one millionth of the expression in which it has been neglected.

From equation (1)

$$R_x = \frac{R_0\,(C_x + C_1)^2}{C_x^2}. \tag{3}$$

This equation shows that it is desirable to have the capacity C_1 as small as possible in order that R_0, which is measured by the bridge, may be nearly as large as R_x, the equivalent series resistance of the condenser under investigation. Consequently the high potential plates of the condenser C_1 were made detachable and when removed the capacity of the terminals and supports alone remains. This capacity C_1 amounts to 32 $\mu.\mu.$f. Taking C_x as 900 $\mu.\mu.$f., the correction factor in equation (3), viz., $(C_x + C_1)^2/C_x^2$, is 1.072.

Procedure.

These experiments were carried out in a constant temperature room of Cruft High Tension Laboratory. The oscillator was not in the same room as the bridge and resonance circuit. The capacity and equivalent series resistance of the condensers were determined by the bridge for frequencies from 500 to 3,000. The procedure was as follows: The condenser C_x was connected to the bridge in parallel with C_1 which was set at its minimum value. The bridge was roughly balanced and the secondary circuit tuned to resonance. The frequency was then determined and an exact balance obtained in the bridge by adjusting the capacity C_2 and the resistance R_2 to the point of silence in the phones. The capacity C_x was obtained by direct substitution of the Precision Condenser. The condensers C_1 and C_2 of the bridge were then made equal and the bridge itself was balanced. A few ohms were generally required for a perfect balance and they were added to or subtracted from R_2 obtained above. The result, substituted for R_0 in equation (3), gave the equivalent series resistance R_x of the condenser under investigation. A series of values was obtained in a similar way covering the range from 500 to 3,000 cycles.

For the high frequency values the secondary circuit was resonated with the resistance set at zero and the condenser C_x in the circuit. The coupling was then adjusted to nearly full scale deflection of the galvanometer. After carefully resonating and noting the deflection, the double-throw switch was thrown over to include the precision condenser in place of the condenser C_x. Resonance was again obtained by varying the precision condenser, and the required deflection by varying the resistance. The resistance introduced represents the difference between the equivalent series resistances of the two condensers. The effect of this resistance upon the capacity of the circuit becomes important at high frequencies. The equivalent capacity was, therefore, obtained by resonating the circuit by means of the precision condenser with the resistance box at zero. After each set of readings, the value of the resonance current with the condenser C_x was checked to ensure that it had remained steady.

Sources of Error and Accuracy.

The sources of error in the bridge itself may be classified as follows:

(*a*) Errors in the ratio of R_3 and R_4.
(*b*) Inductance or capacity of the ratio arms R_3 and R_4.
(*c*) Inductance or capacity of the balancing resistance R_2.
(*d*) Electrostatic capacity between the bridge and its surroundings.

The error in the ratio of R_3 and R_4 can be made extremely small, especially when these resistances are large. Before the condensers were added to the bridge, the equality of the ratio arms was checked by balancing and interchanging two variable air condensers.

If the balancing resistance R_2 were a pure resistance, its introduction into the bridge arm would cause a change in the power factor of approximately $R_2\omega C_2$. When R_2 is equivalent to a pure resistance shunted by a capacity k_2, the change in power factor becomes $R_2\omega C_2[1 - (R_2\omega k_2)^2]$. As stated above, the value of k for 5,000 ohms was found to be 4.7 $\mu.\mu.$f. This represents a decrease in the power factor of about .0002 per cent. at 1,000 cycles.

The effects of the inductance or capacity of the ratio arms and of the electrostatic capacity of the bridge may be practically eliminated by the substitution method. These effects will enter into the measurements of the two condensers in the same manner since they are both measured in the same arm of the bridge. And if the capacities are equal, the errors will be sensibly the same and will not affect measurements of capacity or resistance.

The theory of the bridge circuit given above applies only in the case of a sinusoidal electromotive force and current. Harmonics generally occur in an audion circuit and in order to minimize their effect, the loose coupling and resonance previously described was used. Especially above 1,000 cycles, a sharp balance and silence in the phones was obtained and at high frequencies the resonance peak was extremely sharp. The effect of harmonics was, therefore, considered unimportant.

In order to ensure that the input transformer did not affect the observations, a set of readings was taken with and without this transformer in the circuit. These readings were consistent and showed no effect of the removal of the transformer. Without it, however, the value of the resistance required to balance the bridge itself was generally very much larger.

For the high frequency measurements, the two condensers were symmetrically placed with respect to the rest of the resonance circuit and the observations were checked with the condensers interchanged. The resistance, capacity and inductance of the circuit were varied by means of lever arms, which were also used with the bridge since the presence of the hand near it affected the balance.

It is well known that the equivalent resistance of condensers varies with the temperature. In the present investigation, the condensers were kept in the constant temperature room for at least several days before they were examined. After a series of observations had been made, the results at about 1,000 cycles were checked in order to ensure that no change had taken place due to change in temperature or from any other cause.

The surface leakage of dielectrics exposed to air changes with the humidity. To minimize any possible surface leakage, nearly all the condensers studied were dried and dipped in molten paraffin or ceresin wax.

The percentage accuracy of the bridge measurements depended upon the equivalent series resistance of the condenser examined. Around 1,000 and 3,000 cycles, where the bridge was most sensitive, the resistance balance could be determined to the nearest half-ohm. Around 2,000 cycles, a balance to the nearest ohm could be obtained, while at frequencies below 800 or 900 the sensitivity was not so good. These values were practically independent of the equivalent resistance of the condenser. Since this resistance was obtained from the difference of two readings, the possible error above 1,000 cycles would be from .5 to 1 ohm. When the losses were above three or four hundred ohms, the observations were made to the nearest ohm only.

The capacity balance was much more sensitive than that of the resistance. The capacities of the condensers examined were about 900 $\mu.\mu.$f. and a change in capacity of 0.2 $\mu.\mu.$f. could easily be detected. This corresponds to about 0.02 per cent. A fine adjustment of the capacity was necessary in order to obtain the resistance balance.

The frequencies up to 3,000 were obtained by a frequency meter composed of a variable oil condenser and a set of inductance coils. The meter was calibrated with a set of tuning forks and checked on different occasions. Resonance was determined by a pair of phones connected to one terminal of the condenser. Over the range used, one division of the condenser scale changed the corresponding frequency by about 1 per cent. When a constant frequency was required for some time, a tuning fork was employed.

The high frequencies were determined by means of the Cruft wavemeter. The error in the frequency measurements throughout should not be more than about 1 per cent.

The measurement of equivalent resistance at high frequencies was probably most accurate around 250,000 cycles, where 0.1 ohm corresponded to about 8 or 10 small divisions on the galvanometer scale. Repeated observations agreed to within half a division.

Tests of the Bridge.

The following tests of the bridge were made to ensure that the results obtained are reliable:

(*a*) To show that the addition of a condenser with no dielectric loss would not affect the balance of the bridge: A brass plate about 8 inches in diameter was fastened by a screw to the high potential terminal. This plate was surrounded by a firm brass case connected to the low potential terminal and supported on the table. The capacity of this condenser was about 600 $\mu.\mu.$f. The bridge condenser C_1 was reduced by the same amount and no change in the resistance balance was observed.

(*b*) To show that the equivalent resistance observed was approximately correct: The precision condenser and a resistance box were connected in series with the bridge terminals. The bridge was balanced with the resistance box at zero. A certain resistance was then included and the bridge again balanced. The following results are typical:

Capacity of added precision condenser: 900 $\mu.\mu.$f.

Resistance correction factor:

$$\frac{(900 + 32)^2}{900^2} = 1.072.$$

f.	B_1.	R_a.	B_2.	$(B_2 - B_1) \times 1.072$.
1,000	16	1,000	949	1,000
1,000	16	100	108.5	99.1
1,000	16	50	62.5	49.8
2,000	13	1,000	944	998

f = frequency. R_a = added resistance.
B_1 = balancing resistance when $R_a = 0$.
B_2 = balancing resistance with R_a added.

By equation (3), $(B_2 - B_1) \times 1.072$ should equal the added resistance. This test also serves as a check on the accuracy of the correction factor.

(*c*) To show that correct results could be obtained with the bridge condenser at different settings: It is well known that equation (1), is applicable to an ordinary air condenser which may be considered as a variable pure capacity in parallel with a constant imperfect condenser. For a constant frequency, R_x as well as C_x is constant and the equation may, therefore, be written in the form:

$$R_0 = \frac{\text{const.}}{C^2}, \tag{4}$$

where R_0 is the equivalent series resistance of the condenser and C its total capacity. Hence,

$$\log R_0 = \text{constant} - 2 \log C. \tag{5}$$

The results obtained for the variable precision condenser for a constant frequency of 1,024 cycles per second were as follows:

Capacity ($\mu.\mu$.f.).	Resistance (Ohms).
900	13
800	16
700	21
600	29
500	42
400	64
300	112.5

The graph of equation (5) for these results is a straight line with a slope of -2.

Description of Condensers Examined.

Glass No. 1.—This was a commercial condenser made by the Wireless Specialty Apparatus Co. It consisted of copper coats electrolytically deposited on a glass jar. To reduce its capacity, the condenser was cut in two and the cylindrical part examined.

Glass No. 2.—This condenser consisted of a glass plate about 1/16 inch thick, which was sprayed with a metal coating after having been pitted by a sand blast. A heavy layer of copper was applied over a thin coat of lead. A one-inch margin was left all around. After the leads were attached, the condenser was dipped in paraffin.

Pyrex.—A thin-walled tube of pyrex was made in the form of a test-tube and supported inside another tube. After being carefully cleaned, the pyrex tube and the space between the tubes were filled with mercury to within about an inch and a half from the top.

Paraffin.—This condenser consisted of light copper plates separated by thin layers of paraffin. The condenser was also thickly coated over with paraffin.

Ceresin.—The high and low potential aluminum plates of a small air condenser were supported in a copper case. The case was filled with melted ceresin and allowed to cool in a vacuum. The supports were removed leaving the ceresin alone as the dielectric between the plates.

Mica No. 1.—This condenser consisted of a clear sheet of mica, the surface of which was slightly roughened and then sprayed with lead from an oxy-hydrogen gun. The condenser was padded with mica and clamped between brass plates. It was placed in a bath of melted ceresin under a bell jar. After the air had been exhausted, it was left to cool.

Mica No. 2.—A clear sheet of mica 5 cm. square was lightly coated with a rosin-beeswax mixture over which sheets of copper foil were pressed. The condenser was padded with sheets of mica and bakelite and clamped between heavy brass plates. It was then heated in an oven at about 100° C. and while still hot was clamped in a vise. After cooling, the clamping screws were again tightened and the condenser was coated with shellac.

Composition No. 1.—This was a commercial condenser manufactured by the W. J. Murdock Co. It consisted of copper sheets embedded in the composition. In order to reduce its capacity, the condenser was cut in two.

Composition No. 2.—This condenser was similar to Composition No. 1.

Glass No. 1, Mica No. 2 and Composition No. 2 were used by Lawther in experiments referred to in the introduction.

Data and Curves.

In these experiments observations were made at about twenty different frequencies covering the range from 500 to 1,000,000 cycles. Only a few of these, however, are included in the following tables in which f is the frequency in cycles per second; R, the equivalent series resistance of the condenser in ohms and C its capacity in farads. P.F. represents the power factor.

Precision Condenser and Leads.
Capacity: 900 $\mu.\mu.$f.

Observed.		Obtained from Curve.	
f.	R.	f.	R.
606	37.5	14,000	1.20
890	25	100,000	.141
1,270	16.5	250,000	.053
2,040	10	500,000	.025
2,890	7	1,000,000	.012

By plotting log R against log f, a straight line was obtained. This line was produced and some of the high frequency values obtained from the curve are shown in the fourth column above.

Glass No. 1.

f.	R.	$C \times 10^9$.	P.F.
498	1,383	.9131	.00395
1,015	634	.9115	.00368
2,820	203	.9096	.00327
14,000	34.2	.9073	.00276
100,000	4.14	.9043	.00236
500,000	.75	.9031	.00213
1,000,000	.35	.9019	.00198

Temperature: 20.8° C.

Glass No. 2.

f.	*R.*	$C \times 10^9$.	P.F.
498	5,842	.9222	.0169
1,015	2,562	.9149	.0149
3,130	697	.9052	.0124
14,000	123.2	.8977	.00974
100,000	13.74	.8875	.00766
500,000	2.39	.8803	.00660
1,000,000	1.12	.8773	.00617

Temperature: 20.8° C.

Pyrex.

f.	*R.*	$C \times 10^9$.	P.F.
596	4,281	.8543	.0137
1,010	2,354	.8506	.0127
2,920	719	.8442	.0111
14,000	120.2	.8359	.00884
100,000	14.3	.8275	.00743
500,000	2.58	.8209	.00666
750,000	1.76	.8203	.00678

Temperature: 20.4° C.

Paraffin.

f.	*R.*	$C \times 10^9$.	P.F.
607	188.5	.9124	.000655
1,000	106	.9122	.000607
2,935	31.5	.9118	.000530
14,000	5.2	.9115	.000417
100,000	.54	.9121	.000309
500,000	.09	.9115	.000257

Temperature: 20.3° C.

Mica No. 1.

f.	*R.*	$C \times 10^9$.	P.F.
590	1,947	.9433	.00680
1,020	988	.9415	.00595
2,890	303.5	.9385	.00516
14,000	51.9	.9355	.00427
100,000	6.1	.9295	.00356
500,000	1.12	.9259	.00326

Temperature: 20.3° C.

The data for Ceresin, Mica No. 2 and Composition No. 2 have been omitted. The two samples of mica were very similar and also the two samples of composition, while the results for ceresin are somewhat similar to those of paraffin.

Composition No. 1.

f.	R.	$C \times 10^9$.	P.F.
510	2,744	.8881	.00780
1,000	1,226	.8840	.00681
3,200	314	.8815	.00556
14,000	59.2	.8791	.00458
100,000	7.59	.8746	.00417
500,000	1.81	.8716	.00495
1,000,000	1.03	.8713	.00563

Temperature: 20.7° C.

Discussion of Results.

In order to interpret the results obtained, the effect and probable importance of the different losses in condensers will first be considered. These losses are classified as follows:

(*a*) The leakage loss due to ordinary conduction through the dielectric or along its surface.
(*b*) The loss due to the phenomenon of dielectric absorption.
(*c*) The loss due to resistance in the metal plates or leads.
(*d*) The loss due to brush discharge which occurs only at high voltages and need not be considered here.

A condenser having leakage may be represented by a pure capacity with a constant resistance in parallel. The resulting phase difference ψ is given by the equation: $\tan \psi = 1/R_p\omega C$.

This constant parallel resistance is equivalent to a series resistance R_s, which is equal to $1/R_p\omega^2C^2$ and hence decreases rapidly with increase of frequency. Previous experiments[1] have shown that with good dielectrics the leakage loss is small, being less than one per cent. of the total at a frequency as low as 50 cycles. Above 500 cycles, therefore, this loss may be entirely neglected.

The loss due to dielectric absorption is represented by an equivalent series resistance which decreases with increase of frequency. The resistance of the metal plates or leads is also a series resistance. The equivalent circuit of a condenser with dielectric loss or plate resistance is shown in Fig. 1. For this circuit, $\tan \psi = R\omega C$, which shows that the effect of a constant series resistance upon the power factor increases with the frequency. The resistance of the plates and leads is practically constant, although at high frequencies it may increase slightly on account of skin effect. In a well-designed condenser, this resistance is extremely small and is generally negligible in comparison with the equivalent resistance of the dielectric. At very high frequencies, however, it may become important.

[1] F. Tank (Ann. d. Physik, 48, 1915) and others.

In the present investigation, since the phase difference ψ is small, $\sin\psi$ was taken equal to $\tan\psi$, giving the power factor P.F. $= \sin\psi = \tan\psi = R\omega C$, and for the power loss P: $P = \omega CE^2 \sin\psi = E^2R\omega^2C^2$. $R\omega^2C^2$ therefore represents the power loss when the impressed voltage E is equal to unity.

Curves were plotted in order to show the variation of the equivalent series resistance and the power loss with frequency. With the exception of the Composition condensers and Mica No. 2 above about 14,000 cycles, the log resistance-log frequency curves are apparently straight lines. When this is true,

$$\log R = \text{const.} - k \log f,$$

and

$$R = \frac{A}{f^k}, \tag{6}$$

where A and k are constants.

The values of k are as follows:

Condenser.	k.
Precision (and leads)	1.08
Glass No. 1	1.095
Glass No. 2	1.13
Pyrex	1.10
Paraffin	1.15
Ceresin	1.13
Mica No. 1	1.11
Mica No. 2	1.39
Composition No. 1	1.16
Composition No. 2	1.18

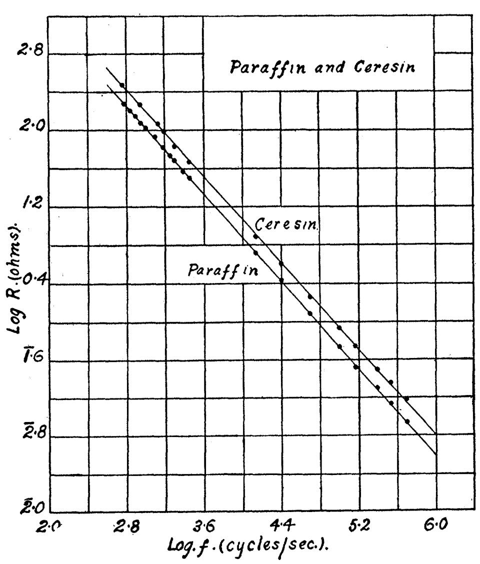

Fig. 6.

As previously explained, the values of k for the precision condenser and leads were applied as corrections to all values of resistance obtained by the resonance method. A check on the accuracy of these corrections is given by the curve for the paraffin condenser which showed the smallest loss. Above 100,000 cycles the correction was over 22 per cent. of the total loss and yet the points lie well on the same straight line as the low frequency ones. The curves for paraffin and ceresin are shown in Fig. 6.

The deviation from a straight line at high frequencies in the case of the Composition condensers and Mica No. 2 represents a larger power loss. The composition was rather brittle and it is probable that the plates were loosened when the condensers were cut in two. While in the circuit, Composition No. 2 emitted a singing note throughout the audible range. For these condensers, which will be omitted from further discussion, the values of k given above were obtained from the slope of the line below 14,000 cycles.

The log of the power loss for unit voltage, which is equal to $R\omega^2C^2$, is plotted against the log of the frequency in Figs. 7 and 8. These curves are also apparently straight lines and therefore

$$\log P = \text{const.} + n \log f$$

or

$$P = Bf^n. \tag{7}$$

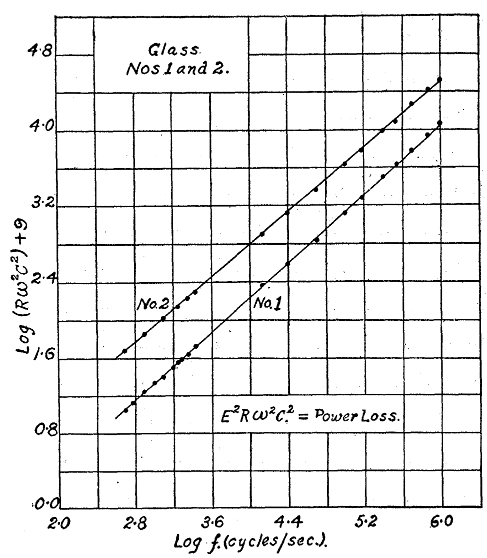

Fig. 7.

The values of n are as follows:

Condenser.	n.	$n+k$.
Glass No. 1	0.90	1.995
Glass No. 2	0.86	1.99
Pyrex	0.885	1.985
Paraffin	0.85	2.0
Ceresin	0.86	1.99
Mica No. 1	0.88	1.99

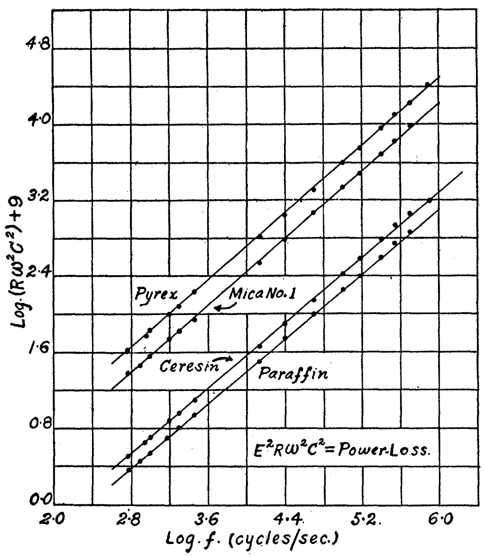

Fig. 8.

The sum of n and k is approximately equal to 2, the difference being due to the slight decrease in the capacity as the frequency is increased. This decrease and also the constant n may be accounted for by the phenomenon of residual charge which has attracted the attention of many eminent physicists.

The experimental work of Kohlrausch,[1] E. v. Schweidler,[2] and others has shown that the residual charge current i is given by the equation:

$$i = EC_0\beta t^{-n}. \tag{8}$$

E represents the constant impressed electromotive force and C_0 the capacity for infinite frequency, which is sometimes called the geometrical capacity; t is the time and β and n are constants. n lies between zero and one.

[1] R. Kohlrausch, Pogg. Ann., 1854.

[2] E. v. Schweidler, Ann. d. Phys., 24, 1907.

E. v. Schweidler has shown that when

$$E = E_0 \sin \omega t,$$

$$i = \omega C_0 E_0 (G \cos \omega t + H \sin \omega t), \tag{9}$$

where

$$G = \omega^{n-1} \beta \Gamma(1 - n) \cos \frac{(1-n)\pi}{2}$$

and

$$H = \omega^{n-1} \beta \frac{\pi}{2\Gamma(n) \cos \frac{(1-n)\pi}{2}}.$$

$\Gamma(n)$ is the Gamma function.

The normal charging current I_0 is given by:

$$I_0 = \omega C_0 E_0 \cos \omega t \tag{10}$$

and the normal conduction current j by

$$j = \frac{E_0}{W} \sin \omega t. \tag{11}$$

The total current I is therefore:

$$I = \omega C_0 E_0 \left[(1 + G) \cos \omega t + \left(\frac{1}{\omega C_0 W} + H\right) \sin \omega t\right]. \tag{12}$$

Hence the apparent capacity C is given by:

$$C = C_0(1 + G) \tag{13}$$

and the power loss P by:

$$P = 1/2\ \omega C_0 E_0^2 \left(\frac{1}{\omega C_0 W} + H\right). \tag{14}$$

Substituting for H:

$$P = \omega C_0 E^2 \left(\frac{1}{\omega C_0 W} + \omega^{n-1} \beta \frac{\pi}{2\Gamma(n) \cos \frac{(1-n)\pi}{2}}\right), \tag{15}$$

where E is the effective voltage.

The first term in the brackets arises from the conduction or leakage current and the second term from the residual charge current or dielectric absorption. In the present investigation, the first is negligible in comparison with the second, and equation (15) reduces to

$$P = \text{const. } E^2 \omega^n.$$

For a constant voltage:

$$P = B f^n. \tag{16}$$

This equation is identical with equation (7) given above to represent the variation of power loss with frequency as shown by the curves. The exponent n of the power loss equation is the n which appears in the equation for the current due to residual charge. This was shown for a frequency of 50 cycles by the experiments of Tank[1] in which the constants β and n of the residual charge current equation were obtained for small values of the time t, by means of a Helmholtz pendulum.

Another method of determining n is suggested by equation (13) which, on substituting for G, is:

$$\begin{aligned} C &= C_0\left[1 + \omega^{n-1}\beta\Gamma(1 - n)\cos\frac{(1 - n)\pi}{2}\right] \\ &= C_0(1 + \text{const. } \omega^{n-1}) \\ &= C_0\left(1 + \frac{M}{f^{1-n}}\right), \end{aligned} \tag{17}$$

where M is a constant, and f is not zero.
Hence

$$\log C = \log C_0 + \log\left(1 + \frac{M}{f^{n-1}}\right)$$

and

$$\Delta \log C = \Delta \log\left(1 + \frac{M}{f^{1-n}}\right). \tag{18}$$

By equation (18), the following values were found from the $\log C - \log f$ curve for Glass No. 2, using the points: f = 1,000, 10,000 and 100,000:

$$n = 0.855, \qquad M = 0.182, \qquad C_0 = 857.0\ \mu.\mu.f.$$

So far as known, no values of n determined by this method have been published. Between 1,000 and 100,000, the capacity change was 27.5 $\mu.\mu.f.$, which may be measured to within one per cent.

Equation (17) shows that this change is a function of M as well as of n, and M is proportional to β. No idea of the change in capacity can, therefore, be obtained from the value of n alone and it is only when M is fairly large that accurate values of n can be obtained by this method. The same remark applied, of course, to the determination of β and n from the current due to residual charge. For example, neither method could be used with paraffin condensers in which there is practically no change in capacity with frequency.

The two values of n obtained for Glass No. 2, viz.,

(1) From variation of power loss: $n = 0.86$,
(2) From variation of capacity: $n = 0.855$,

[1] *Loc. cit.*

are in good agreement. By substituting the first in equation (15) together with the value of C_0 obtained above, the constant β was found to to be .0370. The value of the power loss P was taken from the curve in Fig. 7. By substituting the second value of n in equation (17), β was found to be .0379. These values agree to within 2.5 per cent. and are important as a check on the accuracy of the power loss actually obtained.

The results of this investigation are, therefore, in agreement with the theory given above. They show that the power loss P may be expressed by the equation: $P = Bf^n$ over the wide range of frequency used and there is apparently no reason why this equation should not hold at higher and lower frequencies. Although the actual losses may vary considerably, the value of n does not. For instance, the loss in Glass No. 2 is twenty-four times as great as that in paraffin and yet the corresponding values of n are 0.86 and 0.85, respectively. The condenser with the smaller loss may also have the larger value of n.

A comparison of the energy losses of various dielectrics may be conveniently made by means of their power factors, since the power factor is a function of the dielectric and does not vary with the capacity of the condenser. The power factors of the dielectrics studied are given in

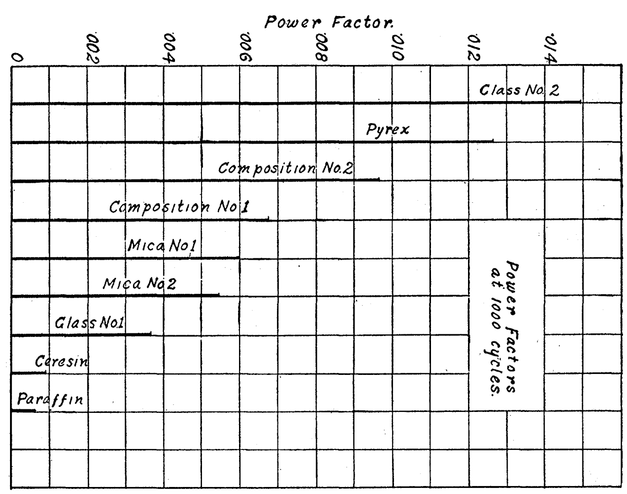

Fig. 9.

Fig. 9 for a frequency of 1,000 cycles. This figure shows clearly the comparatively small loss in paraffin and ceresin wax.

The power factor was taken as $R\omega C$, the tangent of the phase difference. It is greatest in glass No. 2 where the phase difference is about

51 minutes at 1000 cycles. The error caused by taking the tangent in place of the sine is less than .05 per cent.

From the equations for resistance and power loss, it follows that the power factor may be represented by the equation $P.F = D/f^{(k-n)/2}$, where n and k have the values given above and those of D are as follows:

Condenser	D
Glass No. 1	.00714
Glass No. 2	.0373
Pyrex	.0264
Paraffin	.00174
Ceresin	.00225
Mica No. 1	.0132

Since the log C — log f curve is not exactly a straight line, the equation for the variation of the equivalent resistance, viz., $R = A/f^k$, is not absolutely consistent with the power loss equation. However, in the case of Glass No. 2 between 1,000 and 1,000,000 cycles, the deviation from the mean straight line is never more than about 0.22 per cent. It is considerably less in most of the other condensers. The effect of this deviation is well within the limits of experimental error and could not be observed on any of the resistance or power loss curves.[1]

The writer acknowledges with pleasure his indebtedness to Professor G. W. Pierce, under whose direction this work was carried out, for his interest, suggestions and criticism; also to Dr. E. L. Chaffee and Mr. R. F. Field for the frequent loan of special apparatus and for their assistance in other ways.

Cruft Laboratory,
Harvard University,
June 28, 1922.

[1] Note added January 30: It may be noticed, however, that there is a tendency for the points to lie slightly above the straight lines at high frequencies. This may be accounted for by the fact that the resistance of the plates and leads, though small and practically constant, is becoming of importance. By plotting the curves on a much larger scale, it was found that a small constant correction for points above 100,000 cycles brought them practically on the straight line.

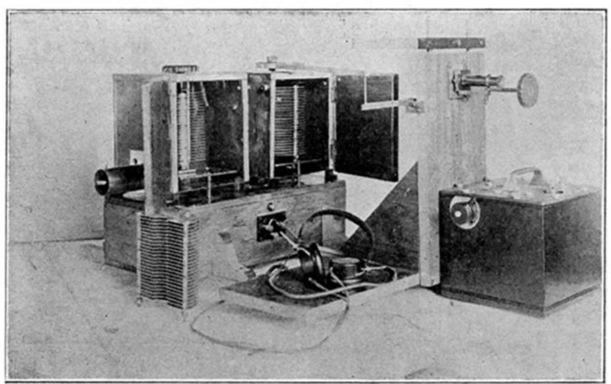

Fig. 2. The bridge.