A METHOD FOR THE NEUTRALIZATION OF ELECTRON SPACE CHARGE BY POSITIVE IONIZATION AT VERY LOW GAS PRESSURES.

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ABSTRACT.

Neutralization of space charge around a hot filament by imprisoned positive ions in gas at very low pressures.—(1) *Design of tube.* If a very small filament, diameter 0.01 cm, is run axially through a cylindrical anode with closed ends, positive ions formed between the electrodes can only rarely escape and will describe orbits around the filament until they lose sufficient energy by collision with gas molecules to enable them to fall into the cathode. The imprisoned ions, during their lives, neutralize a certain amount of the space charge between the electrodes. The effect should increase with the absolute temperature and with the ratio of the cross-section of the anode cylinder to that of the filament. A theoretical calculation indicates that in He at 10^{-5} mm, an ion which misses the filament on its first passage across the tube may circulate around the filament 300 times before discharging to the cathode. (2) *Results in He, H, Ne, and Hg* at pressures ranging from 10^{-2} to 10^{-7} mm are shown in curves. Comparison of the currents when the cylinder ends were connected (1) to the anode and (2) to the cathode gave the effect of the positive ions, and also their number, and therefore the increase in current per ion, α. At pressures so low that the 3/2 power space-charge law holds in an ordinary tube, imprisoned ions may still produce large deviations from this law, in favorable cases the current with the ions being 5 or 10 times the current as ordinarily limited by space charge. The relatively much greater effect in Hg vapor than in He is shown to agree well with the theory. If α depended only on mean free path, however, it should vary as $1/p$, but the results gave $(1/p)^{2/3}$. This difference shows the influence of other factors. However, theoretical mean free paths are of the same order of magnitude as those calculated from α. Moreover, in Hg vapor at 4.2×10^{-7}mm, the time required to reach equilibrium after the positive ions began to accumulate was found by oscillograms to be about the same as the calculated life of an ion, 1.4×10^{-3} sec. The simple theory, then, seems to be pretty well verified.

I. INTRODUCTION.

THE residual gas pressure in a high vacuum thermionic device is usually less than 10^{-4} mm Hg, and the normal free paths of the electrons are of the order of several hundred centimeters. The few positive ions formed carry an insignificant fraction of the current across the tube and cannot ordinarily contribute much to the neutralization of electron space charge as they soon discharge to the cathode or move out from the space between the electrodes.

If, however, the cathode is a straight wire, and the anode is a coaxial cylinder of very much larger radius, many of the positive ions formed near the anode are unable to reach the cathode on the first passage across the tube, because of their initial velocity transverse to the radius due to thermal energy. The escape of such ions from between the electrodes can be prevented to a great extent by making the anode a cylinder with closed ends, through which small holes are pierced to admit the cathode filament. An ion imprisoned in this way executes orbits to and fro across the tube, until in some way it loses its velocity transverse to the radius, and then discharges to the cathode.

II. Theory.

The general case of the motion of an ion in electric and magnetic fields between coaxial cylinders has been investigated by Hull.[1] We shall take the case in which there is no magnetic field and the radius of the anode cylinder is much greater than that of the cathode. Following Hull's notation let V be the potential difference between the electrodes; R and r be the radius of anode and cathode, respectively; e and m be the charge and mass of an ion; and v_0 be the initial velocity of the ion transverse to the radius. The initial radial velocity of the ion can be neglected as it is small compared with the velocity acquired in falling through the potential difference V.

The condition[2] that a positive ion starting from a point immediately inside the anode shall just fail to strike the cathode is that V shall be less than the value given by $eV = 1/2mv_0^2(R/r)^2$.

From this equation we see that:

(1) For given V and R/r the same proportion of ions formed in any gas will fail to reach the cathode, since $1/2mv_0^2$ is the same for all gases at the same temperature.

(2) The number of ions missing the filament increases rapidly with increase of R/r over a certain range of values of this ratio.

(3) The higher the temperature of the gas in the tube, the greater will be the value of $1/2mv_0^2$, and the greater will be the proportion of ions missing the cathode.

(4) The value of R/r necessary so that the average ion will just miss the cathode increases as the square root of V.

III. Experiments.

Apparatus.—Most of the experimental work was done with two tubes denoted respectively by K-26 and K-27.

[1] A. W. Hull, Phys. Rev., 18, 31 (1921).

[2] From Eq. (10) of Hull's paper, putting $H = 0$.

Tube K-26 had a cathode C (Fig. 1) of 4 mil (0.01 cm) tungsten wire, and an anode A of thin molybdenum, 0.75 inch (1.9 cm) in diameter, and 1.4 inches (3.6 cm) long. Two guards to imprison the ions between A and C were mounted at GG. In this tube the guards were made of six spokes of 10 mil tungsten wire evenly spaced around a nickel ring, 1.9 cm in diameter. The spokes did not reach the centre of this ring, and left a space 2 mm in diameter for the cathode to pass through. The filament was welded to stouter leads outside these guards. The electrode system was assembled on a glass frame and then mounted as one unit inside a hard glass tube.

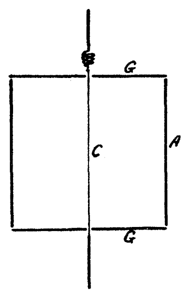

Fig. 1. Design of experimental tube.

Tube K-27 was of the same general construction. It had a 4 mil filament and an anode cylinder 2 inches (5.1 cm) in diameter and 2 inches long. The guards were in this case molybdenum discs, 2 inches in diameter, pierced with 52 mil (0.13 cm) holes to admit the cathode filament.

Gas manipulation.—The tube was thoroughly baked out, and the electrodes bombarded. The desired gas was admitted, and measurements made with the tube shut off by a mercury seal between the liquid air trap and the pump. The higher gas pressures were measured with a McLeod gauge, and the lower with an ionization gauge.

The gases used were helium, hydrogen, neon, and mercury vapor. The first three were kept in contact with charcoal cooled in liquid air throughout the experiment. The pressure of mercury vapor was determined from the temperature of a U tube in which mercury vapor was condensing continually from a bulb at higher temperature. The pump was kept running during the measurements with mercury.

Typical measurements.—Two types of volt-ampere characteristics, as a rule, were taken at each pressure. In the first the guards were connected to the negative end of the filament ($V_g = 0$), and any positive ions formed could therefore discharge at the ends of the cylinder. In the second type of characteristic the guard plates were connected to the anode cylinder, and the escape of positive ions through the ends of the cylinder was thus prevented.

Examples of both types of characteristic are given in Figs. 2 and 3.

Fig. 2 shows the effect of neon in tube K-26. The lower curves are with the guard plates at zero volts ($V_g = 0$), and in the upper curves they are connected to the anode cylinder ($V_g = V_p$). At a pressure of $13(10)^{-5}$ mm the tube shows a fairly good vacuum characteristic with $V_g = 0$, the logarithmic plot of current against voltage between the anode and the negative end of the filament, being approximately a straight line. With $V_g = V_p$ the characteristic shows the effects of

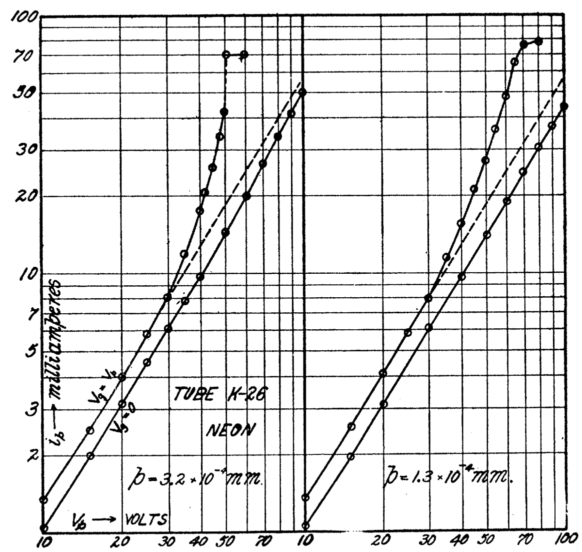

Fig. 2. Logarithmic plot of volt-ampere characteristics of tube K-26 at low pressures of neon.

positive ionization markedly above 30 volts. The $V_g = 0$ characteristic lies below the other on account of the electrostatic screening action of the guards when they are connected to the filament.

Fig. 3 shows the effect at various low pressures of mercury vapor in tube K-27. With this tube the effect is much stronger than with K-26 on account of the greater ratio of anode to cathode diameters. The pressures of mercury vapor ranged from 10^{-7} mm to 10^{-6} mm. The lowest curves, as before, were taken with $V_g = 0$, and hardly show the effects of positive ionization. They lie below the others on account of the electrostatic screening action of the guards, which is much greater in this case than before on account of the greater diameter of the cylinder.

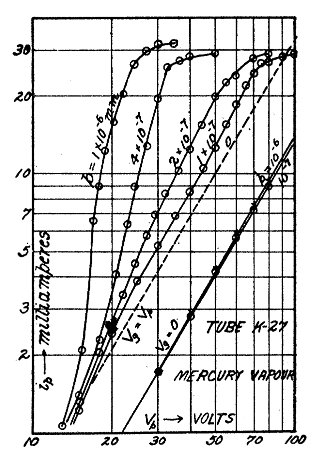

Fig. 3. Logarithmic plot of volt-ampere characteristics of tube K-27 at low pressures of mercury vapor.

These figures illustrate the point that a gas pressure which in an ordinary electron discharge tube is insufficient to disturb the limitation of current by electron space charge may produce almost complete elimination of space charge if the tube is designed so as to imprison the positive ions between the electrodes.

Some measurements were made to estimate the amount of positive ionization occurring in each tube at various gas pressures. The guard plates were made negative to the cathode, and the positive ion current flowing to them measured with various voltages applied to the cylinder. The negative voltage was made high enough to obtain an approximately saturated positive ion current. It should be noted in this connection, however, that practically only those positive ions were collected which missed the cathode on their first passage across the tube, and subsequently drifted towards the ends; for the field due to the guard plates would not be strong enough to pull the other ions out from the center of the cylinder before they struck the cathode. These measurements of ionization current are however significant for our purpose, for it is precisely those ions which do not discharge to the cathode on their first passage across the tube which are most effective in neutralizing electron space charge.

IV. Discussion of Results.

In Figs. 2 and 3 the dotted lines give the current which would flow across the tube in a perfect vacuum with $V_g = V_p$. At any voltage, the difference between the ordinates of the full curve and the dotted line represents the extra electron current which flows as the result of the neutralization of space charge by positive ionization. Denote this increase by Δi_p; and let the positive ion current associated with it be i_g. Then the space-charge effect of Δi_p electrons is neutralized by i_g positive ions, and hence the ratio $\Delta i_p / i_g = \alpha$ represents what may be termed the effectiveness of a positive ion in neutralizing electron space charge.

The factors which limit the lives of the positive ions between the electrodes, and consequently limit the value of α are as follows:

(1) Discharge of the ions to the cathode;
(2) Leakage of the ions through the holes in the guard plates;
(3) Recombination with electrons;
(4) Effects of the collisions of ions with gas molecules.

Rough calculations show that the effects of (2) and (3) are small. Some of the ions strike the cathode on their first passage across the tube. Those that do not should continue to oscillate back and forth across the tube until their orbits are disturbed in some way.

The experimental evidence seems to point to (4), the collisions between ions and molecules, as being the chief cause limiting the lives of the positive ions. The value of α is found to be very closely connected with the gas pressure, and consequently with the mean free path of the ions. This is exemplified in Fig. 4 which shows the logarithm of α plotted against the logarithm of the reciprocal of the gas pressure in microns Hg. The data shown were obtained with helium in tube K-27, with 35 volts

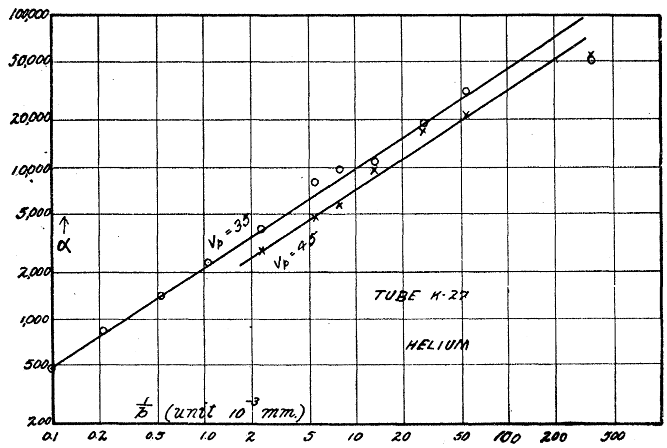

Fig. 4. Logarithmic plot showing the relation between the reciprocal of the pressure and the number of electrons whose space-charge effect is neutralized by one positive ion.

on the cylinder for the upper curve and 45 volts for the lower. The points lie approximately on a straight line of slope 2/3, and the existence of this systematic relation between α and $1/p$ indicates that there is a close connection between the lives of the ions and their mean free path through the gas. The free path cannot, however, be the only factor entering, for, if it were, the slope of the curves in Fig. 4 should be 1 instead of 2/3. There seems to be no simple theoretical reason why the 2/3 power of $(1/p)$ should replace the first, and this difference is probably due to the other factors limiting the lives of the ions as outlined above.

Referring to the previous enumeration we may say that as regards factor (3) a constant fraction of the ions formed at any pressure will recombine with electrons. This cause therefore cannot account for the change in slope. The second factor, however, provides a correction of the right sort, for the rate of flow of ions through the guards will be nearly independent of the gas pressure, and the ions lost in this way will be a larger fraction of the total number formed at low pressures than at high. A rough estimate of this leakage of ions gave a correction of the right kind and one which became quite important at the lower pressures used. Again, as regards factor (1), a constant fraction of the ions formed at any pressure will discharge to the cathode on their first passage across the tube. The ions measured in i_0 were only those which missed the cathode on their first transit, whereas both classes of ions contribute to the neutralization of space charge. It is evident that the contribution of the ions striking the cathode becomes more important the higher the pressure for at these pressures no ion can live long in the tube. This means that the values of α obtained at the higher pressures were too large, and the leakage correction shows that the values of α found at low pressures are too small. If these corrections could be allowed for, the slope of the curve in Fig. 4 would be increased, probably from 2/3 to 1.

Comparison with kinetic theory.—From the electrical experimental data it is possible to make an estimate of the distance traversed by a positive ion during its life between the electrodes, and this may be compared with the average free path of an ion calculated from kinetic theory.

Take the case of a helium ion in tube K-27 starting from a point just inside the anode and moving inwards under a potential difference of 35 volts between anode and cathode. Approximate calculation shows that the time (τ_+) taken to move from the anode to the point of the path closest to the cathode is about 1.4×10^{-6} sec. A similar calculation for the time (τ_-) taken for an electron to move from cathode to anode under a potential difference of 35 volts gives 1.08×10^{-8} sec. Now τ_+ and τ_- give the order of magnitude of the contributions to space charge of an ion and an electron for one passage each between cathode and anode. Hence one ion moving once from anode to cathode will neutralize the space charge of $\tau_+/\tau_- = 130$ electrons, and will permit this number of electrons in excess of the normal electron space-charge current to flow across the tube. It is worth noting that τ_+/τ_- is somewhat greater than the square-root of the ratio of the masses of a helium ion and an electron (86) which would determine their relative velocities after falling through the same potential difference. This is due to the potential distribution between the electrodes.

We have seen above that actually α electrons are released per positive ion. Hence $\alpha/(\tau_+/\tau_-)$ must measure the number of times which a positive ion crosses the tube; and if R be the radius of the anode, $\alpha R(\tau_-/\tau_+) = L_1$ is the distance travelled by the ion before discharging to the cathode.

The average distance (L_2), calculated from kinetic theory, which an ion travels before colliding with a gas atom may be found by multiplying the average free path of a gas molecule by $\sqrt{2}$ to take account of the high velocity of the ion. The temperature of the gas was estimated at 500° K. Fig. 5 shows the logarithms of L_1 and L_2 plotted against the logarithm of the reciprocal of the pressure. The free paths calculated in the two ways are seen to be of the same order of magnitude. The smaller slope of the L_1 curve is probably accounted for by the factors discussed for Fig. 4.

The calculation of L_1 is not sufficiently precise to determine exactly how many collisions with gas atoms an ion makes on the average before discharging to the filament. In order to strike the filament the component of velocity of the ion transverse to the radius must be small,

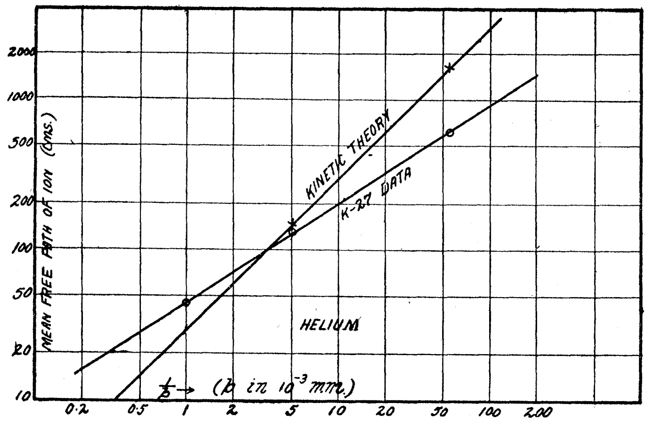

Fig. 5.

and the effect of collisions must be to reduce this component to the necessary value. Horton and Davies,[1] and Saxton[2] have found that positive ions lose energy on colliding with gas molecules. In the present experiments it seems likely that at the first collision the ion will lose a considerable part of its energy, and will not be able to move out as close to the anode as it did before. Its total velocity will be smaller, but its transverse velocity probably will be larger than before the collision. It is therefore unlikely that the ion will strike the filament immediately after the first collision. Succeeding collisions reduce the energy of the ion still further, and force it to follow paths which on the average get closer and closer to the filament. Finally after perhaps three or four collisions the total energy of the ion is so low that it is able to strike the cathode.

Comparison of results for mercury and helium.—The first experiments made with mercury vapor showed at once that a low pressure of mercury gave as great neutralization of space charge as a much higher pressure of helium. This difference is due partly to the lower ionization potential of mercury and partly to the great mass and low velocity of the mercury ions. More ions are formed in mercury at a given pressure than in helium, and each ion stays in the tube for a longer time. From the experimental data we can calculate how long each kind of ion stays in

[1] Horton and Davies, Proc. Roy. Soc., **95**, 349 (1919).
[2] Saxton, Phil. Mag., **44**, 822 (1922).

the tube, and can see what relation this bears to its effectiveness in neutralizing electron space charge. A comparison of the two gases was carried out in this way, for the difference between their effects was so striking that it was thought at first that some new factor must enter in the case of mercury. The comparison showed that the simple theory of the effect described above accounted quite satisfactorily for the difference in the results obtained with the two gases.

The mean free path of an ion at a given pressure was calculated by the kinetic theory formula, and this distance was divided by the average velocity of the ion under the electric field. This gave a time proportional to the life of the ion and this time should be proportional to the effectiveness of the ion in neutralizing space charge as measured by α.

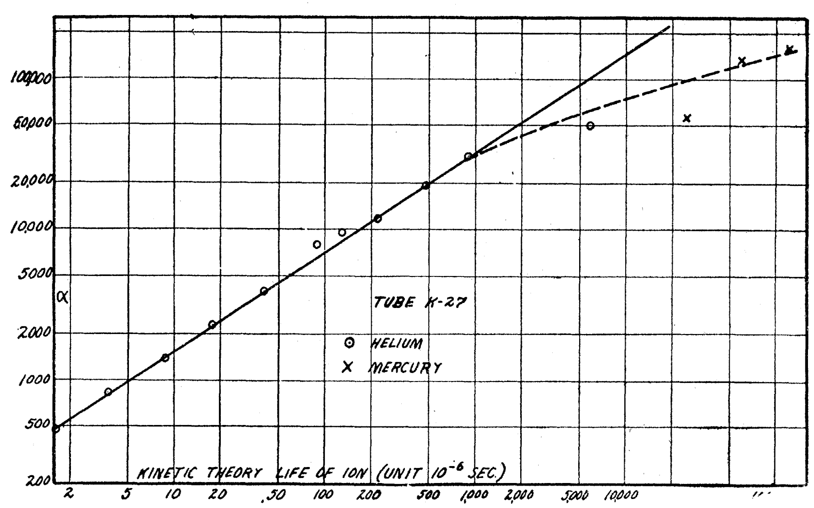

Fig. 6. Comparison of effectiveness of mercury and helium ions in neutralizing electron space charge.

In Fig. 6 the logarithm of the interval between collisions is plotted against the logarithm of α. The points obtained from the helium data lie on a line of slope 2/3 as before (Fig. 4). The mercury points do not fall on this line but lie considerably below it. The agreement is, however, sufficiently good to show that the effectiveness of any kind of ion in neutralizing space charge is proportional to its mean free path and the reciprocal of its velocity. The probable reason why the mercury points lie below the line is as follows. The pressures of mercury used were very low, ranging from 10^{-6} to 10^{-7} mm. Observations could not be made at higher pressures as the plate current reached saturation. The pressures were calculated from the temperature of the mercury, and this made no allowance for the presence of residual gas, the pressure of which was probably of the order 10^{-6} mm. The mercury ions were therefore not able to traverse their calculated free paths. The escape of the ions through the end guards would also tend to lower the values of α at such low pressures.

The paths of the ions.—A rough calculation of the path of an ion moving towards the cathode was made from the equations of motion of the ion. Fig. 7 shows the path of a positive mercury ion moving from just inside an anode cylinder of 1 cm radius towards an axial cathode, under a potential difference of 35 volts. The initial thermal velocity of the ion was taken as that due to a temperature of 300° K and was assumed to be all transverse to the radius.

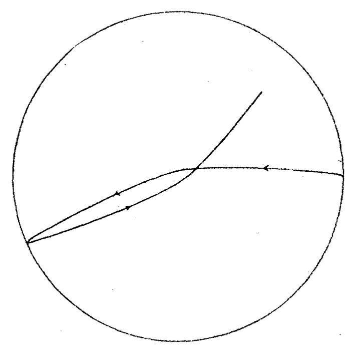

Fig. 7.

From the experimental data for tube K-27 we can form an estimate of the number of times which an ion crosses the tube. Let us assume that all the ions come from near the anode, and that by "crossing the tube" we mean that an ion moves from near the anode to near the cathode. Each electron crosses the tube only once. Then as shown above, $\alpha\tau_{-}/\tau_{+}$ will give the number of times which an ion crosses the tube. For helium in tube K-27 this becomes $7.7(10)^{-3}\alpha$ for an anode voltage of 35.

Fig. 8 shows the logarithm of the number of times of passage plotted against the logarithm of the reciprocal of the pressure for helium. Thus

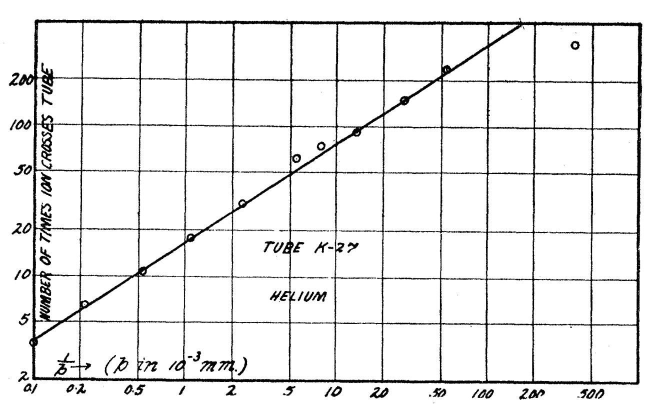

Fig. 8. Logarithmic plot showing number of times which an imprisoned helium ion crosses tube K-27 as a function of gas pressure.

at a pressure of 10^{-5} mm a helium ion may cross the tube 350 times before losing its energy, and discharging to the cathode.

Check by oscillograph records.—If the guard plates in tube K-27 are initially at the same potential as the cathode and then are abruptly raised to the anode potential, the number of positive ions imprisoned between the electrodes will increase to an equilibrium value at which the number of ions formed per second is equal to the number disappearing. The time taken for reaching this equilibrium will be of the same order of magnitude as the life of a single ion. Therefore if the anode voltage is applied intermittently to the guards by a commutator, and an oscillogram is taken of the corresponding changes in anode current, the time required for this current to reach say half its maximum value will be of the same order of magnitude as the life of an ion in the tube.

Oscillograms were taken at low pressures of mercury vapor, and they showed the time-lag of the anode current in rising to its maximum value very clearly. The initial sudden rise in the current due to the electrostatic action of the guards when the anode voltage was applied to them could be distinguished easily on the records, from the slower rise due to the neutralization of electron space charge by the gradual accumulation of positive ions.

The lives of the ions measured in this way were of the same order of magnitude as those calculated by the other methods given above. Thus with mercury vapor at a pressure of 4.2×10^{-7} mm and an anode voltage of 30, the time taken for the anode current to reach half-value was 7.8×10^{-3} sec. whereas the calculated life of an ion under the same conditions was 1.4×10^{-3} sec.

It is a pleasure to express my thanks to Dr. Irving Langmuir for many valuable suggestions during the course of the work. The theory developed here is based on his suggestion that the effects observed were due to imprisoned positive ions.

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