X-RAY ISOCHROMATS OF MOLYBDENUM.

By D. L. Webster and A. E. Hennings.

Abstract.

Variation of intensity of x-radiation from molybdenum with voltage, for wave-lengths 0.246 to 123 A.—The ordinary x-ray spectrum, intensity as function of wave-length, cannot be interpreted in terms of energy because of the unknown variation of several factors with frequency. In the case of isochromats, intensity per cathode ray for a given wave-length as function of voltage, the problem is simpler. *Measurements up to twice the quantum voltage* were made in each case except where that would exceed 70·kv. and both voltage and current were determined to 1/10 per cent. The angle between the x-rays and the beam of cathode rays was about 110°. To avoid correcting for the variation of size of focal spot, a single narrow slit was used near the crystal. The correction for crystal scattering was determined from measurements on each side of the reflected beam; it was less than one per cent. except near the quantum voltage. *The correction for target absorption* was taken to be $\mu\bar{x}$ where $\bar{x}$, the mean depth of penetration, was computed from the data reported in the preceding article. After making these corrections, the isochromats were found to be straight lines except for some downward concavity near the feet. The results for 0.41 to 1.23 Å agree within 1 or 2 per cent with an *empirical equation for isochromats* which is given. The values for the shorter wave-lengths are too unreliable at high voltages to serve as tests. From the above equation, an expression for the *radiation from a vanishingly thin target* is derived which involves b the coefficient of the Thomson-Whiddington law. If this law and the V^2 law for total intensity are correct, the *probability of emission of a quantum* with frequency between ν and $\nu + d\nu$ in distance ds is $idsd\nu/h\nu$, where $idsd\nu$ is the intensity per cathode ray, with the thin target.

I. The Problem.

An x-ray spectrum, as ordinarily plotted, is a graph of ionization against wave-length or glancing angle. To interpret it in terms of energy, we must know several factors, such as the efficiency of reflection of the crystal and the energy value of the ionization unit, all of which vary from one wave-length to another, in ways that are at present unknown. In an isochromat, or graph of intensity against potential for a single frequency, most of these unknown functions of the frequency can be confined to a factor that, while still unknown, is at least constant for any one isochromat. Consequently, the form of the isochromat can be determined, except for a constant factor, much more readily than that of a spectrum.

There are, however, two factors that are variable even here. One is the diffuse scattering of x-rays by the crystal, that results in a slight impurity in the spectrum, like the stray light in an optical spectrometer. The other is the absorption of some of the rays in getting out of the target, that depends on the depth at which they are excited, and consequently on the potential driving the cathode rays. Both factors produce only small errors, often negligible in a first approximation, but important when an accuracy of one per cent. or so is desired.

The purpose of the present paper is to take a set of data, correcting for these factors, on the rays most convenient for this purpose, and to see what such results signify about the mechanism of electron collisions and radiation. For a brief account of the history of this problem, the reader is referred to the Report of the X-ray Committee of the National Research Council.[1]

II. Apparatus.

The apparatus used here was that of the preceding paper, and needs no further discussion, except to emphasize two points: First is the accuracy of the calibration of the high tension D.C. voltmeter, which was not especially important there. The 6-megohm resistance used in calibrating it was measured with apparatus standardized by a set of new 10,000-, 1,000- and 100-ohm coils, certified by the Bureau of Standards. The current through it was measured by the potential drop it produced in the 100-ohm standard inserted in the middle of the line and connected to a new potentiometer and a new Weston cell, both from Leeds and Northrup, and both never put in the hands of students. The milliammeter, like the voltmeter, was calibrated with this apparatus. The calibration of the voltmeter was repeated several times, and although at first there were slight changes, due to elastic after-effects in a new bifilar suspension, the readings at the time these isochromats were taken, allowing for fluctuations and all other disturbing factors, must be better than 1/10 of a per cent.

The other important point is that the x-ray beam was limited by only one slit, placed near the crystal. The slit in front of the ionization chamber was wider than the beam entering it, and the one near the tube was wider than the focal spot, so that neither of them could limit the width of the x-ray beam. These precautions are needed because, as one may predict from space-charge theory, or prove by experiment, the focal spot is wider at low voltages and high currents than at high voltages and low currents.

The cathode rays, allowing for electrostatic deflection, probably strike the target at about 10° to the axis of the tube, and the thin glass window

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[1] Bull. N. R. C., *1*, No. 7, Dec., 1920.

through which the x-rays were taken out was at about 100° from the anode end. This makes the angle of the x-rays from the line of the cathode rays 110°.

III. The Correction for Scattering.

The correction mentioned above, for x-rays scattered by the crystal, is like the correction of an ordinary spectro-photometric measurement for stray light of other colors. Here it was estimated by setting the crystal to reflect some frequency to be used for an isochromat, and then moving the ionization chamber to various positions on each side of the reflected rays, and measuring the rays scattered to these positions. Plotting these scattered rays against the chamber angle, it was usually easy to interpolate graphically for the scattered rays at the reflecting position.

The correction, while often about 10 times as large at twice the quantum voltage as at the quantum voltage itself, was relatively unimportant except near the quantum voltage, where the reflected rays were very weak. Elsewhere it rarely amounted to more than one per cent. of the reflected rays.

There were, however, certain very peculiar effects in this scattering, where lines appear, like weak reflected lines at angles different from the normal reflection. Some of these "ghosts," as we may call them by analogy to those from a grating, could be traced by their excitation potentials and the forms of their isochromats, to small bits of crystal out of the normal alignment, reflecting sometimes rays of the continuous spectrum and sometimes the characteristic lines. Other ghosts, however, followed laws unlike those of such reflections. Duane[1] has described similar effects from a potassium iodide crystal, which he ascribed to fluorescence of the iodine, but these are of some other origin. The ones found here may possibly be due to fragments of crystal that are bent, as well as out of alignment. They are mentioned here chiefly because one of them, unfortunately, moved into coincidence with the reflected beam at small angles. It spoiled the isochromats with quantum voltages above 30 kv. ($\lambda = 0.4117$ Å.), at least for any accurate interpretation. Probably those with quantum voltages 40 and 50 contain errors as large as 3 or 4 per cent., at 70 kv., and are correct only below about 50 and 60 kv., respectively.

An important fact about the continuous scattering, as distinguished from these ghosts, is that some very faint rays are found, at any frequency, below the quantum voltage. Such rays, however, appear to

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[1] W. Duane, Amer. Phys. Soc., Meeting of April, 1922.

be due to scattering only. The importance of this is that if any rays were emitted by the mechanism of the old pulse theory, the only way we could detect them would be by their presence under such conditions. We may now say that the intensities of any such rays are well under one per cent. of the rays found at the same frequencies at twice the quantum voltage. A few measurements with tungsten at 0.3 Å., made to test this point, gave an upper limit 1/10,000 of the corresponding intensity.

IV. The Correction for Target Absorption.

The target absorption, as explained in the preceding paper, is not predictable accurately from any theory now at hand. The measurements recorded there show the absorption at the K limit, but elsewhere we are dependent on rough estimates. Following Ham,[1] and Davey,[2] who found mean depths of excitation, for all frequencies together, increasing in proportion to the voltage, we may as well assume the same here, in a form modified to fit the requirements of a single frequency and the results of the preceding paper. For this, we shall assume a mean depth law for any frequency, with the same algebraic form as at the absorption limit and leading to the same mean depth at a high voltage for rays of any two frequencies. Specifically, the law assumed was

$$\frac{I_{\text{cor}} - I_{\text{obs}}}{I_{\text{obs}}} = \mu\bar{x}, \qquad \text{where} \qquad \bar{x} = gV\left(1 - e^{-15\left(\frac{V}{H\nu} - 1\right)}\right),$$

$$g = 0.049\,\frac{\text{micron}}{\text{kilovolt}} \qquad \text{and} \qquad H = \frac{h}{e}$$

This correction law fits the data at the absorption limit within about one per cent., and is probably as good as anything now available for other frequencies. The absorption coefficients, μ, were taken from data by Richtmyer,[3] and were all found by interpolation except the values at the absorption limit itself, which required short extrapolations. These values were assumed to be 752 cm.$^{-1}$ and 113 cm.$^{-1}$, the ratio between them being 6.65.

V. Reduction to Comparable Scales.

To compare the different isochromats, we have corrected them first for scattering and then for target absorption. Then we have reduced them all to corresponding scales of voltage and intensity. The voltage, V, has been expressed in terms of the ratio, $V/H\nu$ (where $H = h/e$

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[1] W. R. Ham, Phys. Rev., *30*, 1, Jan., 1910.

[2] W. P. Davey, J. Franklin Inst., March, 1911.

[3] F. K. Richtmyer, Phys. Rev., *18*, 13, July, 1921.

$= 4.117 \times 10^{-15}$ volt seconds), the intensity in terms of the ratio I/I_2 where I_2 is the intensity at $V/H\nu = 2$. Thus all the graphs start at the point 1, 0, and go through the point 2, 1.

Beyond this point, it is useless to follow them, because when $V = 2H\nu$, rays are excited whose wave-length is half that of the one in question, and the second order reflection of that wave-length is mixed with the rays to be measured. An attempt was made to eliminate this by the use of a (111) face of fluorite, but no fluorite could be found that was regular enough to use without serious complications. Therefore the present results run only to the point 2, 1. Some, in fact, stop short of that, because gas in the tube prevented the use of any voltages above about 70 kv., and the ones starting at 40 and 50 kv. are unreliable above about 50 and 60, respectively, because of the unknown effect of the "ghost" that is mixed with them above those voltages.

The results are presented diagrammatically in Fig. 1. Each iso-

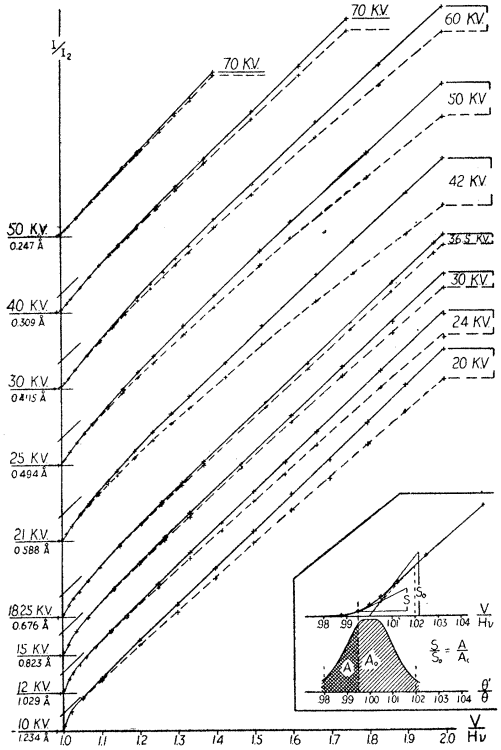

Fig. 1.

Molybdenum isochromats at 110° from the line of the cathode rays. Full lines, corrected for absorption, dotted lines, observed.

chromat is labeled in terms of its quantum potential, $H\nu$, and its wave-length. The wave-lengths are chosen to give round numbers for $H\nu$, except where such numbers would bring the wave-lengths too near the characteristic lines. 18.25 kv. gives a wave-length between the α and β lines, at the point where the stray rays from these lines are a minimum. This graph may be slightly distorted by such rays, making it a little too straight, but the effect is probably not over 1 or 2 per cent., if that much. The 21 kv. wave-length is on the short-wave side of the absorption limit, as will be seen from the size of its absorption correction, as compared to that of 18.25, while the 15 kv. is for a longer wave-length than $K\alpha$.

All these corrected graphs are straight, except for the downward curvature near the foot. The similarity in this respect is especially noteworthy when compared to the differences in the dotted graphs, that represent the same measurements not corrected for absorption.

VI. The Intercepts on the V Axis.

At the foot of each isochromat, the graph should strike the axis at exactly the calculated potential, provided that all the rays used were of exactly the intended wave-length. But of course they can not be so, because of the finite widths of the source and slit. Instead, there is a distribution of wave-lengths over a finite angle, and each isochromat rounds off concave upward, for a short range at the foot.

If the rays were uniformly distributed over their range of glancing angles, $d\theta$, then the width of the foot of the isochromat would be dV, where

$$\frac{dV}{V} = \frac{d\lambda}{\lambda} \text{ or nearly } \frac{d\theta}{\theta} \cdot$$

Thus on the ratio scale, $V/H\nu$, the width of the foot, being $dV/H\nu$, should be inversely proportional to θ, or directly proportional to ν, as one may see it is, qualitatively at least, in Fig. 1.

Practically, we have no such simple distribution over the range $d\theta$ as this. The actual distribution depends on the distribution of cathode rays in the focal spot and the nature of the imperfections, if there are any, in the crystal. But it can be found experimentally, by plotting a short piece of a spectrum that includes the $K\beta$ line. Assuming the line to be truly monochromatic, its form in the spectrum graph represents accurately (although reversed on the scale of angles) the distribution of energy that would occur over the range $d\theta$ if the crystal were set at some point in the continuous spectrum. If this distribution were uniform, then the β line would appear on the graph as a tall rectangle. Practically, however, the β line has a form more like a probability curve, as one might expect from the factors that enter into it. With the rays limited by a 2.5-mm. focal spot and a single slit of 0.55 mm. at 77 cm. distance, the theoretical extreme width is 13.5′ of arc, and the half-value width should be considerably less. Practically, the extreme width was about that amount, but very indefinite, a small trace of line radiation being found many minutes away from the main part of the line, as described in the preceding paper, where it was explained as a result of crystal imperfections. The half-value width was about 7′, and in every case the width of the foot of the isochromat was about as calculated.

To make sure of this point, a few isochromats were taken with two slits, one as before, and the other 12 cm. from the target and 1.0 mm. wide. With the two slits, the theoretical form of the β line is a figure with a flat top and straight, sloping sides. If the slit widths are s_1 and s_2, at a distance D, the width of the flat top is $(s_1 - s_2)/D$, each sloping side s_2/D, the half-value width s_1/D and the extreme width $(s_1 + s_2)/D$. These values, with the above data, are 2.4′, 2.9′, 5.3′ and 8.2′ respectively. The experimental values of the first three were 2.0′, 3.5′, and 5.3′, and for the last, if we cut off the faint spreading at the base of the line by crystal imperfections, we should have 8.5′. This remarkably close agreement indicates a very good crystal. However, another crystal, almost as good, to all outward appearances, gave a rounded β line curve under the same conditions, that did not fit the theory nearly so well. Perhaps this means the crystal we used was better, and perhaps neither of them was as good as the above results indicate.

With the beam thus limited, we could not make any great changes of potential without causing slight errors from changes of the focal spot width, and consequently of the fraction of the cathode rays that were within the lines of the slits. These isochromats were therefore used only to check the lowest few per cent. of the voltage range. At the foot, assuming the graph for truly homogeneous rays to be nearly straight for a short range, we may predict the form of the experimental graph as follows: Transfer the β line graph, unchanged, to the angle θ, where the isochromat is to be taken. It then represents the graph one would get for another line, at that angle, if such a line existed. Then reverse it on the scale of angles, to represent the distribution of rays in a single setting in the continuous spectrum. Next, divide each angle, θ', in this last graph, by θ, thus expressing the distribution of rays in the beam on a relative basis, as in the insert in Fig. 1. Now suppose $V/H\nu$ is given some definite value, say 0.996. The fraction of the rays whose quantum voltage is then exceeded is found by drawing an ordinate in this modified β line graph at $\theta'/\theta = 0.996$, and comparing the area, A, to the left of this ordinate, with the whole area A_0. If, now, the slope of the isochromat for a truly monochromatic beam is S_0, that of a beam in which a fraction A/A_0 of the rays have their quantum voltage exceeded must be S, where

$$\frac{S}{S_0} = \frac{A}{A_0}.$$

Experimentally, this is just what occurs, as illustrated in Fig. 1, which is taken from the data at $H\nu = 40$ kv.

The value of h assumed in this calculation was that given by Birge,[1] in 1919, as an average from all the evidence then at hand. It is

$$h = (6.5543 \pm 0.0025) \times 10^{-27} \text{ erg. sec.},$$

with

$$e = 4.774 \times 10^{-10} \text{ e.s.u.},$$

If

$$c = 2.9986 \times 10^{10} \frac{\text{cm.}}{\text{sec.}},$$

this means

$$H = \frac{h}{e} = 4.117 \times 10^{-18} \text{ kv. sec.}, \quad \text{or} \quad V\lambda_{\min} = \frac{hc}{e} = 12.345 \text{ kv. Å}.$$

The present result depends on the accuracy with which our voltages could be reproduced, allowing for all fluctuations ordinarily produced by all causes. This was at least as good as 1/10 per cent. Another factor was our scale of angles. The zero of this scale was set by taking the middle of the β line to be 5° 59′ 10″, choosing this as a weighted mean of the values 5° 59′ 37″ and 5° 58′ 55″ calculated from data by Overn[2] and by Duane and Patterson,[3] respectively. In view of the great accuracy of the data summarized by Birge, this work adds nothing to our knowledge of h; but it does justify fully our taking the intercept thus calculated as that of the graph for truly monochromatic rays, of the same frequency as the average in the real beam, and considering this point on the graph at least as accurate as any other.

VII. AN EMPIRICAL EQUATION.

An empirical equation for an isochromat, used previously by one of us,[4] is

$$I(V, \nu)d\nu = k(\nu)\{(V - H\nu) + H\nu p(\nu)(1 - e^{-q(\nu)\left(\frac{V}{H\nu} - 1\right)}\}d\nu$$

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[1] R. T. Birge, PHYS. REV., *14*, 361, Oct., 1919.

[2] O. B. Overn, PHYS. REV., *18*, 350, Nov., 1921.

[3] W. Duane and R. A. Patterson, ibid., *19*, 542, May, 1922.

[4] See Bulletin of the N. R. C., *1*, No. 7, Dec., 1920, for a discussion of this equation

In this equation, $k(\nu)$, $p(\nu)$ and $q(\nu)$ are all functions of ν, to be determined, but are of course constant for any one isochromat. The second term in this formula is the one giving the curvature to the isochromat, and it becomes constant in the linear part. To find p and q, we extrapolated the linear part of the curve back to the ordinate at $V = H\nu$. If we call the ordinate of any point on this straight line $I'(V, \nu)$, then

$$I'(V, \nu) = k(\nu)\{(V - H\nu) + H\nu p(\nu)\}.$$

Thus the intercept is

$$I_1' = I(H\nu, \nu) = k(\nu)H\nu p(\nu),$$

while

$$I_2 = I(2H\nu, \nu) = k(\nu)\{(H\nu) + H\nu p(\nu)\}.$$

Solving for p, we have

$$p(\nu) = \frac{I_1'}{I_2 - I_1'}.$$

These intercepts, as found experimentally, are shown in Fig. 1, the 40- and 50-kv. isochromats being omitted because of the unreliability in these data, mentioned above, which affects calculations of this type especially badly.

To find $q(\nu)$, we have

$$I'(V, \nu) - I(V, \nu) = I_1' e^{-q(\nu)\left(\frac{V}{H\nu} - 1\right)}.$$

Thus to find q, we simply looked for the points where this difference equalled $I_1'e^{-1/2}$, $I_1'e^{-1}$ and $I_1'e^{-2}$. The values of $V/H\nu$ at these points should be

$$\left(1 + \frac{1}{2q}\right), \quad \left(1 + \frac{1}{q}\right) \quad \text{and} \quad \left(1 + \frac{2}{q}\right).$$

In general, the values of $1/q$ thus found are not far apart, showing that the formula fits the curves fairly well, and in fact if the mean of the three is chosen for q, the calculated values of I, for the higher frequencies, always agree with the experimental graphs to within an amount equal to $\frac{1}{2}$ per cent. of I_2. At the lower frequencies there is a slight systematic deviation, in the direction of too quick a decrease of $(I' - I)$ at first, and too slow later, but the difference is never over about 1 per cent. of I_2. This systematic deviation is nearly if not quite within the limits of experimental error; but it appeared also in the previous work on rhodium and platinum, and in Wagner's work[1] on copper and platinum at potentials below 10 kv., and in some preliminary work here on copper and tungsten. So it may very well be real; yet it is doubtful,

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[1] E. Wagner Ann. der Phys., *57*, 104, Dec., 1918.

because none of these data, except the present ones on molybdenum, have yet been corrected for target absorption.

To fit the isochromats with a formula containing p and q expressed algebraically, the experimental values of these functions were plotted against $H\nu$, in Fig. 2. The $p(\nu)$ data were fitted fairly well by a formula

$$p(\nu) = \sqrt{\frac{\nu}{\nu_1}},$$

where ν_1 = constant = 9.7×10^{22} sec.$^{-1}$ and $H\nu_1 = 4{,}000$ kv.

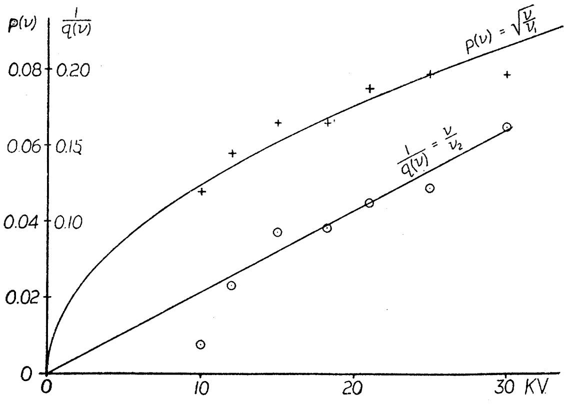

Fig. 2.

Data on $p(\nu)$ and $1/q(\nu)$, plotted against $H\nu$, with graphs of assumed laws.

Likewise the data for $q(\nu)$ were fitted fairly well by a formula

$$q(\nu) = \frac{\nu_2}{\nu},$$

where ν_2 = constant = 4.5×10^{19} sec.$^{-1}$ and $H\nu_2 = 187$ kv. With these values for p and q, the formula does not fit the data quite as well as with the empirical values, but still is rarely off by more than 1 or 2 per cent. of I_2.

VIII. Reduction to Isochromats for an Infinitely Thin Target.

In a thick target, such as the molybdenum block used here, the x-rays are not all emitted by cathode rays of the same speed, even though the potential applied to the tube is constant. Not one cathode ray in a thousand, under most conditions, emits any x-rays hard enough to get out of the tube. Instead of losing energy by large quanta in that way, most of them seem to lose it gradually by ionizing the atoms. Thus the target must contain electrons of all speeds, each one capable of producing x-rays appropriate to its speed. Only in an extremely thin target would we find anything like uniformity of speeds. From the viewpoint of radiation theory, such a target would be the most interesting.

It is also, however, the most difficult to use. Fortunately, by a method outlined in a previous paper,[1] we can calculate what such a target would give, in terms of the V derivative of I. In that paper, the result was expressed for a target of such thickness that any cathode ray would strike just one atom. There is, however, some question as to what constitutes "striking" an object of such indefinite boundaries as an atom, so it may be better to express the result in terms of the *intensity per electron and per unit thickness* from a target of infinitesimal thickness. The symbol $i(V, \nu)$ will be used here for this latter quantity, as $I(V, \nu)$ is used for the intensity per electron from the thick target. Its dimensions are therefore those of $I \div$ length, and it is to that extent different from the i of the previous paper.

By the same sort of reasoning used there, it appears that

$$i(V, \nu) = D_V I(V, \nu)\left(-\frac{dV_s}{ds}\right)_{s=0},$$

where V_s is the potential corresponding to a cathode ray's energy after penetrating a distance s, this s being measured not along the line of x, but along the crooked path of the individual cathode ray.

Although the Thomson-Whiddington law fails to give the depth of penetration, x, except for the few electrons that go straight, it is fairly probable that the path distance, s, can be found by this law. Assuming this, and stating the law in the form

$$V_s^2 = V^2 - bs,$$

we have

$$\frac{dV_s}{ds} = -\frac{b}{2V_s}.$$

Then differentiating the expression for $I(V, \nu)$ given in the last section, we find

$$D_V I(V, \nu) = k(\nu)\left\{1 + p(\nu)q(\nu)e^{-q(\nu)\left(\frac{V}{H\nu}-1\right)}\right\}$$

or, with the present values of p and q,

$$i(V, \nu) = \frac{bk}{2V}\left\{1 + \frac{\nu_2}{\sqrt{\nu_1\nu}}e^{-\frac{\nu_2}{\nu}\left(\frac{V}{H\nu}-1\right)}\right\}.$$

In this expression, the only unknown quantity is $k(\nu)$. To find it,

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[1] D. L. Webster, Phys. Rev., *9*, 220, March, 1917.

we may use a method previously described,[1] dependent on the fact that the total x-ray energy is given by the equation $E(V) = \int_0^{V/H} I(V, \nu)d\nu$.

If $E(V)$ is accurately proportional to V^2, as is usually assumed, and if $p(\nu)$ and $q(\nu)$ were constants, this equation could be satisfied only by making $k(\nu)$ a constant. The value of k would then be given by the equation

$$E(V) = k\frac{V^2}{H}\left\{\frac{1}{2} + \frac{p}{2} - p\int_0^1 xe^{-q\left(\frac{1}{x}-1\right)}\,dx\right\}$$

or approximately

$$E(V) = \tfrac{1}{2}k\frac{V^2}{H}\left\{1 + p - 2\frac{p}{q}\right\}.$$

With p and q variable, as we have them in this case, these formulas could apply only if the values used for p and $1/q$ are certain weighted means of the values at all frequencies involved. Such values, however, would be small, that for p, up to 30 kv., for example, being probably about 0.06, with $1/q$ about 0.10. This would make

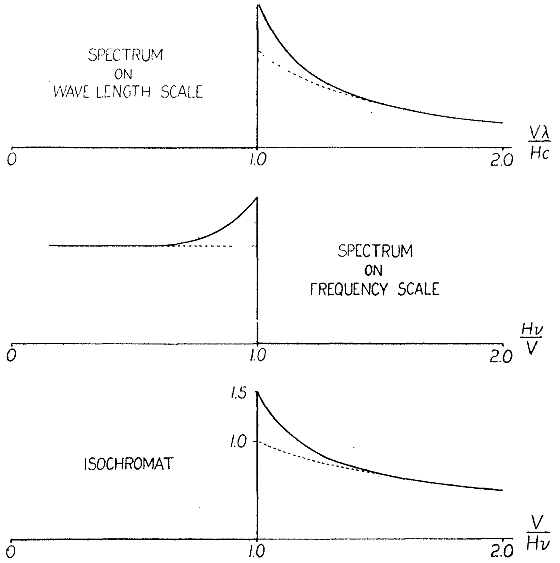

Fig. 3.

Typical thin target spectrum and isochromats, with values of p and q taken from isochromat at $\lambda = 0.412$ Å.

$$E(V) = \tfrac{1}{2}k\frac{V^2}{H}\{1 + 0.06 - 2 \times 0.06 \times 0.10\}$$

$$= \tfrac{1}{2}k\frac{V^2}{H}\{1 + 0.05\}.$$

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[1] D. L. Webster, Proc. Nat. Acad., *5*, 163, May, 1919.

The whole influence of the p, q term in I is therefore to change k by only about 5 per cent. Thus even if p and q vary considerably, the effect on k is small and may easily be as small as the errors in the V^2 law for E. So it hardly seems practical to attempt a more accurate estimate of k from the present data.

Accepting k as a constant, the thin target function may be expressed by the equation given above, or in terms of λ,

$$j(V, \lambda)d\lambda = \frac{bkc}{2V\lambda^2}\left\{1 + \frac{\sqrt{\lambda_1\lambda}}{\lambda_2}e^{-\frac{\lambda}{\lambda_2}\left(\frac{V\lambda}{Hc}-1\right)}\right\}d\lambda,$$

where $\lambda_1 = 0.0031$ Å. and $\lambda_2 = 0.066$ Å. This gives spectra and isochromats of the general type shown in Fig. 3.

IX. Electron Collisions and Radiation.

Assuming that each collision of a cathode ray with an atomic electron must either cause radiation of a whole quantum or none at all, we can find the probability of radiating any particular frequency. For if an electron penetrates a distance ds, radiating energy $i(V, \nu)d\nu ds$ in the range $d\nu$, the number of radiating collisions is

$$\frac{i(V, \nu)d\nu ds}{h\nu}.$$

Presumably, the closer the cathode electron approaches to an atomic electron, the greater will be the chance of exciting the radiating mechanism, whatever it may be. Let us assume tentatively that the distance of approach is the only factor involved; and let us specify it in terms of the distance r from the atomic electron to the straight line that the cathode ray was moving on before the collision. Thus we avoid any assumptions as to what forces deviate it from that line. In a target of thickness ds, with n electrons per unit volume, the number of electrons lying at distances between r and $r + dr$ from the straight line will be, on the average,

$$2\pi nrdrds.$$

In terms of probabilities, we may say that the probability of any electron radiating a frequency ν to $\nu + d\nu$ in a distance ds is

$$\frac{i(V, \nu)d\nu ds}{h\nu} = \frac{bk}{2h}\,\frac{\left\{1 + \frac{\nu_2}{\sqrt{\nu_1\nu}}e^{-\frac{\nu_2}{\nu}\left(\frac{V}{H\nu}-1\right)}\right\}d\nu ds}{V\nu}.$$

Defining r now as the atomic distance giving this frequency and noting that

$$\nu = \frac{V}{H}$$

only on a direct hit, with $r = 0$, we have

$$\int_0^r 2\pi nr dr = \int_\nu^{V/H} \frac{i(V, \nu)d\nu}{h\nu} .$$

Neglecting the variability of k, p and q, and approximating in the integration much as we did for $E(V)$, this becomes

$$\pi nr^2 = \frac{bk}{2h} \frac{\log \frac{V}{H\nu} + p\left(1 - e^{-q\left(\frac{V}{H\nu}-1\right)}\right)}{V} .$$

In conclusion it must be noted that the above expression for the probability of radiating a frequency ν in the distance ds depends on the V^2 law for $E(V)$ and the Thomson-Whiddington law as applied to the crooked path of an individual cathode ray. The expression connecting r with V and ν therefore involves these assumptions as well as the assumption that ν is determined by V and r alone. If the V^2 law and the Thomson-Whiddington law prove incorrect, the above experimental results and methods of calculation can be used with whatever modified forms of these laws are accepted. Meanwhile any theory that uses these laws in attempting to explain this spectrum should start with a probability law approximately equivalent to that given here.

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