THE PENETRATION OF CATHODE RAYS IN MOLYBDENUM.

By D. L. Webster and A. E. Hennings.

Abstract.

Penetration of cathode rays into a target.—(1) *X-ray spectrum method.* The farther a ray penetrates into the target the more the x-rays excited by it are absorbed by the target material before coming out. X-rays just harder than the K absorption limit are absorbed more strongly than those just softer. If the intensity of the spectrum emitted, except for the characteristics, is a continuous function of the wave-length and if the discontinuity in absorption coefficient is μ_K and the observed ratio of intensities on each side of the discontinuity is R, then the logarithmic mean depth of penetration $\bar{x} = \mu_K^{-1} \log R^{-1}$, for voltages above the minimum necessary to excite radiation of the absorption limit frequency, 20 kv. in case of Mo. (2) *Results for molybdenum.* To reduce the effect of stray x-rays, a calcite crystal was used and narrow slits which limited the incident beam to 8' 30". The voltages needed to make the mean depth of excitation of this frequency equal to 1, 2 and 3 μ were found to be about 24, 44 and 67 kv. respectively. The increased difficulty of penetration beyond the first micron is probably due to the deflection of the cathode rays by the atoms.

Definite time exposure method of measuring ionization currents.—Instead of using a stop-watch to measure the time for a given deflection, an instrument was devised which would open the lead shutter in front of the ionization chamber for a definite time interval, from 1 to 120 seconds, with great accuracy. This instrument contained a ratchet wheel driven by an escapement actuated by current impulses from a master clock.

I. The Problem.

THE penetration of cathode rays in matter is important for three reasons:

First, it gives evidence on the forces acting on an electron that comes near an atom.

Second, the penetration of cathode rays in a metal is important in x-ray experiments, because the x-rays will be produced below the surface, and will be absorbed by the material of the target, in getting out, to an extent depending on the depth of their production and the frequency of the rays considered. This makes a correction to be applied to all x-ray measurements before they can be used for any accurate theoretical work.

Third, since each cathode ray goes through many atoms before stopping, the intensity of the x-rays depends on this number of atoms; or conversely, the point of greatest theoretical interest is not the whole intensity, but the intensity per atom penetrated at any given speed.

For the first point, on electronic forces, and the third, on intensity

reduction, we need data on the length of an electron's path, the number of atoms per unit length, and the deflection and loss of speed per unit length. For the length of path and its deflections, the most obvious method is by stereoscopic photographs with C. T. R. Wilson's apparatus for visible electron tracks. For the loss of speed, this should be combined with measurements of speeds after passing through very thin films, such as were first made by Whiddington, but they should be extended to other speeds than the maxima in the emerging rays, and combined with deflection measurements.

Such a set of measurements, when complete, could be used to calculate x-ray absorptions in the target as well. But for that purpose alone, we can fortunately get some fair approximations, with less extensive data, by measuring the absorption of the emerging rays directly. At the same time, calculating back from these absorptions, we can get some rough data on the average depths reached by the cathode rays. It is this type of measurement, applied to molybdenum, that forms the object of the present paper.

II. History.

The first measurement of the mean depth of production of x-rays was that of W. R. Ham.[1] His work, in 1910, was necessarily done with unresolved x-rays, due to cathode rays of variable voltage, and depended on measurements of the relative strengths of x-rays coming out at different angles to the target surface, but with all other conditions the same. W. P. Davey,[2] in 1911, used the same apparatus, and L. G. Davey[3] used a modification of the same method. Ham found the depth in a lead target to change from 0.42 to 0.64 micron while the potential changed from 14 to 21.3 kv. W. P. Davey found for silver 0.54 to 0.92 micron at 10 to 17 kv.; and L. G. Davey, with a platinum target, got 2.0 microns at 43 kv.

J. J. Thomson[4] had previously worked out a theory, which made no attempt at more than a rough approximation, but led to the result that the speed of a cathode ray, starting at v_0, would be reduced in a distance x to v_x, where

$$v_x^4 = v_0^4 - ax,$$

a being an undetermined constant.

Whiddington[5] tested this law with thin films of gold and aluminum,

[1] W. R. Ham, Phys. Rev., *30*, 1, Jan., 1910.

[2] W. P. Davey, J. Franklin Inst., March, 1911.

[3] L. G. Davey, Phys. Rev., *4*, 217, Sept., 1914.

[4] J. J. Thomson, Conduction of Electricity through Gases, 2d ed., p. 378.

[5] R. Whiddington, Proc. Roy. Soc., A*86*, 360, Apr., 1912.

in leaf form, and an arrangement of magnetic fields for measuring his speeds. He found the cathode rays emerging from the leaf to be rather heterogeneous in speed, and measured only the fastest, on the theory that these were the ones that had come through straight, while the others had been deflected and perhaps returned to their original direction, thus having larger values of x. In this way he deduced the values

$$a_{Al} = 732 \times 10^{40} \frac{\text{cm.}^3}{\text{sec.}^4}$$

and

$$a_{Au} = 2540 \times 10^{40} \frac{\text{cm.}^3}{\text{sec.}^4}.$$

Expressing the equation in equivalent potentials, we have

$$V_x^2 = V_0^2 - bx,$$

with values

$$b_{Al} = 0.585 \times 10^6 \frac{\text{kv.}^2}{\text{cm.}}$$

and

$$b_{Au} = 2.03 \times 10^6 \frac{\text{kv.}^2}{\text{cm.}}.$$

From this, setting $V_x = 0$, we should calculate maximum depths for gold, at 21.3 kv., 2.24 microns, and at 43, 9.1, which are about four times the mean values for lead and platinum found by Ham and Davey respectively.

A part of this difference is doubtless explainable by the fact that Whiddington's law gives maximum depths, while the others give weighted means. Balancing off the greater x-ray efficiency at higher speeds against the greater penetrating power, one might reasonably expect this weighted mean to be about half the maximum depth. To reduce the mean to a quarter, there must be some other factor, which one may guess from C. T. R. Wilson's experiment to be the deflection of the cathode rays into erratic paths. Whether or not this is the reason, it is at least evident that the practice of using Whiddington's data directly, for the calculation of target absorption, is open to serious question, and more exact data are needed.

III. THE PRESENT METHOD.

From Whiddington's data, one would expect a great discontinuity at the K absorption limit of the emitting substance, because rays of a slightly shorter wave-length are absorbed several times as much as those slightly longer. Such a discontinuity fails to appear in most cases. It was indeed found by one of us in some spectra[1] of rhodium, but since these

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[1] D. L. Webster, PHYS. REV. *6*, 599, June, 1916.

were obtained with rays emerging at a very small angle from the target surface, and the surface was quite visibly rough, it could not be interpreted as evidence of noticeable penetration. On the other hand, if one were to apply Kirchhoff's law to x-rays, one might expect a discontinuity in the other direction, the intensity of the shorter wave-lengths being the greater, by a factor of about 6.5 in rhodium or molybdenum. This difference might of course be cancelled by absorption in some cases, but with the variety of conditions used, the absorption certainly could not always be of exactly the right amount. Since x-rays are not a temperature effect, however, this application of Kirchhoff's laws is mere guesswork, and the absence of any very noticeable discontinuity is good evidence for the belief, which will be adopted here as a fundamental postulate, that *the continuous spectrum is produced by a mechanism independent of the characteristic.*
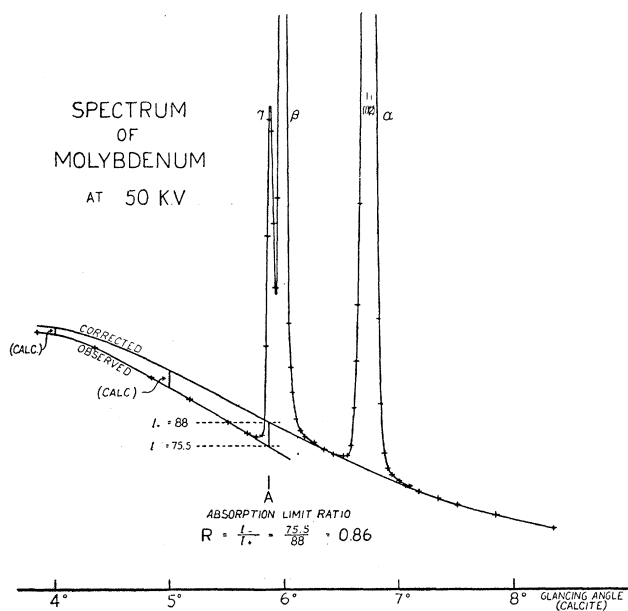

Fig. 1.

Molybdenum spectrum at 50 kv., illustrating discontinuity at K absorption limit.

The K characteristic is almost surely produced by the removal of electrons from the K positions in the atoms and their replacement by other electrons from the L, M, etc. On the other hand, in the higher frequencies of the continuous spectrum the quanta are greater than can be obtained by replacing even these inner electrons from anywhere else. Therefore it would be unreasonable to assume that these quanta

could be obtained any more readily from the K electrons than from any others, and the independence of the continuous and characteristic spectra would be predicted theoretically.

The present method is therefore to make a more careful search for a discontinuity in the observed spectrum, and to interpret it as due entirely to absorption, and then to try to separate the parts due to absorption by surface irregularities and to true penetration. A typical spectrum, illustrating the method, is shown in Fig. 1.

IV. The D.C. Generating Plant.

The potential for the tube was obtained from a D.C. outfit patterned in general after that of A. W. Hull, but modified so as to make use of both voltage waves, one in one condenser and the other in another, as shown in Fig. 2. This makes the potential of the D.C. line practically twice
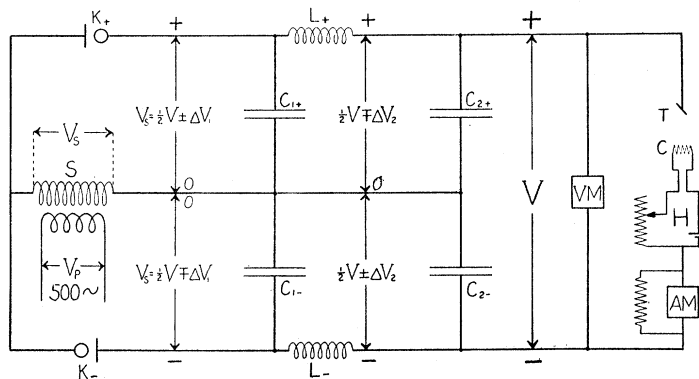

Fig. 2.

Essential features of high potential D.C. outfit. S, secondary of 500-cycle transformer; K_+, K_-, kenotrons; C_{1+}, C_{1-}, C_{2+}, C_{2-}, condensers; L_+, L_-, choke coils; VM, voltmeter; AM, milliammeter; H, cathode heating circuit; C, cathode; T, target. Potentials denoted by V's with appropriate subscripts.

the peak voltage of the transformer. It also puts the 500-cycle fluctuations in such a phase relation that, if the two sides are symmetrical, their effects on the potential difference between the + and − lines must cancel. Only the first harmonic is then appreciable. If any point of such an outfit is to be grounded, it should be the middle line, because if either the positive or negative is grounded, the distributed capacitance of the transformer is liable to introduce fluctuations. It is not, however, necessary to ground any point, as the essential function of the middle line is to connect the transformer to the condensers.

The voltage fluctuation, ΔV_2, on the second condenser, of the primary

A.C. frequency, is not exactly calculable, but can readily be shown to lie between 1 and 2 times

$$\Delta V_2 = \frac{\pi i}{C_1 C_2 L \omega^3},$$

where in this apparatus C_1 (either C_{1+} or C_{1-}) = 2.8 × 10^{-3} mfd., C_2 = 5.6 × 10^{-3} mfd., L = 12,000 henries, and ω = 480 × 2π sec^{-1}. This makes it 1 or 2 times 0.4 volt per milliampere of tube current, which is quite negligible.

The only appreciable fluctuations in voltage were those due to changes of speed of the generator, which were always slow enough to show on the voltmeter and enable us to discard readings where they were greater than normal. Errors in voltage would normally be less than 1/10 per cent., although they might produce much larger changes than that in the x-ray intensities at wave-lengths near the short-wave limit.

The high tension voltmeter was of an electrostatic type previously described by one of us,[1] and was calibrated by current through a 6-megohm resistance.

The milliammeter was a Weston, model 1. It was enclosed, along with its shunt and the whole filament heating system, in a wire cage connected to the high potential source, back of the milliammeter. This precaution is needed at high voltages, even with the best practicable insulation, to prevent errors in the current measurement, due to leakage.

V. A Device for Timing Ionization Currents Accurately.

The x-ray spectrometer was of the Bragg type, with a quadrant electrometer, mounted above the crystal. It needs no special description, except for one point which we found a great convenience, as well as an improvement in accuracy, by the elimination of the stop-watch usually used in ionization work.

Instead of timing the motion of the electrometer, from the instant the grounding key was lifted, to some other point, we used a lead shutter, placed in the path of the x-rays before they reached the crystal, and opened or closed by an electric current from a clock. The grounding key would first be lifted and the zero of the electrometer scale set on the spot of light from the electrometer. Then the shutter would be opened, kept open for a definite number of seconds, and closed. The electrometer would swing during this time and come to rest momentarily at a certain definite time after the shutter was closed. The time of the exposure, as we may call it by analogy to photography, was always kept the same for all readings on any one spectrum or isochromat, and there-

[1] D. L. Webster, loc. cit.

fore the time of swing after the end of the exposure, and the fraction by which the reading increased in this time, was constant. The resulting reading was therefore proportional to the intensity of the rays. This proportionality, which one might predict theoretically, was checked by tests with different tube currents at the same voltage and frequency.

For timing the exposures accurately, we used current impulses from a good pendulum clock, with a secondary clock, driven by the first, to select the impulses to be used. This selector clock was a single ratchet wheel driven by an escapement, which in turn was driven by a bell magnet with current from the master clock. On the wheel were two pins, so placed as to make contact with a brush at the release of the magnet, and therefore to have the contact already made before the arrival of the next current impulse, that worked the shutter. This arrangement is most important, because it makes the accuracy of the timing depend on the master clock, and not on the selector. By placing the pins in different holes on the wheel, we could use different exposures, varying from 1 second, in runs with strong rays, to 120 seconds with weak. In no case could any error be found, due to any inaccuracy in the exposure, even with the 1-second intervals, which would have been wholly worthless if taken with a stop-watch.

VI. THE CRYSTAL.

For this problem, it is especially important to get accurate data at wave-lengths near the absorption limit, and to avoid as far as possible any influence of stray x-rays belonging to the K series lines. This means, first, the use of fairly narrow slits.

Theoretically, we should expect x-ray emission lines to be very narrow. The Doppler effect due to atomic motion is quite negligible, and Jauncey [1] has shown that the width due to damping by loss of energy should be only 1.2×10^{-4} Å. at any wave-length, if the damping follows the classical theory. This width means about 4 seconds of arc as reflected from calcite. Compton,[2] however, finds experimentally in the tungsten L series, widths of the order of 18' of arc with either calcite or rock salt. Most of this width he explains as due to slit width and imperfections of the crystal with a small addition for complexity of the lines.

In the present experiments the beam was limited by two slits, between the tube and the crystal, of such widths as to give the beam a maximum angular width of 8' 30''. If the crystal were perfect, and the rays homogeneous, this should represent practically the whole width of each

[1] G. E. M. Jauncey, PHYS. REV., *19*, 64, Jan., 1922.
[2] A. H. Compton, PHYS. REV., *19*, 68, Jan., 1922.

spectrum line. Practically, however, the spectrum lines are all widened at the base, as shown in Fig. 1, above, by stray rays whose intensity is very low compared to that in the centers of the lines, but not negligible compared to the continuous spectrum. With no slit beyond the crystal, except the thin aluminum window of the ionization chamber, which subtends about a degree, this widening at the base is so great as to overlap from the α lines to the β, and to make an accurate estimate of the intensity between the lines impossible. This is true either with a perfectly clear polished piece of calcite, a clear piece split and not polished, or one split and ground to remove the step-like layers on the surface. Presumably all these crystals must contain many very minute pieces, slightly out of line with the general shape of the crystal, and reflecting some rays at slightly wrong positions.

With a third slit, just in front of the ionization chamber, subtending an angle of about 25' from the crystal, and just wide enough to be sure to include the whole reflected beam from a true surface, the stray rays were greatly reduced, and it became possible to measure the continuous spectrum, as in Fig. 1, with much less error from this cause. It must be remembered, however, that some error from stray line radiation is still present, and that it always acts to make the estimated discontinuities and depths of cathode ray penetration too large.

VI. RESULTS.

Plotting spectra at various potentials, with no measurements at angles reflecting the characteristic lines, we get a set of graphs of which typical examples are given in Fig. 3. The discontinuity is not easy to estimate accurately, especially at the lower voltages, because of the extrapolations needed to get to the absorption limit, and the probable error in the ratio R, of intensity on the long wave side to that on the short, is about 1 or 2 per cent.

The method of extrapolation, illustrated by the graphs labeled as "calculated," is as follows: First, extrapolate by guess, and estimate the value of the ratio R; second, from this "1st R," calculate the losses of intensity by the K absorption at some shorter wave-lengths (shown on each graph by vertical lines); third, plot the intensities corrected for K absorption at these wave-lengths, and see if they form a smooth continuation of the graph on the long wave side. If they do, the "1st R" can be accepted as a final value. If not, a second trial value of R is taken, and the process repeated, until a smooth curve is obtained. In a few cases, where the primary voltage had been very unsteady and the data were ragged, no smooth curve could be drawn, and such cases were discarded.

In this calculation for the shorter wave-lengths, the change of absorption coefficient was allowed for, and the mean depth of production of such rays was assumed to follow a law like that of the limit frequency, but on a different scale of potentials. Fortunately this assumption did not have to be accurate, so far as the final values of R were concerned.
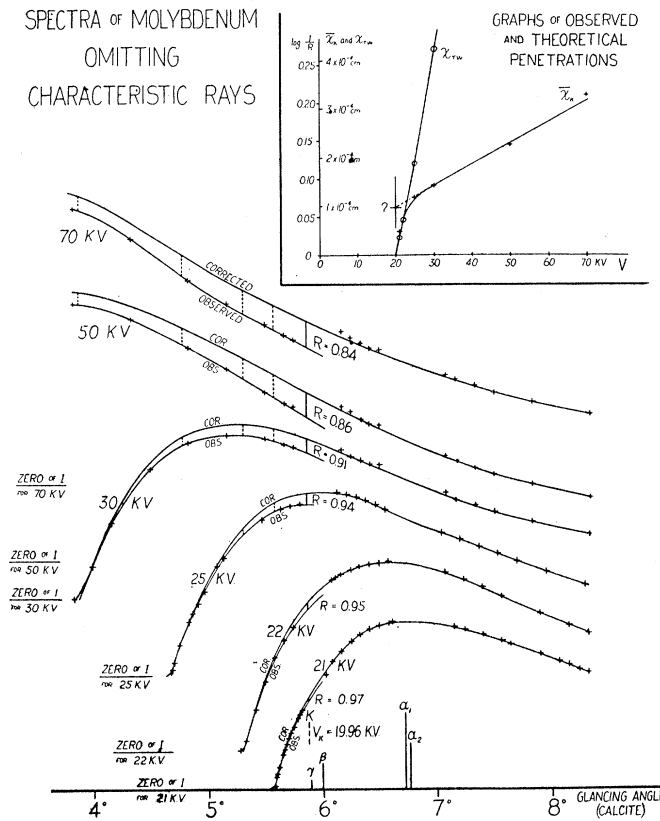

Fig. 3.

Spectra of molybdenum at various potentials with different intensity scales, adjusted for approximate agreement at the K limit; also graph of mean depth, $\bar{x}_K$, calculated from the K limit, and maximum depth x_{TW}, calculated from the Thomson-Whiddington law.

To find the mean depth, measured along the line of the cathode rays, we must note that the target was tipped at 45° to the axis of the tube, and the cathode rays, allowing for electrostatic deflection by the field near the target, struck it at some direction probably between 50 and 60° to its surface. The x-rays were taken out through a thin glass window, blown into the tube, at about 80° from the cathode end of the tube axis.

This made an angle between the x-rays and the target surface of about 55°, or practically the same angle for both cathode rays and x-rays. Therefore we shall take the depths along the line of the cathode rays to be the same as on that of the x-rays.

Let $\mu_{L'}$ stand for the absorption coefficient due to the L, M, $N \cdots$ electrons alone, μ_K that due to the K electrons, $\bar{x}_K$ the weighted mean depth of production of the K limit frequency, I_K the unabsorbed intensity at the absorption limit λ_K, I_+ the intensity at wave-lengths just above the limit and I_- just below. Then, if the mean $\bar{x}_K$ is weighted logarithmically,

$$I_+ = I_K e^{-\mu_{L'}\bar{x}_K}$$

and

$$I_- = I_K e^{-(\mu_{L'}+\mu_K)\bar{x}_K}.$$

From this we find

$$\mu_K \bar{x}_K = \log \frac{I_+}{I_-} = \log \frac{1}{R}.$$

μ_K can be calculated from Richtmyer's[1] data, as 639 cm.$^{-1}$.

The graph in the upper right hand corner of Fig. 3 is taken by these formulas from the averages of a few determinations from different sets of data for each voltage, weighted 1, 2 or 3 according to the general appearance or the steadiness of the primary voltage in each case. The spectra at 21 and 22 kv. are weighted low, not because of any unsteadiness, but because the strong curvature of the graphs made extrapolation difficult and the results correspondingly uncertain. The values of R are given in Table I.

TABLE I.

V.	Weight.	R.	Mean R.	Mean log 1/R.	$\bar{x}_K$.
21 kv.	1	0.97			
"	2	0.97	0.97	0.0305	0.5 micron
22	2	0.95			
"	2	0.955	0.953	0.048	0.75
25	1	0.925			
"	3	0.94			
"	3	0.915	0.927	0.076	1.2
30	3	0.91			
"	1	0.92			
"	1	0.925			
"	3	0.91	0.913	0.091	1.4
50	3	0.86			
"	1	0.89			
"	3	0.86	0.865	0.145	2.3
70	3	0.84			
"	2	0.79			
"	3	0.805	0.81	0.21	3.3

[1] F. K. Richtmyer, PHYS. REV., 18, 13, July, 1921.

VII. Conclusions.

It is difficult to compare these data accurately with those of previous observers, whose measurements were on different quantities and with different metals. The nearest to these are those of W. P. Davey, who found mean depths of 0.54 to 0.92 micron for silver at 10 to 17 kv., using x-rays of all frequencies together. His values are of the same order of magnitude as the present ones.

Whiddington's maximum penetrations, for aluminum and gold, may be used by interpolation to give rough values for molybdenum. For this metal we have taken as a rough guess,

$$b = 1.2 \times 10^6 \frac{\text{kv.}^2}{\text{cm.}},$$

and have plotted the predicted maximum depths of excitation of this frequency against the values of V, as the graph x_{TW}. The curve agrees well with that of $\bar{x}_K$ at first, but keeps on rising more and more rapidly, while $\bar{x}_K$ rises only very slowly beyond 25 kv. At 70 kv., $x_{TW} = 37$ microns, more than 11 times the observed depth, $\bar{x}_K$.

This rapid change in the behavior of $\bar{x}_K$ needs explanation. The data at 21 and 22 kv. are very hard to get accurately, and no great confidence can be placed in them, because of the weakness of the rays at those voltages and the curvature of the spectrum graphs in the region extrapolated. At first, before special efforts were made to get these points, it was supposed that there must be a constant absorption by irregular humps on the target about one micron high, as indicated by the intercept marked " ? " on the small diagram of Fig. 3. With a smooth surface, such as the target was, it did not seem reasonable to expect such large humps, yet it was not certain that they did not exist. The later data, however, including all those of Fig. 2, were taken with a perfect mirror surface, made by melting the focal spot with 5 times the normal current, and then cutting off the potential while it cooled. No humps could be expected here as large as half a wave-length of light, and certainly not a whole micron. We must therefore eliminate any allowance for surface absorption, and conclude that Fig. 2 gives a fair approximation to the true penetration law.

The explanation is presumably that the cathode rays go in approximately like Whiddington's straightest rays for the first micron or so, but by that time get so badly thrown out of line that higher voltages accomplish very little in further penetration. This conclusion is in accord with the known tortuous character of C. T. R. Wilson's electron tracks.

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