THEORY OF THE ELECTRIC ARC.

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Abstract.

Theory of the electric arc.—(1) *Thermionic emission from the cathode.* The fundamental phenomena of the arc are the cathode fall and the copious emission of electrons from the cathode. J. J. Thomson first suggested that this emission is chiefly of thermionic origin. But is thermionic emission sufficient to account for observed primary arc currents? Using the best data as to the emission from carbon and tungsten, the computed thermionic currents in the case of the carbon arc, the tungsten arc in hydrogen, and low-voltage, low-pressure arcs in various gases are found to be of the right order of magnitude. Bräuer attempted to measure the thermionic emission directly by suddenly reducing the arc voltage to below the ionizing potential and obtained low values; but from a discussion of the effect of space charge on the current from the cathode it is shown that he certainly did not measure the thermionic emission but probably measured only the fraction released as a result of the effect on the space charge of positive thermionic emission from the anode. These facts and others all favor the thermionic theory as against the photo-electric and canal-ray theories of the origin of the electronic emission. (2) *Current carried by positive ions, and cathode fall.* If i is the current density of electronic emission, there must be a current density of positive ions $j = i/242M^{1/2}$ to neutralize the space charge due to i, where M is the atomic weight of the ions, and also an additional positive current density J to maintain the excess positive space charge. From theoretical considerations it is found that $J = 1.47(10)^{-7}V_c^{3/2}\lambda^{1/2}/M^{1/2}c^{5/2}$, where V_c is the cathode fall in volts, λ the electronic mean free path, and c is the cathode dark space. Incidentally, since c is approximately equal to λ, J varies as the square of the pressure. An application of this expression to several cases, the carbon, hydrogen, and mercury arcs, leads to reasonable values for J. The ratio of positive to negative currents, $(j + J)/i$, is computed also from energy considerations at the cathode for various cases, with results that are consistent with the values obtained independently from the above equations. This is good evidence for the general correctness of the theory. (3) *Ionization in the region between the electrodes* of a carbon arc must be sufficient to neutralize the space charge due to the electrons. Reasons are given for concluding that neither emission from the anode nor ionization by collision will account for this ionization but that it is primarily of thermal origin. An expression for the current is derived and it is found that if the temperature is 4000° K or over, and if the ionizing potential is about 8.6 volts or less, the thermal ionization estimated by applying Nernst's Heat Theorem, alone is sufficient. Photo-electric ionization if present would be secondary, a result of the radiation accompanying recombination. (4) *The anode fall* is accounted for in a qualitative way by a deficiency of positive ions near the anode due to decreased recombination. Thermionic emission from the anode may also play a part.

Limitation of electronic current from a hot cathode by space charge.—The equation for the case of elastic impacts: $i_e = 5.24(10)^{-6}(V^{3/2}/d^2)(\lambda/d)$ amp./cm², becomes $i_i = 5.45(10)^{-6}(V^{3/2}/d^2)(\lambda/d)^{1/2}$ amp./cm², for inelastic impacts.

THE complexity of the phenomena occurring in the ordinary electric arc has caused a wide divergence of opinion as to which phenomena are fundamental to its operation. Child [1] has given an excellent summary of experimental investigations and theories prior to 1913, and a number of papers have been published within the past few years. The data at our disposal include values of the cathode drop of potential, the anode drop, the electric intensity in the arc, the influence of the size and material of the electrodes, the nature and pressure of the surrounding gas, the current, the temperature of the electrodes, the rate of consumption of the electrodes, the spectrum and the intrinsic brightness of the arc, the mechanical reaction and the back electromotive force at the electrodes. The conclusions regarding the fundamental explanation of the arc have been conflicting.

In 1890 Fleming [2] suggested that the arc is due to a stream of negatively charged carbon atoms from the cathode, and experiments on the consumption of carbon in the electric arc led Duffield [3] to the same view.

Sir J. J. Thomson [4] suggested that the current from the arc is chiefly of thermionic origin, consisting of the electronic emission from the cathode plus a relatively small current arising from ionization of the gas by these electrons. This ionization is essential because positive ions are necessary in sufficient number to create a positive space charge around the cathode, thus permitting the escape of the electrons from it. All physicists now concur in the view that electronic emission from the cathode is of primary importance in the arc. This view has recently received strong additional support in the interesting work of Duffield, Burnham and Davis,[5] who showed that the mechanical reaction on the cathode in an arc was of the magnitude to be expected if due to the thermionic emission of electrons, but was much too small to be attributed to the expulsion of particles with values of e/m of atomic size. But J. J. Thomson's view that these electrons are of *thermionic* origin has not been universally accepted.

Child [6] calls attention to the fact that the observed current density at the hot spot of the cathode is of the order of 318 amperes per sq cm, whereas Richardson's measurements of thermionic emission from carbon,[7] if extrapolated to the temperature of the arc, predict a current density nearly a million times greater. These early experiments of Richardson

[1] Electric Arcs.

[2] Roy. Soc. Proc., A, 47, p. 123, 1890.

[3] Roy. Soc. Proc., A, 92, p. 122, 1915.

[4] Conduction of Electricity through Gases, 2d ed., p. 604.

[5] Roy. Soc. Proc., A, 97, p. 326, 1920.

[6] Electric Arcs, p. 164.

[7] Phil. Trans., 201 A, p. 497, 1903.

were, however, made under quite unsatisfactory conditions, so that he has revised his values for the constants in the thermionic equation. The best present evidence on thermionic emission from carbon, as will be discussed later in the paper, indicates that thermionic currents of the order of the observed arc currents are to be expected at the temperature of the arc, so that this objection to Thomson's theory seems unfounded.

A second objection is suggested in the fact that, in the mercury arc, the cathode would vaporize at a temperature far below that required to give appreciable thermionic emission. It has been pointed out by Stark,[1] however, that there is an incandescent spot on the surface of the mercury cathode, indicating that local high temperature can be reached before the atoms have time to evaporate from the liquid surface. Thus this argument is inconclusive.

Pring[2] performed experiments which appeared to indicate that the electronic emission from incandescent carbon is dependent on the presence of gases capable of chemical action on the hot carbon and vanishes to the extent to which such gases are removed. Richardson,[3] however, showed that the nature of Pring's results was entirely accounted for by the effect of the magnetic field of the current used in heating his carbon cathode, together with the effect of gases on the mean free paths of the ions. Thus Pring's experiments do not really bear on the vital problem of the arc.

Ionization of the cathode as a direct result of impacts of positive ions which have fallen through the cathode drop of potential is a suggested source of electronic emission from the cathode.[4] It is argued that, although bombardment of cold cathodes produces no detectable electronic emission at voltages similar to those in the arc, yet such emission may occur if the cathode is at a high temperature. However, the fact that the work done during the escape of an electron from the surface of a conductor is found to be practically independent of the temperature[5] would seem to make this view untenable.

The theory discussed in the preceding paragraph has recently been upheld by Bräuer,[6] who describes experiments and calculations which lead him to conclude (1) that thermionic emission accounts for only a few per cent of the total arc current, and the ionization produced by thermoelectrons constitutes a negligible fraction of the current and (2)

[1] Phys. Zeit., 5, p. 750, 1904.
[2] Roy. Soc. Proc., 89 A, p. 344, 1913.
[3] Roy. Soc. Proc., 90 A, p. 174, 1914.
[4] Child, Electric Arcs, p. 165.
[5] Richardson, Emission of Electricity from Hot Bodies, pp. 164–178; Koppius, Phys. Rev., 18, p. 443, 1921.
[6] Ann. d. Physik, 60, p. 95, 1919.

that the electronic emission is due principally to the impact of positive ions against the cathode, the high temperature of the cathode so reducing the work necessary to extract an electron that each impinging positive ion is able to eject a large number of electrons.

I think Bräuer's interpretation of his results can be shown to be impossible, and that they can readily be explained in a manner quite compatible with Thomson's theory of the thermionic origin of the primary current of the arc.

In the remainder of this paper I shall discuss (1) the adequacy of thermionic emission as the source of the electron current in an arc, (3) the interpretation of Bräuer's experiments, (4) the theory of the arc. In connection with Bräuer's experiments it will be necessary to discuss (2) the limitation of thermionic emission by space charge.

I. The Adequacy of Thermionic Emission as the Source of the Electron Current in an Arc.

(*a*) *Carbon Arcs.*

The thermionic emission from the cathode of an arc may come from its entire surface, but by far the greater part of it must proceed from the cathode "spot," whose temperature is higher than that of the rest of the cathode. Its temperature is 3,140° K according to Reich,[1] and its diameter is such as to lead to estimates of the total current density at this point of the arc varying from 180 to 318 amperes per sq. cm.[2] Child's estimate of thermionic current density at this temperature, giving a value of $2(10)^8$ amps. per sq. cm, was based on constants of thermionic emission which are now known to be much in error.[3] Bräuer, on the other hand, calculates the thermionic emission using data published in a preliminary note by Langmuir and finds the expected emission to amount to only 1.2 amps. per sq. cm at 3,500° K. Unfortunately, however, these data are also incorrect, owing, presumably, to a printer's error, since they were added in a note with the proof.

Langmuir's values for the constants A and b of Richardson's equation for the thermionic current

$$N = A\sqrt{T}\,\epsilon^{-b/T} \qquad (1)$$

were determined under conditions of the best vacua attainable in a "baked out" tube immersed in liquid air and with a vaporized metallic deposit on the glass bulb.[4] They are by far the most reliable data

[1] Phys. Zeit., 7, p. 73, 1906.

[2] Granqvist, Phys. Zeit., 4, p. 537, 1903; Reich, Phys. Zeit., 7, p. 73, 1906.

[3] Richardson, Emission of Electricity from Hot Bodies, p. 75.

[4] Am. Electrochem. Soc. Trans., 29, p. 125, 1916; Richardson, Emission of Electricity form Hot Bodies, p. 69.

available. From them are calculated the thermionic current densities for pure carbon in Table I.

TABLE I.

Carbon. $A = 1.49(10)^{25}$. $b = 48,700$.		Lime-Impregnated Cathode. $A = 3.3(10)^{26}$. $b = 42,000$.	
$T°$ K.	Amps./cm².	$T°$ K.	Amps./cm².
2,700	1.9	2,700	500
3,000	13.2	3,000	2,390
*3,140	26.7	3,140	4,400
3,300	54.7		
3,500	127		
4,000	775		

Two points must be borne in mind. In the first place the temperature of the cathode bright spot is not known very accurately. Bräuer makes his comparison on a basis of 3,500° K. It is known in a qualitative way only that the temperature is higher when pure carbon electrodes are used than when the carbon contains impurities of an electropositive nature. In the second place, arc carbons are inevitably contaminated by materials of a more electropositive nature, such as calcium and sodium, which are known greatly to increase the thermionic emission even when present in minute quantity. In a carbon arc, the volatilization of the cathode continually presents new surface, so that the electrode never can be completely freed from these impurities by continued heating, as could be done with the filaments used in Langmuir's experiments. Table I. also shows calculated current densities from cathodes coated with CaO, the constants of the Richardson formula being averages of the closely agreeing values given by Deininger, Wehnelt and Jentzsch.[1]

In view of these considerations it seems safe to estimate the thermionic emission from the cathode of a carbon arc as lying between the values given in Table I. at the appropriate temperature. It is evident that the thermionic current is at least an important part of the total current, and there is no reason for believing that it is not practically the total current.

(*b*) *Arcs between Tungsten Electrodes.*

Bräuer again belittles the rôle played by thermionic emission by calculations based on experiments of Mackay and Ferguson[2] on arcs between tungsten electrodes in hydrogen at pressures of about 400 mm. They state that the arc was about 0.5 mm in diameter when the current

[1] Richardson, Emission of Electricity from Hot Bodies, p. 81.

[2] Frank. Inst. Jour., 191, p. 209, 1916.

was 25 amps. Thence Bräuer calculates a current density of 3,200 amps./cm^2, which much exceeds the possible thermionic current density.

Mr. Mackay has kindly informed me, however, that the dimension of 0.5 mm referred to the luminous line of the discharge between the electrodes and not to the size of the hot tungsten cathode, which was a tungsten button whose surface area was approximately 0.5 cm^2 and whose temperature at the hottest part was not less than 3,300° K. Although the most intense thermionic emission must have come from this hottest spot, the remaining larger area of the hot cathode must have contributed considerably to the total current. If the current had come equally from the entire area, the current density would have been 50 amps./cm^2 instead of 3,200. By determining pyrometrically the actual distribution of temperature over the surface of the cathode, the investigators concluded that the expected thermionic currents equalled the arc currents except in the case of the peculiar intense line discharge obtained at high gas pressures, in which case the arc current was larger than the expected thermionic current, but of the same order of magnitude.

Table II. gives actual thermionic currents from tungsten, using Langmuir's data.[1] More recent data for pure tungsten give currents a little lower, but those given in the table are probably applicable to comparison with the arc currents between the tungsten buttons used.

Table II.

Tungsten: $A = 1.55(10)^{26}$.
$b = 52,500$.

T^0K.	Amps./cm^2.
2,400	0.365
2,800	8.98
3,200	96.9
3,540 M.P.	509

(c) Low Voltage Arcs.

In this laboratory recent experiments on low voltage arcs in helium, hydrogen, mercury vapor, nitrogen and iodine, stimulated by thermionic bombardment from an independently heated tungsten cathode, have led us to the conclusion that the current in this type of arc is the thermionic current, limited to a greater or less extent by the space charge. The ionization in the arc functions principally in providing positive ions, each of which permits the liberation of many additional electrons because it moves relatively slowly through the region of the space charge. On this view, the maximum current through a low voltage arc (one in which

[1] Mackay and Ferguson, loc. cit.

the voltage is insufficient to permit an electron to ionize more than once on its path between the electrodes) should never exceed twice the saturation value of the thermionic emission, and should ordinarily be less than this.

The following experimental results may be cited as examples. In mercury vapor at 3 mm pressure an arc current of 15 amps. was observed at 8 volts difference of potential between the electrodes. The cathode was a coil consisting of three close turns of 20-mil tungsten wire, wound around a 20-mil iron wire which was then dissolved away. Beyond the coil, on each side, was a 3-mm length of the tungsten which was welded to stout molybdenum leads which carried in the heating current of about 24 amperes. Because of its shape, the coil of three turns was heated to a higher temperature than the straight connecting wires, so that we may assume, to a first approximation, that the total thermionic emission comes from this coil. The effective emission area may be calculated approximately by considering it to be a drum of 60 mils length and 60 mils external diameter, giving a surface area of 0.106 cm^2. The temperature of the coil was not measured, owing to the difficulty of using any method of measuring such high temperature while the arc current was flowing. The middle one of the three turns must have been near the melting point of tungsten, however, since the wire became thinner and soon burned out at this point. (It is not likely that this burning out was due to chemical action on the filament, since, at a slightly lower temperature, the resistance of the wire remained constant for hours.) In view of some experience in measuring temperatures of similar coils by their resistances in the absence of ionization, I feel safe in estimating the mean temperature of the coil as between 3,200° K and 3,400° K. The current density was about 142 amps./cm^2, which falls within the range to be expected from thermionic emission from tungsten at the estimated temperatures.

Similar experiments were made in helium at pressures up to 10 mm. In this case the maximum current densities obtained were about 60 amps./cm^2. Here the temperature was not quite as high as in the case of mercury, because it was desired to conserve the filament for a series of spectroscopic observations.

Current densities of about the same magnitude, though slightly smaller, were observed in hydrogen, nitrogen and iodine arcs. In the case of nitrogen, the currents became abnormally large when the "flare," due to dissociation of nitrogen, set in.[1] This was probably due to the decreased proportion of the applied voltage which occurred outside the region of the cathode drop, owing to the increased conductivity of the gas, or to

[1] Duffendack, PHYS. REV., 20,665, 1922.

the chemical action of atomic nitrogen on the filament. Even in this case, however, the current densities did not exceed the possible thermionic currents given in Table II.

It may be noted, in passing, that the considerations advanced above are not vitiated by the effect of gases on the constants of thermionic emission from tungsten, for Langmuir[1] has shown that these effects, which are large at lower temperatures, become of less importance at higher temperatures and are probably negligible at the highest temperatures reached by the tungsten.

In view of these facts it is difficult to escape the conviction that thermionic emission from the cathode is adequate to account for the primary current of the arc. At any rate the "burden of proof" to the contrary must rest upon those who advocate a different source of current. Bräuer interpreted his experiments as constituting such a proof. Before discussing his experiments it is necessary to examine conditions which limit the current from an electrode at a high temperature in a gas.

II. The Limitation of Current by Space Charge.

In his book "Conduction of Electricity through Gases," Sir J. J. Thomson called attention to the effect of the space charge, due to ions, on the distribution of potential between electrodes. Langmuir[2] applied this principle to the emission of electrons from a hot cathode, and showed that the current density i_v between plane parallel electrodes *in vacuo* is limited to the value

$$\begin{aligned} i_v &= \frac{\sqrt{2}}{9\pi}\sqrt{\frac{e}{m}}\frac{V^{3/2}}{d^2} \\ &= 2.33(10)^{-6}\frac{V^{3/2}}{d^2}\text{ amps./cm}^2. \end{aligned} \tag{2}$$

Richardson and Bazzoni[3] showed that the current density i_e in a gas in which the electrons make elastic impacts with the molecules is similarly limited to

$$\begin{aligned} i_e &= \frac{\sqrt{2}}{4\pi}\sqrt{\frac{e}{m}\frac{\lambda}{d}}\frac{V^{3/2}}{d^2} \\ &= 5.24(10)^{-6}\frac{\lambda}{d}\frac{V^{3/2}}{d^2}\text{ amps./cm}^2. \end{aligned} \tag{3}$$

In these equations V is the difference of potential between parallel plates distant d apart, λ is the mean free path of an electron through the gas and e/m is the ratio of the charge to the mass of an electron.

[1] Phys. Rev., 2, p. 457, 1913.
[2] Phys. Rev., 2, p. 457, 1913.
[3] Phil. Mag., 32, p. 426, 1916.

In a gas in which electrons make inelastic impacts with molecules the limiting current density i_l may be calculated as follows:

$$\frac{d^2V}{dx^2} = 4\pi\rho,$$

where x is measured outward from the surface of the cathode. The density of space charge ρ at any point is $\rho = i/v$, and the average velocity of the electrons v is easily shown to be

$$v = \sqrt{\frac{\pi e\lambda}{2m}\frac{dV}{dx}}.$$

With these substitutions,

$$\sqrt{\frac{dV}{dx}}\frac{d^2V}{dx^2} = 4\pi i\sqrt{\frac{2m}{\pi e\lambda}}.$$

This is integrated twice and the integration constants determined by the fact that the current is limited to such a value as makes $dV/dx = 0$ when $x = 0$ and by the relations $V_{x=0} = 0$, $V_{x=d} = V$. In this way, and putting $i = i_l$, we obtain

$$\begin{aligned} i_l &= \frac{1}{4\sqrt{2\pi}}\left(\frac{10}{9}\right)^{3/2}\sqrt{\frac{e}{m}}\sqrt{\frac{\lambda}{d}}\frac{V^{3/2}}{d^2} \\ &= 5.45(10)^{-6}\sqrt{\frac{\lambda}{d}}\frac{V^{3/2}}{d^2}\ \text{amps./cm}^2. \end{aligned} \quad (4)$$

III. Interpretation of Bräuer's Experiments.

Bräuer suddenly short-circuited an arc through a resistance of such size as to reduce the difference of potential between the electrodes to a value, such as 10 volts, too small to maintain ionization of the gas between the electrodes. An oscillograph in the arc circuit measured the current before and after thus suddenly reducing the voltage. It was found that the current between the electrodes dropped, practically instantaneously, to a small percentage of its value before the short-circuiting switch was thrown. This percentage was 1 to 3.5 with pure carbon electrodes, 4 to 5 with cored carbons containing some salts and oxides, 8 to 12 with carbons impregnated with CaF_2 and 1.5 to 1.8 in the case of a mercury arc.

Bräuer believes that the small current which persists after short-circuiting the arc is the true thermionic current, which he therefore takes to be but a small fraction of the total arc current. He points out that the electrodes are still at practically the normal arc temperature, and argues that these small currents are true saturation thermionic currents.

The conclusion that they are saturation currents is based on the experimental fact that they are practically independent of changes in the arc length which would vary the electric intensity by a factor of two or three.

That this interpretation of the results is impossible, I believe is shown by Table III., which shows the maximum possible current densities permitted by the space charge under the conditions appropriate to equations (2), (3) and (4). The mean free path λ of an electron is known to be given with considerable accuracy by the relation $\lambda = 4\sqrt{2}\,L$, where L is the mean free path of a gas molecule. Taking L as $9.6(10)^{-6}$ cm for air under standard conditions, and reducing to a temperature of 2,730° K, as a rough estimate of the temperature of the gas near the cathode after the arc has been shut off, we find $\lambda = 0.54(10)^{-3}$ cm.

TABLE III.

Limiting Current Densities: i_v in a vacuum,
i_e in an elastic gas,
Taking V = 10 volts. i_i in an inelastic gas.

d (cm).	i_v (amps./cm²).	i_i (amps./cm²).	i_e (amps./cm²).
0.2	$1.87(10)^{-3}$	$0.223(10)^{-3}$	$0.0112(10)^{-3}$
0.4	0.47	0.039	0.0014
0.6	0.21	0.014	0.00041
0.8	0.12	0.007	0.00018
1.0	0.075	0.004	0.00009

Even in a perfect vacuum the maximum possible thermionic currents would have been at least a thousand times less than those which Bräuer observed, because of the limitation by space charge. Under his actual conditions, in air, the currents would have been much less still, ranging between the values of i_i or i_e according to the degree of elasticity of impact of electrons with air molecules at high temperatures. As far as our evidence goes, the values of i_i are probably the more nearly appropriate. It is evident, therefore, that *Bräuer did not measure the thermionic current of an arc, and hence that his experiments cannot be taken as evidence against the theory that the principal part of an arc current is of thermionic origin.*

A reasonable explanation of Bräuer's results can be suggested along the following lines, which are quite in conformity with the idea of the thermionic origin of the primary arc current. The electrodes are at a sufficiently high temperature to give a large thermionic current, but this is limited by space charge. The only way to obtain currents larger than those given in Table III. is by supplying positive ions, each of which permits the escape of many additional electrons, as the following calculation shows.

Ions, whose mean free path l is considerably less than the distance between the electrodes, are drawn through a gas by an electric intensity E with an average velocity of drift [1]

$$v \propto l\Omega eE,$$

where Ω is the average translational velocity of the ion. At any point in the field the product eE is identical (except in sign) for electrons and positive ions, since it is safe to assume the ions to be singly charged under the conditions of the experiments. Ω for an electron is $\sqrt{1{,}836M}$ times Ω for an ion, M being the molecular weight of the ion. l for an electron is $4\sqrt{2}$ times l for an ion, the factor 4 arising from the negligible size of the electron and the factor $\sqrt{2}$ from the fact of its much greater speed. Thus the electrons drift through the gas $4\sqrt{3{,}672M}$ times faster than do the positive ions. Since the contribution of an ion to the space charge varies inversely as its velocity of drift, it is seen that each positive ion neutralizes the space charge of $242\sqrt{M}$ electrons and thus permits the escape of that additional number, provided the cathode is hot enough to supply them.

In Bräuer's experiments with the carbon arc the voltage was insufficient to produce positive ions in the gas. The hot anode, however, was certainly a source of a plentiful supply, since the greatest care and long heat treatment are required to eliminate positive emission even from thin strips of carefully purified metals. In his experiments with impregnated carbons, the positive emission must have been particularly large.

It is impossible accurately to calculate the positive emission under the conditions of Bräuer's experiments, but a rough estimate of its probable order of magnitude may be made. For platinum in air approximate values are given in Table IV. For heated salts the currents are

TABLE IV.

Platinum in Air.[2] $A = 7(10)^{18}$.
$b = 24{,}600$.

$T°$ K.	i_+ amps./cm².
2,000	0.00022
2,500	0.0029
3,000	0.017
3,500	0.058

usually larger, especially at the lower temperatures. The data for carbon are unknown, but the currents would probably be somewhat less than

[1] See equation (12) of this paper.
[2] Richardson, Emission of Electricity from Hot Bodies, p. 218.

these if the carbon could be purified, but would be of this order of magnitude if the carbon contained impurities.

Now the area of the anode crater is of the order of 1 cm^2, but only those positive ions which happened to enter the cathode in the region of the "bright spot" would be effective in neutralizing space charge. Probably the effective positive currents would be of the order of 1/10 of the values given in Table IV. The average atomic weight of the positive ions would certainly be close to 40, as shown by Richardson's experiments. Thus the positive emission from the anode would give rise to an electronic current from the cathode about 1,500 times as large as itself and quite adequate to account for Bräuer's experiments.

It is evident, therefore, that an experimental arrangement such as Bräuer employed would measure not the thermionic emission from the cathode but the positive emission from the anode, magnified about 1,500 times by the resultant liberation of electrons.[1] His observation that the currents were approximately independent of arc length is to be expected in view of the fact that the space charge remained negative, so that the positive emission would reach approximately its saturation value for all arc lengths.

IV. Theory of the Arc.

It is difficult to formulate a theory of the arc because the complexity of the phenomena makes it difficult to distinguish its essential features from those of secondary or accidental significance. A study of the simplest arcs, low voltage arcs between independently heated non-vaporizing electrodes in gases at reduced pressures,[2] has made it fairly certain that the essential feature of an arc is an emission of electrons from the cathode which produce sufficient ionization of the surrounding gas to give a positive space charge just outside the cathode, thus permitting approximately saturation electron emission at relatively low voltage. All other characteristics of arcs appear to be either consequences of this emission and ionization or prerequisites to it under the particular physical conditions in which the arc is produced. Thus it is possible to produce arcs in which the anode drop in potential is practically eliminated, in which the potential gradient in the gas between the electrodes is nearly zero, in which there is no chemical action or consumption of the electrodes or the gas, or in which the gas and anode temperatures are low. The cathode drop and its emission of electrons are, however, indispensable.

[1] Recent experiments by Professor A. Trowbridge and the writer have proved the correctness of this explanation of Bräuer's results.

[2] Mackay and Ferguson, loc. cit.; Compton, Olmstead and Lilly, Phys. Rev., 16, p. 282, 1920; Duffendack, Phys. Rev., loc. cit.

(a) Conditions at the Cathode.

If i is the current density of the negative emission from the cathode, there must be a current density j of positive ions sufficient to neutralize the space charge of the electrons. As we have seen,

$$i = 242\sqrt{M}\, j, \tag{5}$$

where M is the molecular weight of the positive ions. Besides these, there must be an additional positive current density J to give the excess positive space charge, to which the cathode drop is due. Let ρ be the density of the space charge.

$$\frac{d^2V}{dx^2} = -4\pi\rho = -4\pi\frac{J}{v}.$$

v is the average velocity of drift of the positive ions and is given by

$$v = \sqrt{\frac{\pi e}{2m}}\, l \frac{dV}{dx},$$

where e/m is the ratio of charge to mass and l is the mean free path of the positive ions. This expression is based on the assumption that the ion loses its forward velocity at each collision which must, on the average, be approximately true since ions and molecules have approximately equal masses. Thus, putting

$$B = \frac{4\pi}{\sqrt{\frac{\pi e}{2m}}\, l},$$

$$\sqrt{\frac{dV}{dx}}\frac{d^2V}{dx^2} = -BJ,$$

$$\left(\frac{dV}{dx}\right)^{3/2} = -\tfrac{3}{2}BJx + C_1.$$

The integration constant C_1 is determined by the condition that $dV/dx = 0$ (approximately) at the boundary of the cathode drop—at a distance from the cathode which we shall call c. Thus

$$\frac{dV}{dx} = [\tfrac{3}{2}BJ(c - x)]^{2/3}.$$

Integrating again,

$$V = -(\tfrac{3}{2}BJ)^{2/3}\tfrac{3}{5}(c - x)^{5/3} + C_2.$$

Since, when $x = 0$, $V = 0$ and when $x = c$, $V = V_c$, where V_c is the cathode drop in potential, we have

$$V_c = \tfrac{3}{5}(\tfrac{3}{2}BJ)^{2/3}c^{5/3}.$$

Substituting the value of B, we have

$$V_c = \frac{3}{5}\frac{\left(\frac{8\pi}{3}\right)^{2/3} J^{2/3}c^{5/3}}{\left(\frac{\pi}{2}\frac{e}{m}l\right)^{1/3}} \tag{6}$$

as the value of the cathode drop of potential.

It has been well established that the minimum value of V_c is the minimum voltage through which an electron must fall in order to ionize the surrounding gas. This is not always the "minimum ionizing potential," which refers to the voltage necessary to ionize at a single impact, since ionization may be a cumulative effect of two or more impacts. If the gas molecules are multi-atomic, this minimum voltage is probably identical with the "minimum ionizing potential." [1] If they are monatomic, it may be the "minimum radiating potential" [2] or the difference between the minimum ionizing and minimum radiating potentials.[3] In any case, the minimum values of V_c are pretty well established in several cases (5.5 volts in the mercury arc, 8.6 volts in the carbon arc, 16.2 volts in a tungsten arc in hydrogen, etc.). These minimum values of V_c are found when the arc operates under conditions favorable to ease of operation and passage of good sized currents. Under these favorable conditions, therefore, we may consider V_c as a known quantity.

As far as the writer is aware, no accurate measurements of the thickness of the region of cathode drop c in arcs have been made. Theoretical considerations, however, indicate that it should be about equal to the mean free path of the electrons from the cathode, since these have their best chance of ionizing at their first impact, owing to the fact that the electric intensity diminishes with distance from the cathode. This is almost certainly true in gases in which electrons lose their energy at each impact, such as multi atomic gases or monatomic gases whose molecules are partially ionized by previous impacts or absorption of radiation. This conclusion is strengthened by measurements of the thickness of the cathode drop in Geissler tube discharges, in which the width is found to be equal to the electronic free path with probably as much accuracy as the free paths are known at the large velocities involved. We shall, therefore, assume that c equals the electronic mean free path λ, *i.e.*, that $c = 4\sqrt{2}\, l = \lambda$.

This leaves J as the only unknown quantity, and its value may there-

[1] Duffendack, loc. cit.

[2] Compton, Olmstead and Lilly, loc. cit.

[3] Yao, PHYS. REV., 21, 1, 1923.

fore be determined. We find

$$J = \frac{3}{16\pi}\left(\tfrac{5}{3}V_c\right)^{3/2}\left(\frac{\pi}{2\sqrt{2}}\frac{e}{m}\right)^{1/2}\frac{1}{\lambda^2} \qquad (7)$$
$$= 1.47(10)^{-7}\frac{V_c^{3/2}}{M^{1/2}\lambda^2}\text{ amps./cm}^2,$$

where V_c is expressed in volts, M is the molecular weight of the positive ion, and λ is the electronic mean free path.

An application of this expression to various cases leads to reasonable values for J. It must be remembered that some of the data are not known with certainty, so that proper orders of magnitudes only can be expected.

1. *Carbon Arc.*—At atmospheric pressure and 3,300° K, which is close to the temperature at the cathode, we find $\lambda = 0.66(10)^{-3}$ cm. We have $V_c = 8.6$ volts and we may take $M = 16$ to at least the right order of magnitude. We then find, by equation (7), that $J = 2.5$ amps./cm². Since 250 amps./cm² is an average value for the current density of the arc current at the cathode, we have about 0.01 of the total current due to the *excess* positive current, which is in addition to the fraction $1/242\sqrt{M}$, or about 0.001, which just neutralizes the electronic space charge. Thus about 0.011 of the total current is carried by positive ions.

2. *Hydrogen Arc.*—(*a*) Taking data from the work of Mackay and Ferguson, we have pressure = 400 mm., $T = 3{,}300°$ K, whence $\lambda = 2.4(10)^{-3}$ cm according to kinetic theory. In hydrogen, however, we know from the work of Loeb[1] that the actual free path of an electron is only 0.53 of its theoretical value, so that our best value for λ is $1.27(10)^{-3}$ cm. $V_c = 16$ volts and $M = 2$. Thus $J = 4.1$ amps./cm². The observed current density was about 75 amps./cm², so that about 0.055 of the total current was due to *excess* positive ions and $1/242\sqrt{2}$ to *neutralizing* ions. Thus about 0.058 of the total current was carried by positive ions.

(*b*) From data taken in this laboratory with an arc in which the cathode was independently heated, we had, at 5 mm. pressure of hydrogen, an estimated temperature of 3,000° K and a current density of about 20 amps./cm², giving $J = 8(10)^{-4}$ amps./cm²; whence 0.00004 + 0.003 = 0.003, approximately, is the fraction of the total current which was carried by positive ions.

3. *Mercury Arc.*—The temperature of the vapor near the cathode of an arc is unknown. If the arc is maintained by an independently heated tungsten cathode, or by a self-heated tungsten cathode as in one type of

[1] Phys. Rev., 20, p. 106, 1922.

Cooper-Hewett quartz-mercury arc, the temperature is probably at least 2,800° K. In an ordinary arc with a mercury cathode, the temperature of the "bright spot" is unknown. It is probably at least 2,000° K. An assumed temperature of 1,000° K cannot be wrong by a factor more than three. $V_c = 5.5$ volts and $M = 200$.

If $p = 1$ mm, $\lambda = 0.049$ cm, and $J = 0.056(10)^{-3}$ amps./cm². The area of cross section of the cathode bright spot is unknown, but, in a well-known type of arc such as the Hereaus, cannot exceed 0.05 cm². If the total current is 3 amperes, the current density is at least 60 amps./cm², so that the *excess* positive current would be $9(10)^{-7}$ of the whole. The *neutralizing* current is $1/242\sqrt{200} = 0.29(10)^{-3}$ of the total. Thus about 0.0003 of the total current is carried by positive ions.

If $p = 100$ mm, $\lambda = 0.00049$ cm and $J = 0.56$ amps./cm². In this case 0.0093 of the total current is carried by positive ions.

If $p = 760$ mm, $\lambda = 0.000065$ cm and $J = 32.5$ amps./cm². Here a little more than half the assumed current would be carried by positive ions. This is physically impossible, which probably means than an arc would not maintain itself at 3 amperes under 760 mm. pressure—a lower pressure or higher current being required.

From the above examples it is seen that the least fraction of the current carried by positive ions is $1/242\sqrt{M}$, and that this occurs only at very low gas pressures. At the highest pressures, on the other hand, the large value of J sets a lower limit to the possible arc current, since the total arc current must exceed $2J$.

Energy Considerations at Cathode.

In addition to satisfying the space charge relations, the conditions at the cathode must satisfy the principle of conservation of energy. This was long ago pointed out by J. J. Thomson,[1] but has not always been taken into account. In the light of our present knowledge the energy relations at the cathode may be formulated as follows:

Let n_1 and n_2 be the numbers, respectively, of electrons leaving the cathode and of positive ions reaching it per second. If $e\phi$ is the heat of evaporation of an electron from the cathode,[2] the cathode loses heat at the rate of $n_1 e\phi$ as a result of thermionic emission. It likewise gains heat at the rate of $n_2 e V_c$ from the energy gained by the positive ions in falling through the cathode drop of potential V_c.

$(n_1 - n_2)$ electrons fall through the cathode drop without ionizing. Their energy is, of necessity, dissipated ultimately in the form of heat.

[1] Conduction of Electricity through Gases, 2d ed., p. 614.

[2] Richardson, Emission of Electricity from Hot Bodies, p. 164.

Since this occurs close to its surface, the cathode gains half of this energy, or $\frac{1}{2}(n_1 - n_2)eV_c$, provided the mean free path is small in comparison with the linear dimensions of the emitting surface of the cathode. In general we have $f(n_1 - n_2)eV_c$, where f is the fraction of the total solid angle, about a point distant λ from the cathode, which is subtended by the cathode.

When positive ions are absorbed into the cathode, some energy is liberated in addition to the kinetic energy with which they strike. This includes (1) heat of condensation of positive electricity, (2) heat of condensation of the uncharged material of the ion, (3) possible chemical action. Its amount may be taken account of by imagining the absorption of the positive ions to proceed in the following way. An electron escapes from the cathode, absorbing heat $e\phi$, and combines with the positive ion outside the surface, liberating heat eV_i where V_i is the ionizing potential of the gas. The neutral system is then absorbed by the cathode, liberating heat L, where L is the latent heat of condensation per molecule, or the heat of chemical combination per molecule. Thus $n_2[e(V_i - \phi) + L]$ is the rate at which heat is liberated by absorption of positive ions.

Finally, there are other heat losses which may be grouped as $(C + C' + R - H)$, where C is loss by conduction through the cathode, C' is loss through the gas, R is net loss by radiation and H is heat supplied by independent heating of the cathode, if it be electrically or otherwise heated. In a self-maintained arc $(C + C' + R)$ represents a positive heat loss. If the cathode is independently heated, $(C + C' + R - H)$ may be positive or negative, depending on conditions.

Grouping all these items, we find the equilibrium condition to be given by

$$n_2eV_c - n_1e\phi + f(n_1 - n_2)eV_c + n_2e(V_i - \phi) + n_2L - C - C' - R + H = 0,$$

or

$$\frac{n_2}{n_1} = \frac{\phi - fV_c + \dfrac{C + C' + R - H}{en_1}}{V_i + fV_c - \phi + \dfrac{L}{e}}. \tag{8}$$

In some cases the quantities in this expression are known or may be calculated with sufficient accuracy to permit an estimate of the ratio n_2/n_1. It is of interest to compare these estimates with those previously made from space charge considerations.

1. *The Carbon Arc.*—In this case $H = 0$. It is found that chemical action, as with introduction of oxygen, has an effect on the arc but is

not essential to its maintenance, and probably does not ordinarily play an important rôle. In such cases L/e has to do with heat of condensation. In all cases where the value of L/e is known it is much smaller than the other terms in equation (8), so that we shall neglect it. $V_c = 8.6$ volts and $\phi = 3.92$ volts for carbon. V_i is uncertain, because the kind of atom which is principally ionized in the ordinary arc in air is unknown. For most substances, including oxygen and nitrogen, V_i is about 16 volts. We will therefore put $V_i = 16$ volts provisionally. $f = 1/2$.

If we neglect the losses $(C + C' + R)$ we obtain a minimum value for n_2/n_1 of $n_2/n_1 > -0.0244$, which means, of course, that $n_2/n_1 > 0$.

A probable maximum limit to n_2/n_1 can be found by using Richardson's conclusion that, at temperatures above 3,000° K the cooling effect due to evaporation of a saturation current of electrons exceeds that due to radiation, which is easily shown to be the major term in $(C + C' + R)$. Thus

$$\frac{C + C' + R}{en_1} < \phi,$$

and an upper limit to n_2/n_1 is given by

$$\frac{n_2}{n_1} < \frac{2\phi - \frac{1}{2}V_c}{V_i + \frac{1}{2}V_c - \phi} = 0.215.$$

Thus

$$0 < \frac{n_2}{n_1} < 0.215.$$

A closer estimate may be made by attempting to evaluate $(C+C'+R)$. Taking a current of 10 amps., the area of the bright spot is about 0.04 cm^2, the temperature gradient is of the order of 2,500 deg./cm and the conductivity is about 0.01. Reducing to appropriate units we find $C/n_1e = 0.04$ volt, approximately.

Since the temperature of the surrounding gases exceeds that of the cathode, C' is negative, but rough calculations show it to be negligible compared with C and R. If the cathode is cooled by convection, as by blowing it, C' may become positive and large. It seems safe, however, to neglect C' under ordinary conditions.

The loss by radiation is calculated by assuming both anode and cathode to radiate as black bodies at the rate of $5.5(10)^{-5}T^4$ ergs/cm^2 sec. Taking the cathode to be at 3,140° K we find its loss to be at the rate of 21.3 watts, approximately. This loss is reduced, however, by a gain of heat from the anode, whose crater usually subtends about one third of the hemi-solid angle about the cathode spot, and whose temperature is about 3,700° K. Thus, in exchange for the 7.1 watts radiated to the anode crater, $7.1\left(\frac{3,700}{3,140}\right)^4 = 13.8$ watts are returned. The net loss from the cathode is thus $21.3 - 13.8 = 7.5$ watts, whence $R/n_1e = 0.75$ volt, approximately.

Combining these terms $(C + C' + R - H)/n_1e = 0.79$ volt. Substituting this value in equation (8) gives

$$\frac{n_2}{n_1} = 0.025, \qquad \text{whence} \qquad \frac{n_2}{n_1 + n_2} = 0.0245,$$

as the best estimate which we can make. The previous method gave a value 0.011. The difference is well within the limits to be expected in view of the uncertain data upon which the calculations are based.

2. *Hydrogen Arc between Tungsten Electrodes.*—(*a*) Here the data are more accurately known. $V_i = 16$ volts, $\phi = 4.4$ volts, $V_c = 16$ volts. In the work of Mackay and Ferguson, $T = 3{,}300°$ K, current = 25 amps. and effective area = 0.375 cm², approximately. Thus $R = 141$ watts and $R/n_1e = 5.6$ volts, approximately. Owing to the construction of the apparatus C may be neglected, C' is small compared with R and H is zero. Thus we have

$$\frac{n_2}{n_1} = 0.102, \qquad \text{whence} \qquad \frac{n_2}{n_1 + n_2} = 0.091$$

as compared with 0.058 calculated for the same case by the previous method.

(*b*) In dealing with the second example of hydrogen, we must take $f = 1/6\pi$, approximately, since the mean free path at $T = 3{,}000°$ K and 5 mm pressure, which is 0.09 cm, caused the energy of the electrons to be liberated at a distance at which the wire (15 mil) subtended about $1/6\pi$ of the total solid angle. The emitting area of the cathode was, in this case, estimated as 0.3 cm². We thus calculate R/n_1e to be approximately 2.23 volts.

If we neglect the independent heating H we find

$$\frac{n_2}{n_1} = 0.52, \qquad \text{or} \qquad \frac{n_2}{n_1 + n_2} = 0.34.$$

This value is nearly a hundred times greater than that calculated by the previous method, *viz.*, 0.003, which means that the arc could not be maintained under these conditions of current and pressure without independent heating of the cathode,—as was the case. If we assume that the two methods, including the value of H, should yield identical results, we find that

$$\frac{n_2}{n_1} = 0.003 = \frac{4.4 - \frac{16}{6\pi} + 2.6 - \frac{H}{n_1 e}}{16 + \frac{16}{6\pi} - 4.4},$$

whence $H = 36.66$ watts. Actually, the cathode filament was heated by a current of 18 amperes with a potential drop of 3.1 volts, or 56 watts, not all of which was dissipated across the emitting part of the cathode surface. Again the two equations give consistent results.

3. *Mercury Arc.*—In this case $V_i = 10.4$ volts and $V_e = 5.5$ volts are accurately known. Probably ϕ is in the neighborhood of 4 volts. We cannot calculate $(C + C' + R)$ owing to ignorance of the real conditions at the surface of the cathode. If we neglect these terms, we get the lower limit for n_2/n_1 as

$$\frac{n_2}{n_1} > 0.123, \qquad \text{whence} \qquad \frac{n_2}{n_1 + n_2} > 0.109.$$

We previously estimated a fraction 0.0003 at 1 mm pressure, 0.0093 at 100 mm pressure and 0.54 at 760 mm pressure. The conclusion is that an arc would not maintain itself under the assumed conditions (3 amperes, cathode cross-section 0.05 cm^2, minimum cathode drop of 5.5 volts) unless the vapor pressure were greater than 100 mm or unless the cathode were independently heated. This agrees with experience.

From the results of these tests of the equations (7) and (8) I think there is good evidence that the essential features of the arc are those which have been incorporated in this theory of the conditions at the cathode.

(*b*) *Conditions in the Region between the Electrodes.*

In this region the conditions may vary considerably, depending on the nature of the arc. In a low-voltage arc maintained by an independently heated cathode in a gas at a few millimeters pressure, there appears to be no ionization of gas in this region or at the anode. The region is therefore one of negative space charge, but the drop of potential across it may be very small if the anode is of large area and close. If the gas pressure is raised, the mobility of the electrons diminishes, and the space charge and potential drop increase. If the pressure is diminished considerably, there is insufficient ionization at the cathode to maintain the arc without increased voltage and, at still lower pressures, ionization by impact occurs in the region between the electrodes, and the discharge becomes one of the vacuum tube type.

Carbon Arc.

In the carbon arc the conditions are less simple. Experiments show that the potential gradient in this region is approximately constant, which proves that there is no space charge here. Therefore there must be sufficient ionization in this region to supply enough positive ions to neutralize the space charge of the electrons coming from the region of the cathode. We shall inquire into the origin of these positive ions by considering the various possible sources. We have seen that each positive ion neutralizes the space charge of $242\sqrt{M}$ electrons, or, roughly, 1,000 electrons. The greatest current density is near the cathode, where it amounts to about 250 amps./cm^2. In the middle of the arc the current density is much less, and is probably of the order of 50 amps./cm^2. Thus it is necessary to account for a positive current density ranging from 0.05 to 0.25 amp. per cm^2 in this region.

Positive Ions from Anode.—If these are present, they must have been produced by electron impacts on the anode and not as a result of its high temperature, since the anode may be cooled without affecting very much the conditions in the arc. But we have no independent evidence of the emission of positive ions from solids subjected to electronic bombardment at these low voltages which is at all adequate to produce the necessary positive current.

Furthermore the conditions in this region are favorable to recombination, since the potential gradient is only about 27 volts/cm and the mean free path is extremely small. The existence of recombination is proved by the luminosity of this region. Now if the positive ions originated at the anode, their number would diminish as they approach the cathode, so that the space charge could not be zero at more than one surface. Thus the anode cannot be the chief source of positive ions.

Ionization by Collision.—The mean free path of electrons in this region is about $0.74(10)^{-3}$ cm, assuming the temperature to be 3,700° K. With a potential drop of 27 volts/cm, an electron would fall through only 0.02 volt in an average free path. The probability of ionizing at a collision would be $\epsilon^{-V/0.02}$, where V is the ionizing potential of the gas. If $V = 16$ volts, only $2.76(10)^{-347}$ of the collisions would result in ionization. Each electron makes on the order of 1,000 collisions in its path between the electrodes, so that the positive current from this cause, even neglecting all recombinations, would amount to only about $2.76(10)^{-344}$ of the electronic current.

Possibly the effective ionizing potential, by cumulative action or otherwise, is as low as 8.6 volts, as suggested by the size of the cathode drop. If we put $V = 8.6$ we find the proportion to be about $(10)^{-184}$. In either case, ionization by collision is evidently of no importance.

Photo-electric Ionization—This must certainly occur to some extent, since it can occur, theoretically, wherever recombination occurs. It is well known that the temperature radiation from the electrodes produces no detectable photo-electric ionization of the ordinary gases. Thus any photo-electric action, if present, must result from radiation of a very absorbable nature arising from the gas itself, and is taken account of in the following treatment.

Thermal Ionization of Gas.—The degree of thermal ionization may be calculated by applying Nernst's Heat Theorem after the manner of Saha.[1] We have

$$\log \frac{x^2}{1 - x^2} P = - \frac{U}{4.57T} + 2.5 \log T - 6.5, \tag{9}$$

where x is the fraction of molecules ionized, P is the pressure in atmospheres, U is the energy required for ionization in calories per gram molecule and 6.5 is calculated from the chemical constants.

$$U = \frac{eVN}{300J} = 2.29(10)^4 V,$$

where V is the effective ionizing potential in volts. V is certainly not greater than 16 volts and is more likely about 8.6 volts, as evidenced by the cathode drop.

The temperature T of the gases in the arc is not known at all accurately. It is agreed that it is higher than that of the electrodes.[2] Thus it may be even higher than 4,200° K.[3]

After calculating x by equation (9), the number of positive ions per cm³ can be calculated by multiplying x by the number of molecules per cm³, considering the pressure to be 1 atmosphere. Thus

$$\nu = 2.75(10)^{19} \frac{273}{T} x \tag{10}$$

is the number of positive ions per cm³. It is then possible to calculate the current density j of the positive ions from the relation

$$j = \nu e u E, \tag{11}$$

where u is the average velocity of drift of positive ions in unit field and E is the intensity of the field.

The mobility u may be calculated by the principles of the Kinetic

[1] Phil. Mag., 40, p. 472, 1920.

[2] J. J. Thomson, Conduction of Electricity through Gases, 2d ed., p. 604.

[3] Lummer and Pringsheim, see Child, Electric Arcs, p. 44.

Theory. The coefficient of interdiffusion D_{12} of gas 1 into gas 2 is [1]

$$D_{12} = \frac{\pi}{8N}(N_1L_1\Omega_1 + N_2L_2\Omega_2),$$

where N_1, N_2 are the numbers of molecules per unit volume; L_1, L_2 are the mean free paths; Ω_1, Ω_2 are the average translational velocities; $N = N_1 + N_2$. In this case the number of ions N_1 is so much smaller than the number of neutral molecules N_2 that we may neglect the first term, leaving

$$D = \frac{\pi}{8} L\Omega,$$

where L and Ω refer to the molecules.

J. J. Thomson has shown [2] that the coefficient of diffusion D is related to the mobility u according to the equation

$$u = D\frac{Ne}{p},$$

where p is the pressure of the gas.

Eliminating D between these equations we have

$$u = \frac{\pi}{8}\frac{L\Omega Ne}{p}. \qquad (12)$$

Substituting from (10) and (12) into (11) we find, for the positive current density,

$$j = 2.94(10)^{21}\frac{L\Omega Nxe^2E}{pT}, \qquad (13)$$

or, if j is in amperes and V in volts,

$$j = 3.27(10)^9\frac{L\Omega Nxe^2E}{pT} \text{ amps./cm}^2$$

in which all the constants are accurately known and the fraction x may be calculated by equation (9).

The following table shows results of calculations under several conditions, considering L and Ω to be for air molecules at the temperature T and atmospheric pressure. We saw that it was necessary to account

TABLE V.

T° K.	V Volts.	x.	ν.	j amps./cm².
3,700	16.0	$2.0(10)^{-10}$	$4.0(10)^{8}$	$5.0(10)^{-8}$
3,700	8.6	$2.2(10)^{-5}$	$4.4(10)^{13}$	$5.5(10)^{-3}$
4,200	8.6	$1.4(10)^{-4}$	$2.5(10)^{14}$	$3.5(10)^{-2}$
4,500	16.0	$2.5(10)^{-8}$	$4.2(10)^{10}$	$5.8(10)^{-6}$
4,500	8.6	$3.35(10)^{-4}$	$5.6(10)^{14}$	$7.8(10)^{-2}$

[1] Meyer, Kinetische Theorie der Gase, 2d ed., p. 261.

[2] Conduction of Electricity through Gases, 2d ed., p. 43.

for currents of the order of 0.05 ampere in the central portions of the arc. Thermal ionization appears adequate to give such currents at reasonable arc temperatures, provided the effective ionizing potential is about 8.6 volts, but not if the ionizing potential is 16 volts. This value, 8.6 volts, may represent a "radiating" potential of the gas, ionization being a two-stage cumulative effect. Both oxygen and nitrogen have radiating potentials in this region. It is more likely, however, a critical potential of carbon vapor, about which we have no direct experimental evidence.

Of the possible explanations of ionization in this region of the carbon arc, only thermal ionization appears possible. Of course the impact of electrons contributes to the attainment of the necessary high temperatures.

(*c*) *Conditions at the Anode.*

No success has attended the effort to obtain explicit quantitative values for the magnitude of the anode drop. It is easy to account for it, however, in a qualitative way. By thermal ionization in the gas, ions are produced at the rate N/τ per unit volume, $1/\tau$ being the average frequency with which a molecule is thermally ionized. If t is the average life-time of a positive ion, $Nt/\tau = \nu$, where ν is the number of positive ions per unit volume. Thus $t = \nu\tau/N$ is the average life-time of a positive ion. The average distance which a positive ion moves between formation and recombination is then given by

$$d = uEt = uE\frac{\nu\tau}{N}\cdot$$

Outside the regions of the anode or cathode drops, ν is given by equation (10) and u by equation (12). Inside the region of the anode drop, ν has a smaller value than that predicted by equation (10) since the ions are swept away too fast to permit the assumption of equilibrium conditions between production and recombination of ions at a specified point. In any case, the positive ions formed near the anode travel a distance d somewhat greater than that predicted by the above equation before disappearing. It is therefore evident that, if the space charge is zero in the main body of the gas, there must be a deficiency of positive ions within a distance d from the anode. The concentration of positive ions varies from 0 at the surface of the anode to ν at a distance d from it. Within this distance there exists a negative space charge and the electric intensity rapidly increases with nearness to the anode.

Probably a second factor contributing to the drop at the anode is a space charge, very near its surface, due to the suppressed thermionic emission from it, when it is hot.

Under some conditions, also, positive ions may be produced by electron impact against the anode, or against vapors emitted by the anode when it is heated.

Varying the nature and geometrical relation of the anode to the cathode may vary any or all of the above factors, and thus account for the fact that the anode drop is not a constant in a given type of arc, but appears to be essential only for the reason that the anode must receive the electronic current which leaves the cathode.

Conclusions.

The arc seems dependent on an adequate supply of electrons at the cathode, whose escape from it is made possible by sufficient ionization of gas near it to form a positive space charge. The calculations which are given support this view and indicate that the thickness of the region of the cathode drop is of the order of the electronic mean free path in the gas.

There is no reason for doubting that any origin of sufficient electronic emission from the cathode would serve to maintain an arc, but these currents appear actually to be of thermionic origin. It seems unlikely that photo-electric emission can be obtained large enough to support an arc, and chemical action or bombardment by positive ions are effective only as they contribute to the general or local high temperature of the cathode.

It is possible that, in certain cases, true thermionic emission may be obtained in larger amount than would be predicted by Richardson's thermionic equation (1). Impacts of positive ions may produce local high temperatures which cannot be allowed for in applying Richardson's equation. Furthermore, in gases at high pressure, the region of the cathode drop may be so thin as to give rise to such a high electric intensity as to draw out of the cathode electrons which would not otherwise be included in the saturation current. In other words, the field may extract electrons from the region of the "electron atmosphere." This would be equivalent to a reduction in the ordinary value of the work-function ϕ for the material of the cathode, as is at once evident when it is remembered that ϕ includes the effect of the attraction of the positive charge induced on the surface of the cathode by the escaping electron. Millikan and Shackelford [1] were unable thus to extract electrons from metals by fields lower than 4,000,000 volts per cm. At the high temperature of an arc cathode, however, the electron atmosphere should extend farther from the cathode surface and an effect of the sort suggested may be appreciable.

[1] Phys. Rev., 15, p. 239, 1920.

We have one bit of positive evidence in some work in this laboratory in which abnormally large thermionic currents were obtained from a hot tungsten filament when the anode was brought very close, perhaps within the region of its electron atmosphere.

A recent paper by Dushman[1] has shown that this effect, first suggested by Schottky, is appreciable in ordinary measurements of thermionic emission. It should be much greater in the case of high pressure arcs.

Finally, it may be mentioned that equation (7) suggests that the ionization should increase as the square of the gas pressure. This is qualitatively in agreement with the observation that the intensity of spectral excitation in an arc increases very rapidly with increase in gas pressure.

Palmer Physical Laboratory,
Princeton, New Jersey,
September 25, 1922.

[1] Dushman, Rowe and Kidner, Phys. Rev. 21, 207, 1923.