AN EXTENSION OF THE PRINCIPLE OF THE DIFFRACTION EVOLUTE, AND SOME OF ITS STRUCTURAL DETAIL.

By G. G. Becknell and John Coulson.

Synopsis.

Diffraction Patterns inside Shadows due to Point Sources of Light.—(1) *Conic section shadows.* In confirmation and continuation of the results in the preceding article, the patterns are found to depend only on the shape of the shadow, the Arago spot being obtained at the center of the circular shadow even when this was cast by a spiral edge of large pitch ground to the form of a truncated cone, when the point source was placed at the apex. Patterns inside the shadows of hyperbolic and parabolic plates were also obtained and are reproduced. As in the case of the ellipse, the predominant figure in each case is the evolute of the edge of the shadow. (2) *This general relation between diffraction caustic and shadow* is found to hold even in the case of the *shadow of the involute of a circle,* when the diffraction figure was identical with the generating circle which was, of course, the evolute of the edge of the shadow. A series of photographs of elliptical shadows show that the diffraction caustics are not continuous curves. The changes of detail and of color in the patterns with change of ellipticity of the shadow are described at some length.

In a previous paper[1] reported in this journal it was shown that for circular and ellipitcal plates illuminated by a point source, whatever the position of the plates with reference to the wave-normal, a predominant figure was produced behind the object which proved to be the evolute of the geometrical shadow.

Knowing the general mathematical relations between plane curves and their evolutes, it did not seem probable that the example there discussed of the relation between an elliptical shadow and its diffraction figure could be merely an isolated case of a curve and its evolute. This paper presents illustrations of diffraction patterns produced by parabolas, hyperbolas, and other diffracting objects.

Accurate parabolic and hyperbolic plates were produced by a lathe method similar to that discussed in our former paper for elliptical plates. In this case a cone was used instead of a cylinder. By holding thin plates of metal between proper sections of the cone it was possible to turn out both shapes at one operation. However, in the case of the parabola it was necessary that the cone be sliced well back from the apex. Otherwise, the bevelled edge of the plate caused the shadow to be that of two overlapping parabolas. It is also worth noting that it is far more difficult to turn out a true parabola than a true hyperbola since the para-

[1] See page 594, this issue.

bolic section must be accurately parallel to the generator of the cone. Much of the success in producing these plates is due to the careful workmanship of Mr. G. S. Shallenberger, our department mechanician.

The apparatus and its arrangement were the same as used previously. The diffracting plates were supported on a rod extending along the axis of symmetry of the figure in such a way as not to interfere with the diffraction pattern. The plates were then oriented until the patterns were symmetrical with respect to the axis just mentioned.

Plate I., No. 1, shows the shadow and diffraction within the shadow of a hyperbolic plate whose height along the axis was 1.85 cm., and whose base was 3.6 cm. No. 2 shows the corresponding photograph for a parabola of height 2.7 cm., base 2.4 cm. The curves are precisely the ones anticipated, and measurements taken from the plates prove them to be true evolutes.

The equation of the hyperbola referred to its vertex is

$$(x/a)^2 + 2(x/a) - (y/b)^2 = 0.$$

And the equation of the corresponding evolute is

$$[a(a + x)]^{2/3} - (by)^{2/3} = (a^2 + b^2)^{2/3}.$$

In order to determine the values of a and b for the hyperbola an outline of the plate was projected upon coördinate paper and corresponding values of x and y for a number of points were recorded. The best values for the a and b corresponding to the geometrical shadow were then found

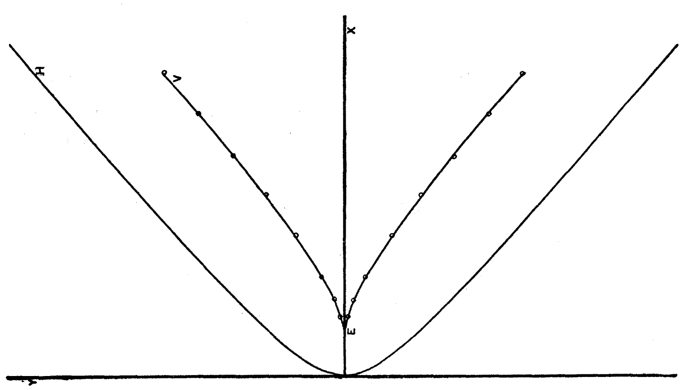

Fig. 1.

by the method of least squares to be 1.26 cm. and 1.08 cm. respectively. These values show upon computation that the hyperbola and its evolute do not intersect. The photograph indicates, also, that the same is true of the diffraction figure and the outline of the shadow.

Fig. 1 shows the hyperbola and corresponding evolute computed for the geometrical shadow using the values of a and b above. The circles indicate points taken at random from the diffraction curve on the photographic plate (Plate I., No. 1).

Fig. 2 illustrates similar curves for the parabolic shadow. The equation of the evolute for the parabola $y^2 = 4ax$, is

$$y^2 = 4(x - 2a)^3/27a.$$

These curves always intersect at the point $x = 8a$, $y = 4a\sqrt{2}$. This is in agreement with the photograph (Plate I., No. 2.)

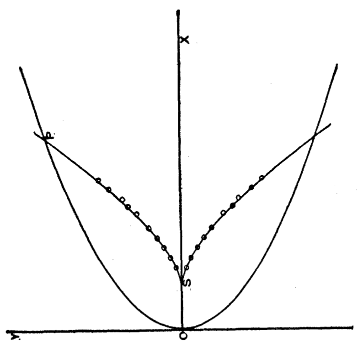

Fig. 2.

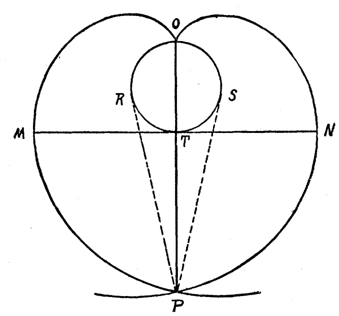

Fig. 3.

In order to apply this general principle of the diffraction evolute to other figures than conic sections a plate was cut in the form of the heart-shaped portion of the involute of a circle. This was done by unwinding a waxed thread from a small cylinder. A needle fastened to the free end of the thread was used to cut the outline of Fig. 3 from a sheet of heavy lead foil. From the construction it follows that the evolute of this curve is the original circle. Plate I., No. 3, reproduces the shadow of one of these heart-shaped plates and the diffraction figure accompanying it. The plate was about 2.5 cm. in length between the points of the heart, and was produced from a cylinder 8 mm. in diameter. The diameter of the circle as measured from the photograph agrees with that of the predicted evolute within 2 per cent.

It may be instructive to point out that there is not a one-to-one correspondence between the points of the circular evolute and those of the heart-shaped outline shown in Fig. 3. However, such correspondence does hold between the circle and the outline *MON*. This has been observed by placing a straight-edged screen over the portion *MPN* of the diffracting plate. In this case the whole circle was clearly visible. When the portion of the plate lying above *MTN* was covered the diffraction figure was only the arc *RTS*.

Examination of the photographic illustrations in this and in our preceding paper show that the shadows are crossed by a multitude of bright lines running normal to the edge. These lines are clearly developed in Plate I., No. 4, which was produced by a heart-shaped plate cut from paper, and the diffraction figure is seen to be a true envelope or caustic. From this it is probable that shadows and the diffraction patterns lying therein may be ascribed to the diffraction normals at the boundary of the shadow.

It seems, then, to be true in general *that whenever an object illuminated by a point source produces a diffraction pattern within its shadow, the predominant diffraction figure is the evolute of the geometrical shadow.*

The production of the Arago spot has always been associated with the disc or sphere as the diffracting object. However, it was shown in our first paper on this subject that an elliptical plate producing a circular shadow also has associated with it a true Arago spot. A still more striking case is furnished by a diffracting object in the form of a spiral. Plate I., No. 6, is the circular shadow and Arago spot produced by a ¾-in. (1.90 cm.) auger bit of 1-in. (2.54 cm.) pitch ground to the form of a truncated cone and illuminated by a point source at the apex of the cone (see Plate I., No. 7).

If the axis of the cone is not accurately perpendicular to the wave front the shadow cast is not strictly circular, although it may apparently be so. In such a case the diffraction figure consists of two caustics. These are illustrated in No. 8. When the cone was reversed so that the apex was turned away from the point source, no adjustment whatever was capable of producing a true Arago spot. The nearest approach to a circular spot was crescent shaped with a dark shadow on the concave side, similar to one of the caustics in No. 8. It should be observed in this instance that the geometrical shadow could never be circular. The explanation of an Arago spot produced by such a device as the one here discussed, that is, by means of a spiral object, is a decided departure from the concept of Fresnel zones.

It may be said that the experiments discussed here and in the preceding paper seem to indicate the following general principle: *Any particular form of geometrical shadow is always associated with the same diffraction pattern whatever be the diffracting object.*

The Detail in the Structure of the Evolutes.

It seems desirable to call attention to certain peculiarities of the diffraction evolutes of the conic sections. In order to make a more minute study of the formation of the evolutes of ellipses and circles the

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Plate I.
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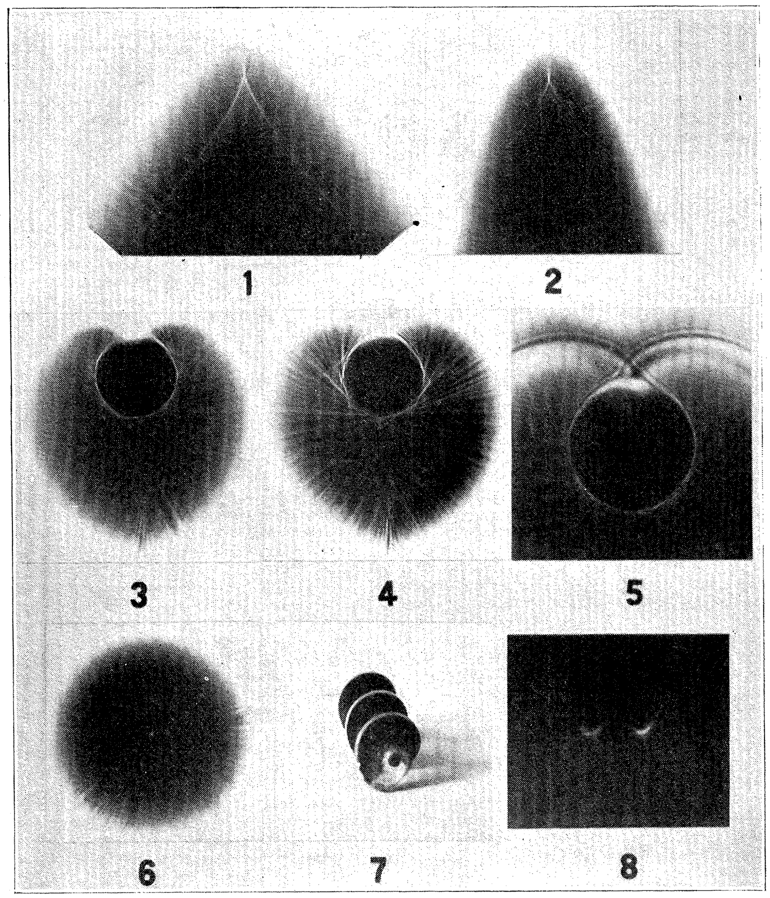

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distance from pinhole to photographic plate was increased from the usual distance of 7 meters to 20 meters. The diffracting plate was placed 3 meters from the pinhole instead of 2 meters. This arrangement produced about double the magnification previously used.

Plate II., No. 9, shows the diffraction pattern due to an ellipse of major diameter 1.6 cm. when placed with its plane perpendicular to the wave-normal. It will be observed that a series of bands lies *inside* the evolute and parallel thereto; and that the intensity of the bright bands diminishes rapidly as they recede from the evolute. The evolute consists of a number of separate flame-like streaks or "flares" which are merely the terminals of the bright bands within. As the eccentricity of the elliptic shadow diminishes the number of flares in a quadrant of the evolute decreases; and, therefore, likewise the number of bands within the evolute. This is illustrated by No. 10, taken for the ellipse above rotated through an angle of 25°. A similar decrease in the number of bands accompanies a reduction in the size of the plate when distances are kept constant.

A disc casting an elliptical shadow produces diffraction patterns identical with those just described when the dimensions of the shadows correspond. Plate II., Nos. 11, 12 and 13, were made with a disc .98 cm. in diameter, and correspond to angles of rotation from the normal position of 30°, 60° and 75° respectively. These illustrations bring out still more clearly the discontinuous nature of the diffraction evolute.

In the case of elliptical shadows there is always a set of bright and dark bands lying in the direction of the major axis. These become more distinct with increasing eccentricity of the shadow, as shown by Nos. 12 and 13. Similar strong bands appear in the patterns produced by an ellipse. When such a plate is rotated about its minor axis toward the position producing a circular shadow these bands gradually vanish, reappearing upon further rotation of the plate, but with an orientation in direction of 90°. That is, the direction of these bands in all cases is that of the *major axis* of the elliptical shadow.

Nos. 14, 15, 16 and 17, were taken with a carefully turned disc 5 mm. in diameter, the distance from source to photographic plate being increased to 33 meters. No. 14 shows the Arago spot and accompanying rings. Precisely the same pattern has been produced by an elliptical plate when casting a circular shadow. When the disc is rotated 20° from this position about an axis in its own plane No. 15 is produced. For angles up to 10° the transition in the pattern is almost imperceptible, but becomes more rapid beyond a rotation of 20°.

No. 16 corresponds to an angle of 30°. Here the bright spot at the center has entirely disappeared and is replaced by a black figure resembling a pair of crossed dumb-bells with white knobs faintly appearing at the ends of the bars. These knobs become brighter upon further rotation of the disc. At approximately 45° a bright spot reappears at the center of the figure with a pattern as shown in No. 17. It will be observed in the last two figures that the rings which surrounded the original Arago spot have changed decidedly in shape. In No. 16 they appear elliptical while in No. 17 they are oval. The major diameter of each figure is along the minor axis of the elliptical shadow.

Very definite transitions in color were observed as the disc was rotated beyond the position represented by No. 15. The central spot remains white up to about 22.5°, shading into pink with increasing angle and becoming distinctly red at 27.5°. At 30° the spot has entirely disappeared. On further rotation of the disc a central spot gradually reappears, showing green at 35°, surrounded by four white spots in the form of an upright cross as in No. 16. Four distinctly red knobs appear on the crossed dumb-bells in this figure. At 45° the central dot changes to red, closely surrounded by four streaks of green, as in No. 17. At 50° the center again is black, and the four streaks have changed to white. The four flares on the axes always remain white.

Attention has been called to the fact that in the case of elliptical shadows a set of "secondary" bands lie *inside* the evolute and parallel to it, while another set extend along the major axis. The hyperbola and parabola, also, give rise to a similar series of bands. However, in Plate I., No. 3, the secondary bands occur on the *outside* of the circular evolute, and none on the inside which appears to be absolutely dark. The latter point is explained by the fact that the circular area is not traversed by any of the diffraction normals from the edge of the shadow. No. 5 shows that these secondary bands intersect at the cusp of the heart and become continuous with the usual diffraction bands lying outside the shadow.

In the case of all shadows in the form of conic sections the evolute is the envelope of the normals to the edge of the shadow which emanate from the opposite side of the axis. Thus, the areas outside the evolute are traversed by only one set of these normals; and, therefore, the secondary bands cannot exist within these areas.

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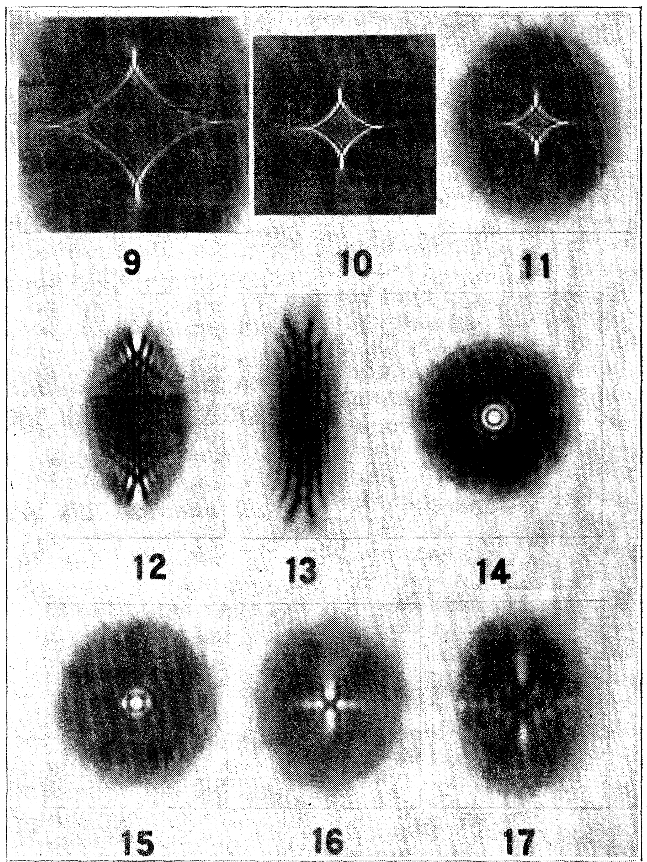

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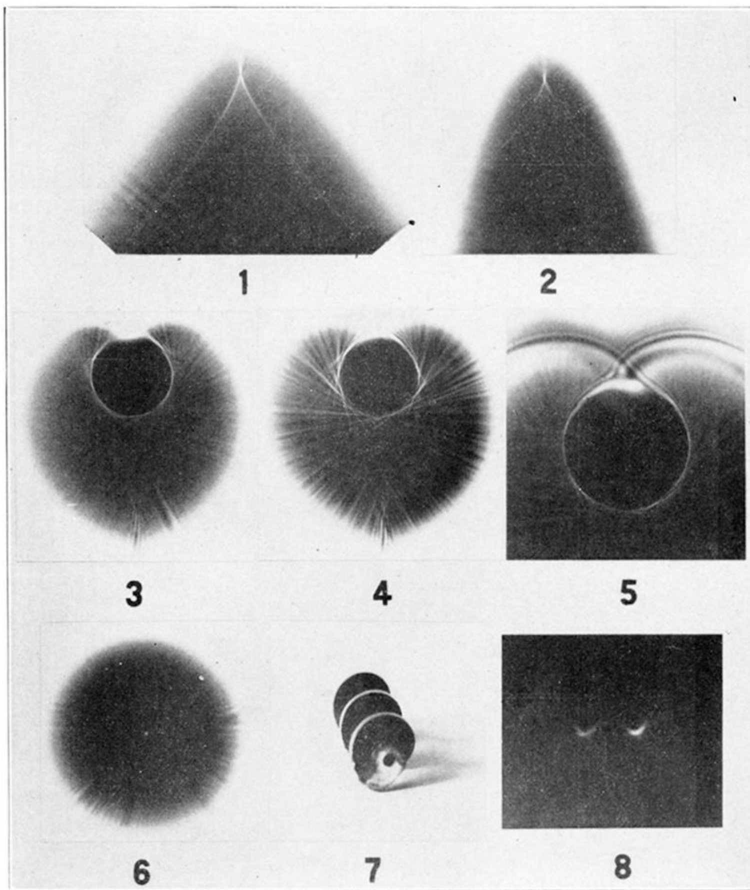
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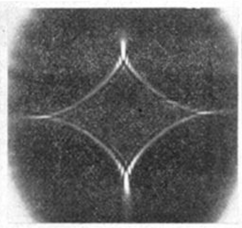

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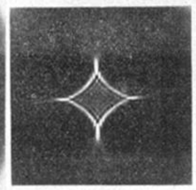

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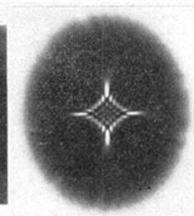

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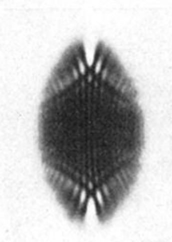

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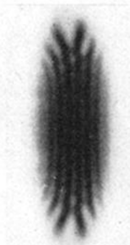

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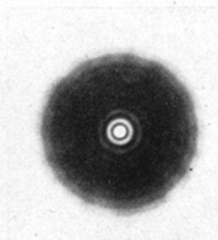

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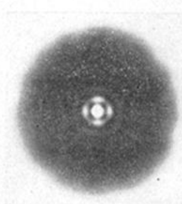

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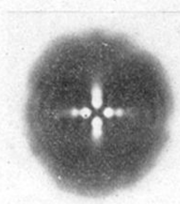

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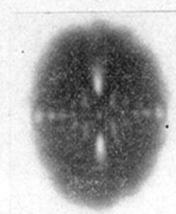

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