ACOUSTIC WAVE FILTERS.

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SYNOPSIS.

Acoustic Wave Filters Composed of a Series of Like Sections.—(1) *Theory.* Taking the impedance of any part of an acoustic circuit to be equal to the complex ratio of the applied pressure difference to the rate of change of volume displacement, it is shown that, neglecting dissipative forces, it is possible to construct a filter having limiting frequency values of no attenuation determined by the formulæ $Z_1/Z_2 = 0$ and $Z_1/Z_2 = -4$, where Z_1 is the impedance of the transmitting conduit circuit and Z_2 of each branch of each section. The impedance of any section depends on the inertance M of dimensions mass per unit area squared, and the capacitance C which has the dimensions of stiffness per unit area squared. If M and C are in parallel, $Z = iM\omega/(1 - MC\omega^2)$, whereas if they are in series, $Z = i(M\omega - 1/C\omega)$. For instance, in the case of a closed chamber or resonator, M and C are in series and are equal to ρ/C and $V/\rho a^2$ respectively where ρ is the density of the medium, c is the conductivity of the mouth, a the velocity of sound and V the volume. Formulæ are derived for various assumed cases. On account of the uncertainty as to whether a tube may be considered sa having the equivalent inertness and capacitance connected in parallel or in series, the application of these formulæ to actual cases is somewhat empirical. (2) *Construction and test of filters of three types.* Low-frequency-pass filters were made, for example, by two concentric cylinders joined by walls equally spaced and perpendicular to the axis. Each chamber thus formed had a row of apertures in the inner cylinder which served as the transm!ssion tube. In one case the volume of each chamber was 6.5 cm.^{3}, the radius of the inner tube 1.2 cm. the length between apertures, 1.6 cm. A chamber and one such length of the inner tube is called a section. Four such sections were found to transmit 90 per cent. of the sound from zero to approximately 3,200 d.v. where the attenuation became very high, resuiting in zero transmission up to about 4,600 d.v. where transmission aga!n appeared, Other similar filters of different dimensions attenuated through wider or narrower ranges. The lower limit of attenuation was found to correspond within 8 per cent. with the formula: $f = (1/\pi)(M_1C_2 + 4M_2C_2)^{-1/2}$. The upper limit was not predicted theoretically. High-frequency pass filters were made with a straight tube for transmission and short side tubes, for example, 0.5 cm. long and 0.28 cm. in diameter, opening through a hole with conductivity 0.08 into a tube 10 cm. long and 1 cm. in diameter. Six sections of such a filter would transmit about 90 per cent. of sounds above 800 but would refuse transmission to sounds of lower frequency. As would be expected, the cut off is not sharp. Filters with other dimensions were found to have an upper limit of attenuation varying from 450 to 2,300 d.v., agreeing with the formula $f = (1/2\pi)(1/4M_2C_1 + 1/M_1C_2)^{1/2}$, within about 13 per cent., on the average. The single-band filters made were a combination of the other two types, having side tubes leading to chambers of considerable volume. For instance, three sections each 5 cm. long and 0.5 cm. in diameter, with side tubes of the same size and 2.2 cm. long leading to a volume of 28 cm.^{2}, transmitted between 270 and 370 d.v. The frequencies of the edges of the band of small attenuation are determined by the following formulae,

$$2\pi f = [M_2C_2(1 + M_2'/M_2)]^{-\frac{1}{2}} \text{ and}$$

$$2\pi f = [M_2C_2(1 + 4M_2/M_1)^{-1}(1 + M_2'/M_2 + 4M_2'/M_1)]^{-\frac{1}{2}}.$$ Such filters

exhibit the same variations from theoretlcai performance as would be expected from a combination of the other two types. However, the agreement of each type with theory is sufficient to enable filters to be designed to fulfill set conditions. The *attenuation secured* with only four sections is very great, the transmission being certainly less than 10^{-7} in the attenuated region, while it may rise to 90 per cent. in unattenuated regions. *Possible applications* of these simple filters in laboratory work and in connection with specking devices, are briefly suggested.

I. General Introduction.

THE selective transmission of an acoustic wave of a given frequency is well known. A Helmholtz resonator with a small ear opening is such a filter. Cylindrical tubes such as shown in Fig. 1 will also serve as filters, (*a*) transmitting chiefly the resonating frequencies of *cd* and (*b*) giving poor transmission especially for the resonating frequencies of *cd*. The acoustic wave filters which this article describes

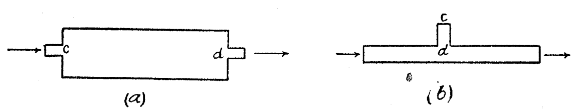

Fig. 1.

are different in principle and in performance in that they do not depend upon resonance itself, but upon the interaction between recurrent similar sections of a transmission "line," these sections containing the elements upon which free vibrations are to depend, and having over-all dimensions that are small in comparison with a wave-length of the sound.

These new filters are remarkable in that selected groups of frequencies, extending over a large range, can be eliminated in the transmission. Up to the present time, three kinds have been constructed and tested. The low frequency pass filter will give approximately zero transmission at all frequencies above, and a fairly good transmission below a certain predetermined frequency. The high-frequency filter will transmit above a minimum frequency. The single-band filter will transmit a group of frequencies. In all cases, the frequency limits are ascertained approximately by calculation from the dimensions so that the filters may be designed to fit specifications. In these filters, the cutoffs are not sharp and the performances are not exactly as just stated, but, as will be shown, the explanation is found in the fact that the experimental conditions only approximate the theoretical.

II. Theory.

A. General.

An exact theoretical discussion of the acoustic wave filter may be possible, but certainly there is much to be gained in securing a theory which, though only approximate, will aid in understanding the operation of the filters and in developing new designs. The well-known electric wave filters suggested to the writer the possibility of analogous effects in acoustics. Naturally, the theory[1] of the electric filters has been of much assistance.

Certain limitations will be imposed in our discussion. We will assume that the length of any selected section of an acoustic "line" or conductor is so small in comparison with a wave-length that no change in phase occurs therein. We will consider only sinusoidal waves. The term "acoustic impedance" will be used. Its absolute value is the ratio of the maximum pressure difference applied to the maximum rate of change of volume displacement. When complex notation is used for simplicity, as in alternating electric current theory, the acoustic impedance is the complex ratio of the pressure difference applied and the rate of change of volume displacement. The mathematical procedure will be based upon three hypotheses which are obviously reasonable approximations. They are: (1) the rate of volume displacement in any selected portion of the line, with a harmonically varying applied difference of pressure, can always be expressed by $Ie^{i\omega t}$ where I is complex; (2) the product of acoustic impedance and rate of volume displacement in any selected portion of the line equals the difference of pressure applied; (3) the algebraic sum of the volume displacements at any junction of lines is zero. By acoustic "line" is meant a bounded region of fluid or solid, forming a tube or channel and capable of transmitting sound waves in the direction of the tube or channel only.

We will consider an acoustic wave filter consisting of equal acoustic impedances in series, divided into sections by acoustic impedances in what might be termed shunt branches. In Fig. 2, let a sound wave of a

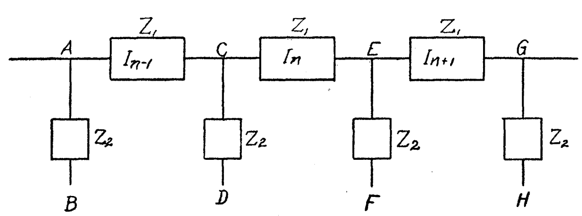

Fig. 2.

frequency $\omega/2\pi$ be transmitted through the line AG, a portion of an infinite line, containing a series of equal impedances, Z_1. Let each branch line AB, CD, EF, GH, etc., contain an impedance Z_2 and terminate in a

[1] U. S. Patent 1,227,113 by George A. Campbell. Chapter XVI. of Pierce's "Electrical Oscillations and Electric Waves," 1920.

volume of gas otherwise at rest, so that there is a common constant pressure at or near these termini. Let $I_n e^{i\omega t}$, etc., represent the rates of change of volume displacement in the corresponding lines, I_n being a complex quantity, and let the positive direction be from A to G. From the three conditions stated in the foregoing paragraph the following equation may be secured:

$$Z_2(I_{n-1} - I_n) = Z_2(I_n - I_{n+1}) + Z_1 I_n$$

or

$$I_{n+1} - \left(2 + \frac{Z_1}{Z_2}\right) I_n + I_{n-1} = 0. \tag{1}$$

This equation does not require an infinite line, but we will for convenience impose that condition. We may then write, using ΔP as the complex pressure difference over a branch,

$$\Delta P_{CD} = I_n(Z_1 + Z_\infty)$$

and

$$\Delta P_{EF} = I_{n+1}(Z_1 + Z_\infty),$$

wherein Z_∞ is the impedance of the infinite network to the right in the figure and has the same value in both equations.[1] Substituting the values,

$$\Delta P_{CD} = Z_2(I_{n-1} - I_n),$$
$$\Delta P_{EF} = Z_2(I_n - I_{n+1})$$

and dividing we have,

$$\frac{I_{n+1}}{I_n} = \frac{I_n - I_{n+1}}{I_{n-1} - I_n}$$

or

$$\frac{I_{n+1}}{I_n} = \frac{I_n}{I_{n-1}}. \tag{2}$$

Let us call the ratio of these successive I's, e^Y, where Y is unknown but in general is complex. Substitute in (1), and we have,

$$e^Y + e^{Y-} = 2 + \frac{Z_1}{Z_2}$$

or

$$\cosh Y = 1 + \tfrac{1}{2}\frac{Z_1}{Z_2}. \tag{3}$$

[1] The filter through its branches terminates in the undisturbed medium. The pressure at E can be expressed by $I_n Z_\infty$. From the equation of motion of a portion of a vibrating medium having a sinusoidal impressed force, and possessing mass, stiffness and dissipation, it can readily be shown that the impedance Z is a function of mass, stiffness, the dissipative factor and frequency only. In as much as these factors are the same for our Z_∞, whether taken from E or from G or any junction point, the assumption of identity of the Z_∞'s is justified.

If Y is a pure imaginary the rates of volume displacement in two adjacent sections differ only in phase. If Y is not a pure imaginary the rate of volume displacement is diminished in transmission, obviously the diminution occurring away from the input end. If Y is a pure imaginary, since $\cosh ix = \cos x$

$$+1 > \left(1 + \tfrac{1}{2}\frac{Z_1}{Z_2}\right) > -1 \tag{4}$$

and hence the limiting values of no attenuation are determined by the following:

$$\frac{Z_1}{Z_2} = 0, \tag{5}$$

$$\frac{Z_1}{Z_2} = -4. \tag{6}$$

We can find the limiting values of no attenuation in a filter by utilizing the actual values of Z_1 and Z_2. It would therefore appear that an acoustic wave filter can be constructed, the only uncertainty being the manner of constructing Z_1 and Z_2. In order to determine upon the practical development of the filter, idealized conditions will first be discussed.

B. Inertance and Capacitance in Parallel.

Consider two idealized diaphragms, a and b, supposed to move as a whole, the former having mass, m_a, and not stiffness, and the latter stiffness, f_b, and not mass. Let them have areas S_a and S_b and displacements ξ_a and ξ_b and let them be connected in parallel as branches of an acoustic line so that they are subject to the same fluid pressure differences, $Pe^{i\omega t}$. Then,

$$m_a \frac{d^2\xi}{dt^2} = S_a P e^{i\omega t} \qquad \text{and} \qquad f_b \xi_b = S_b P e^{i\omega t},$$

the latter being merely the definition of f_b.

If we are concerned with the total volume displacement of the gas at the diaphragms, X_a and X_b, these equations become:

$$\frac{m_a}{S_a^2}\frac{d^2X_a}{dt^2} = Pe^{i\omega t} \qquad \text{and} \qquad \frac{f_b X_b}{S_b^2} = Pe^{i\omega t}. \tag{7}$$

We wish to obtain an expression for dX/dt or the rate of change of volume displacement in the main line at the junction of these two branches. By integration we find,

$$\frac{m_a}{S_a^2}\frac{dX_a}{dt} = \frac{P}{i\omega} e^{i\omega t} \tag{8}$$

the constant of integration vanishing since we are dealing with a variation in dX/dt caused by P and vanishing with P. By differentiation we obtain,

$$\frac{f_b \dfrac{dX_b}{dt}}{S_b^2} = i\omega P e^{i\omega t}. \tag{9}$$

But we know that at the junction of the branches the algebraic sum of the rate of volume displacements must be zero, or

$$\frac{dX_a}{dt} + \frac{dX_b}{dt} = \frac{dX}{dt}. \tag{10}$$

Substituting the values of dX_a/dt and dX_b/dt just found, we obtain,

$$\frac{dX}{dt} = Pe^{i\omega t}\left(\frac{1 - MC\omega^2}{iM\omega}\right), \tag{11}$$

wherein

$$M = \frac{m_a}{S_a^2} \qquad \text{and} \qquad \frac{1}{C} = \frac{f_b}{S_b^2}. \tag{12}$$

We will call M the *inertance*, C the *capacitance*, both defined by (12) and (7). We thus have for the impedance of the combination of the two circuits in parallel,

$$Z = \frac{iM\omega}{1 - MC\omega^2}. \tag{13}$$

Let us now assume that our impedances Z_1 and Z_2 are each composed of two such branches in parallel. The branch having mass, m_a, will vanish when $m_a = \infty$ or $M = \infty$ and the branch having stiffness, f_b, will vanish when $f_b = \infty$ or $C = 0$.

If we apply condition (5) to Z_1 and Z_2 as just determined in (13) and we have as a limiting frequency, f_1:

$$\omega^2 = \frac{1}{M_2C_2} \qquad \text{or} \qquad f_1 = \frac{1}{2\pi}\sqrt{\frac{1}{M_2C_2}}. \tag{14}$$

If we apply condition (6), we have:

$$\omega^2 = \frac{M_1 + 4M_2}{M_1M_2(4C_1 + C_2)} \qquad \text{or} \qquad f_2 = \frac{1}{2\pi}\sqrt{\frac{M_1 + 4M_2}{M_1M_2(4C_1 + C_2)}}. \tag{15}$$

In (14) and (15) the subscripts of M and C correspond to those adopted for the Z's. These frequencies f_1 and f_2 are those that limit the range in which there is no attenuation. It is to be observed that the range of no attenuation is one in which $\left(1 + \frac{1}{2}\frac{Z_1}{Z_2}\right)$ can change from $+1$ to -1.

But, from (13), Z_1/Z_2 is a continuous function of ω or $2\pi f$ and hence there is a continuous frequency range wherein there is no attenuation. A filter in which Z_1 and Z_2 each consists of an inertance and capacitance in parallel is thus possible.

C. Inertance and Capacitance in Series.

Consider each Z_1 and Z_2 to be an inertance and capacitance in series. The current must be the same throughout Z_1 or Z_2. Let P_1 be the pressure difference over the inertance, M_1, and P_2, over the capacitance, C_1. Then, from (7) and (8)

$$\frac{P_1}{i\omega M_1} = I = \frac{P_2}{\dfrac{1}{C_1 i\omega}}\,.$$

Hence

$$\frac{P_1 + P_2}{i\omega M_1 + \dfrac{1}{iC_1\omega}} = I$$

or

$$Z_1 = i\left(M_1\omega - \frac{1}{C_1\omega}\right)\,. \tag{16}$$

If we now ascertain the limits of no attenuation, we have from (5),

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{1}{M_1C_1}} \tag{17}$$

and from (6)

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{C_2 + 4C_1}{C_1C_2(M_1 + 4M_2)}}\,. \tag{18}$$

Again we have the possibility of a filter.

A special and yet a common case of inertance and capacity in series is one wherein M_1 is the inertance of an orifice entering a chamber and C_1 is the capacitance of the chamber. The condition for resonance, or $Z = 0$, occurs as shown by (16) when,

$$M_1\omega = \frac{1}{C_1\omega} \qquad \text{or} \qquad f = \frac{1}{2\pi}\sqrt{\frac{1}{M_1C_1}}\,. \tag{19}$$

But in orifices which are short compared with their diameters we cannot use for M_1 the mass divided by the square of the area, for this expression neglects the end effects of the channel. The well-known formula for the vibration of such a system is,[1] neglecting dissipation[2]

[1] Rayleigh, Theory of Sound, Vol. II., p. 195.

[2] Insertion of dissipation seems to lead to difficulties which are avoided by this approximation.

$$\frac{\rho}{c}\ddot{X} + \frac{\rho a^2}{V} X = Pe^{i\omega t} \tag{20}$$

from the solution of which we get,

$$\dot{X} = \frac{Pe^{i\omega t}}{i\rho\left(\frac{\omega}{c} - \frac{a^2}{V\omega}\right)}. \tag{21}$$

In these equations ρ is the density, c is the "conductivity" of the channel, a the velocity of sound and V the volume of the chamber.

It is evident that the new value of Z_1 viz.,

$$Z = i\rho\left(\frac{\omega}{c} - \frac{a^2}{V\omega}\right), \tag{22}$$

differs from (16) only by the apparent substitution of ρ/C for M_1 and $V/\rho a^2$ for C_1.

D. Inertance and Capacitance of a Tube.

In order to assist in constructing a filter, we will now discuss the nature of a column of gas, which we must use as Z_1 in our conducting line.[1]

Assume an acoustic plane simple harmonic wave passing along a tube in the direction of x. Let ξ be the displacement of a particle from its mean position; ρ the density, a the velocity of sound, and p the excess pressure over the mean.

Then

$$\rho\frac{\partial^2\xi}{\partial t^2} = -\frac{\partial p}{\partial x} \tag{23}$$

(an exact equation) and for small vibrations,[2]

$$\frac{\partial^2\xi}{\partial t^2} = a^2\frac{\partial^2\xi}{\partial x^2}. \tag{24}$$

From (23) and (24)

$$\rho a^2\frac{\partial^2\xi}{\partial x^2} = -\frac{\partial p}{\partial x}. \tag{25}$$

The integration of (25), since

$$\frac{\partial\xi}{\partial x} = 0$$

at all times when $p = 0$, gives

$$\rho a^2\frac{\partial\xi}{\partial x} = -p. \tag{26}$$

[1] Similar discussions occur in Drysdale, Jl. of Inst. Elec. Engs., July, 1920, Vol. 58, p. 591, and Kennelly and Kurowaka, Proc. Am. Acad., Feb., 1921, Vol. 56, No. 1, p. 29.

[2] Lamb's Hydrodynamics, 1916, p. 474.

We may write (26) as follows:

$$\rho a^2 \frac{\partial \xi}{\partial x} \frac{\Delta x}{\Delta x} = - p$$

and if we represent the volume in a length Δx by ∇V, we have,

$$\rho a^2 \frac{\Delta V}{V} = - p$$

or $\rho a^2 = - E$ where E is the modulus of elasticity of volume. Since the compression is due to p and not to changes of pressure along the line, the stiffness can be considered as analogous to stiffness in the walls of the tube upon which the pressure difference p acts. Our tube possesses inertance and capacitance as we know, but the above shows that these are not the equivalent of inertance and capacitance connected either in parallel or in series. Indeed, the capacitance can be thought of as between the inside and outside of the tube instead of along the tube.

Inasmuch as we must use tubes in a practical construction it is essential that we consider whether or not a tube can be used for either Z_1, or Z_2, arranged as in the preceding theory. Consideration shows that Z_1 cannot accurately be composed of a tube, for Z_1 is wholly in the line and not between the line and the outside. Hence we must ascertain whether or not such a substitution would be a sufficient approximation. In short, can we consider a tube as having the equivalent of inertance and capacitance connected in parallel or in series, or may we consider it as having inertance only or as capacitance only?

Assume that any tube we may use will be short as compared with a wave-length. Consider the gas to move as a whole. Then the δp is due to inertance. Or consider the gas to be stationary acting as a cushion. Then δp is due to capacitance. In both cases we have neglected the change of phase along the tube. Whichever case is the better approximation will depend upon the service the tube is rendering, or, in other words, will depend upon the adjacent construction or the composition of the filter. It might appear that the tube can be approximated by lumping the capacitance. This cannot be done at the center of the section for the general theory does not permit of a side branch other than Z_2. It cannot be lumped at Z_2 as can be shown by comparison of the resulting theory with experiment. The assumption of M_1 and C_1 in series does not appear reasonable for the total pressure over the section certainly acts on M_1. We are therefore forced to the policy of using M_1 and C_1 in parallel in the line with the understanding that experiment will determine empirically whether M_1 or C_1 or both shall be used in the computations. As will subsequently be shown, experiments thus far seem to demand the introduction of C_1 in only one of the three types of filters. Thus the somewhat arbitrary manner of introducing C_1 in parallel is minimized. It is fair to say that, in this one case, the assumption seems not without a theoretical justification. It is thus observed that at the outset we are driven to an approximation which demands an empirical selection of formulæ.

III. Construction of Filters.

A. General Limitations.

If we are to construct an acoustic filter that will have good transmission we must avoid any changes in the nature of the medium and in the diameter of the transmitting conduit or conductor. If the filter is to have good attenuation, similar conditions would hold for the branch lines. For first experiments, then, a single medium and a transmitting line of constant cross section are chosen. These selections suggest, from the standpoint of convenience, the use of air and the use of a cylindrical tube as the boundary of the transmission line.

The only limitation to our application is that there be a series of like sections with Z_1 in the main line and Z_2 in the side branch. With no limitation placed upon the constitution of Z_1 or of Z_2, an infinite number of designs may be possible. But, as already stated, considerations lead to the selection of a cylindrical tube containing air as the transmission line. The impedance of a short section of such a line cannot be accurately expressed by an equivalent M_1 and C_1 connected either in series or parallel. We will assume M_1 and C_1 in parallel, for thereby we can make $M_1 = \infty$ or $C_1 = 0$ or remove them from consideration without obstructing the transmission of the line. But even with these limitations the number of filters is infinite, for Z_2 can be formed, theoretically, in any manner one chooses.

B. Low Frequency, High Frequency, and Single Band Pass Filters.

In order to determine the construction of low frequency and high frequency pass and single band filters in as simple a manner as possible, an additional provisional limitation will be made, viz., to filters in which Z_2 consists of an equivalent M_2 and C_2 connected either in series or parallel. We shall then have four quantities M_1, M_2, C_1 and C_2. If either M_1 or M_2 is absent, *i.e.*, removed from consideration, its value is infinity. Under similar conditions the value of a capacitance is zero. We have then to ascertain the possible combinations of

$$M_1 = \infty, \qquad M_2 = \infty, \qquad C_1 = 0, \qquad C_2 = 0.$$

Taken singly there are four combinations in pairs, six in triplets, four with one only, and one with none, making a total of fifteen possible combinations. If we remove those combinations which remove the *line* or the branches entirely, we have left nine designated as follows:

1. All present,
2. $M_1 = \infty$,
3. $M_2 = \infty$,
4. $C_1 = 0$,
5. $C_2 = 0$,
6. $M_1 = M_2 = \infty$,
7. $M_1 = \infty, \quad C_2 = 0$,
8. $M_2 = \infty, \quad C_1 = 0$,
9. $C_1 = 0, \quad C_2 = 0$.

By the preceding, our values of acoustical impedances are limited to the following:

In the transmission line,

$$Z_1 = \frac{iM_1\omega}{1 - M_1C_1\omega^2}. \tag{13 bis}$$

In the branch,

$$Z_2 = \frac{iM_2\omega}{1 - M_2C_2\omega^2} \tag{13 tris}$$

or

$$Z_2 = i\left(M_2\omega - \frac{1}{C_2\omega}\right). \tag{16 bis}$$

If the limits of no attenuation are now ascertained by the application of equations (5) and (6) we find the values in Table I. for the nine cases, each having the possibility of Z_2 in parallel or in series.

The explanation of most of the blanks in the fourth and fifth columns is that, assuming M_2 and C_2 to be in series, our original arrangement of Z_2 providing for the same pressure in common at the termini, requires the following:

1. There must always be an M_2 at the junction point, for otherwise there could be no $\dot{X}$ and hence no use of the side branch. Hence M_2 is at the junction point and C_2 is next in the branch.

2. M_2 cannot be infinity, for if infinity, the side branch would not be used at all.

3. C_2 cannot be zero for this would prevent any value of $\dot{X}$ and therefore any use of the side branch. Thus six of the nine cases are eliminated from consideration leaving only cases 1, 2 and 4. Case 1 leads to the possibility of an imaginary frequency and introduces two values of f_2,

TABLE I.

Case.	Parallel.		Series.	
	f_1.	f_2.	f_1.	f_2.
1	$\frac{1}{2\pi}\sqrt{\frac{1}{M_2C_2}}$	$\frac{1}{2\pi}\sqrt{\frac{M_1+4M_2}{M_1M_2(4C_1+C_2)}}$	0	
2	$\frac{1}{2\pi}\sqrt{\frac{1}{M_2C_2}}$	$\frac{1}{2\pi}\sqrt{\frac{1}{M_2(4C_1+C_2)}}$	∞	$\frac{1}{2\pi}\sqrt{\frac{C_2+4C_1}{4M_2C_1C_2}}$
3	0	$\frac{1}{2\pi}\sqrt{\frac{4}{M_1(4C_1+C_2)}}$		
4	$\frac{1}{2\pi}\sqrt{\frac{1}{M_2C_2}}$	$\frac{1}{2\pi}\sqrt{\frac{M_1+4M_2}{M_1M_2C_2}}$	0	$\frac{1}{2\pi}\sqrt{\frac{4}{C_2(M_1+4M_2)}}$
5	∞	$\frac{1}{2\pi}\sqrt{\frac{M_1+4M_2}{4M_1M_2C_2}}$		
6	0	0		
7	∞	$\frac{1}{2\pi}\sqrt{\frac{1}{4M_2C_1}}$		
8	0	$\frac{1}{2\pi}\sqrt{\frac{4}{M_1C_2}}$		
9	∞	∞		

which are not found in experience. Knowing that the assumption of C_1 in parallel with M_1 is arbitrary, we are justified in acknowledging the incorrectness of the assumption in this case and omitting the formulæ from consideration.

These considerations lead to the development of the three following types of filters:

1. Low-frequency pass filters; case 3, parallel; case 4, series; and case 8, parallel.

2. High-frequency pass filters; case 2, series; case 5, parallel; case 7, parallel.

3. Single-band pass filters; case 1, parallel; case 2, parallel; case 4, parallel.

Conditions of construction suggest that an additional formula for the single-band filter be determined. The three cases in which we have a single-band filter as above stated, are those in which we have M_2 and C_2 in parallel. But in actual construction it is impractical to have truly a C_2 without an orifice into a volume and in this orifice we have an M_2' in series with C_2. A simple construction is thus suggested of taking *Case 1* and adding an M_2' as an orifice into C_2. The impedance of the orifice and C_2 will be, according to (16)

$$Z_2' = i\left(M_2'\omega - \frac{1}{C_2\omega}\right).$$

This impedance is in parallel with the inertance M_2, the impedance of which is,

$$Z_2'' = i\omega M_2.$$

As readily follows from the definition of impedance and the fact that the sum of the two currents is the resultant branch current, the combined impedance, Z_2, is determined from the following relation,

$$\frac{1}{Z_2} = \frac{1}{iM_2\omega} + \frac{1}{i\left(M_2'\omega - \frac{1}{C_2\omega}\right)}$$

or

$$Z_2 = i\frac{M_2\omega\left(M_2'\omega - \frac{1}{C_2\omega}\right)}{M_2\omega + M_2'\omega - \frac{1}{C_2\omega}} = i\frac{M_2\omega(M_2'C_2\omega^2 - 1)}{M_2C_2\omega^2 + M_2'C_2\omega^2 - 1}. \tag{27}$$

According to equation (13) if we make C_1 arbitrarily zero for the sake of simplicity, we have

$$Z_1 = iM_1\omega. \tag{28}$$

If we now use the values of (27) and (28) for the conditions (5) and (6) we will have, respectively,

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{1}{C_2(M_2 + M_2')}}, \tag{29}$$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{M_1 + 4M_2}{C_2(M_1M_2 + M_1M_2' + 4M_2M_2')}}. \tag{30}$$

C. Computation of Inertance and Capacity.

By comparison of (23) with (7) we see that the inertance for a straight tube, assumed to move as a whole, is,

$$M = \frac{m}{S^2} = \frac{\rho l S}{S^2} = \rho\frac{l}{S}. \tag{31}$$

In the case of an orifice, M, as already shown, is expressed by ρ/c where c is the conductivity. The expression for c in terms of the radius, R, and length, l, of the channel is[1]

$$c = \frac{\pi R^2}{l + \dfrac{\pi R}{2}}. \tag{32}$$

With l large in comparison with $\pi R/2$, this reduces to S/l and $M = \rho(l/S)$ as already stated.

As shown in the discussion of (22) we should substitute $V/\rho a^2$ for C_1. Therefore our equations for substitution are:

$$M = \frac{\rho}{c} = \frac{\rho\left(l + \dfrac{\pi R}{2}\right)}{\pi R^2} \tag{33}$$

and

$$C = \frac{V}{\rho a^2}. \tag{34}$$

D. Form of the Filters.

(*a*) *Low-Frequency Pass Filter.*—In a preceding section we found at least three formulæ for a low-frequency pass filter. They are as follows:

$$f_1 = 0, \qquad f_2 = \frac{1}{\pi}\sqrt{\frac{1}{M_1(4C_1 + C_2)}} = \frac{1}{\pi}\sqrt{\frac{1}{M_1C_2 + 4M_1C_1}}, \tag{A}$$

requiring that C_2 be the only portion of the branch;

$$f_1 = 0, \qquad f_2 = \frac{1}{\pi}\sqrt{\frac{1}{C_2(M_1 + 4M_2)}} = \frac{1}{\pi}\sqrt{\frac{1}{M_1C_2 + 4M_2C_2}}, \tag{B}$$

requiring that M_2 and C_2 be connected in series and that C_1 be zero;

$$f_1 = 0, \qquad f_2 = \frac{1}{\pi}\sqrt{\frac{1}{M_1C_2}}, \tag{C}$$

requiring that C_2 be the only portion of the branch and that C_1 be zero.

All these formulas are dimensionally correct and there is no conclusive evidence showing that one is superior to another save by actual experiment. Attention may be called, however, to the difficulty of constructing a C_2 without an M_2 in series. With C_2 only the most favorable cross-section of construction would be similar to Fig. 3. But there would be an M_2 and the justification for neglecting it would be raised.

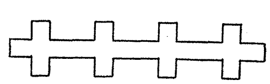

Fig. 3.

[1] Rayleigh, Theory of Sound, Vol. II., p. 181.

If the construction is made as in Fig. 4, the M_2 is better defined and can be determined from equation (33). These considerations point to the use of (B).

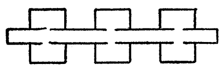

Fig 4.

(*b*) *High-Frequency Pass Filter.* — We previously found three simple cases of a high-frequency pass filter. The formulæ are as follows:

$$f_1 = \infty, \qquad f_2 = \frac{1}{2\pi}\sqrt{\frac{C_2 + 4C_1}{4M_2C_1C_2}} = \frac{1}{2\pi} = \sqrt{\frac{1}{4M_2C_1} + \frac{1}{MC_{22}}} \tag{D}$$

requiring that $M_1 = \infty$;

$$f_1 = \infty, \qquad f_2 = \frac{1}{2\pi}\sqrt{\frac{M_1 + 4M_2}{4M_1M_2C_1}} = \frac{1}{2\pi}\sqrt{\frac{1}{4M_2C_1} + \frac{1}{M_1C_2}} \tag{E}$$

requiring that $C_2 = 0$;

$$f_1 = \infty, \qquad f_2 = \frac{1}{2\pi}\sqrt{\frac{1}{4M_2C_1}}, \tag{F}$$

requiring that $M_1 = \infty$ and $C_2 = 0$. Formula (E) and (F) indicate that it might be possible to construct such a filter with a straight tube and side branches consisting of M_2 only, or consisting of orifices. This plan has been followed in the filters herein described.

(*c*) *Single-Band Filter.*—We previously found four simple single-band filters. The formulæ are as follows:

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{1}{M_2C_2}}, \qquad f_2 = \frac{1}{2\pi}\sqrt{\frac{M_1 + 4M_2}{M_1M_2(4C_1 + C_2)}} = \frac{1}{2\pi}\sqrt{\frac{1}{M_2C_2}\left[\frac{1 + 4\dfrac{M_2}{M_1}}{1 + 4\dfrac{C_1}{C_2}}\right]}, \tag{H}$$

requiring that all of the four elements remain in consideration;

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{1}{M_2C_2}}, \qquad f_2 = \frac{1}{2\pi}\sqrt{\frac{1}{M_2(4C_1 + C_2)}} = \frac{1}{2\pi}\sqrt{\frac{1}{M_2C_2}\left[\frac{1}{1 + 4\dfrac{C_1}{C_2}}\right]}, \tag{I}$$

requiring that $M_1 = \infty$;

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{1}{M_2C_2}}, \qquad f_2 = \frac{1}{2\pi}\sqrt{\frac{M_1 + 4M_2}{4M_1M_2C_2}} = \frac{1}{2\pi}\sqrt{\frac{1}{M_2C_2}\left[\frac{1}{4} + \frac{4M_2}{M_1}\right]}, \tag{J}$$

requiring that $C_1 = 0$. The fourth pair of values requiring that $C_1 = 0$ is found in equations (29) and (30) or

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{1}{M_2C_2}\left[\frac{1}{1+\frac{M_2'}{M_2}}\right]},\quad f_2 = \frac{1}{2\pi}\sqrt{\frac{1}{M_2C_2}\left[\frac{1+4\frac{M_2}{M_1}}{1+\frac{M_2'}{M_2}+\frac{4M_2'}{M_1}}\right]}. \qquad (K)$$

E. Empirical Character of Formulæ.

We are prevented from securing accurate formulæ by the following circumstances:

1. Our dimensions cannot be vanishingly small as compared to a wave-length and therefore appreciable phase differences occur in any section or branch.

2. We cannot get an accurate expression for the impedance of a short tube because our theory assumes that this impedance is strictly in the line, whereas a tube has distributed capacitance virtually between itself and the undisturbed surrounding medium.

3. One hypothesis upon which the theory is based assumes that the junction points are really points, *i.e.*, there can be no accumulation there.

In respect to these circumstances stated, it is fair to say that the accuracy of the approximations cannot be estimated save by experiment. It is allowable, therefore, to take the position that we are to seek, in the case of each construction, that pair of formulæ which seems best to agree with experiment. In other words, the formulæ are somewhat empirical, though their manner of derivation gives a reason for their forms. This attitude of empiricism does not, then, hide the nature of the phenomena but rather decides as to the best approximation.

IV. Comparison of Theory and Experiment.

A. Measurement of Transmission.

An apparatus, Fig. 5, similar to that used by Drs. Gray and Roebuck, but not described in print, gave sufficiently accurate measurements of

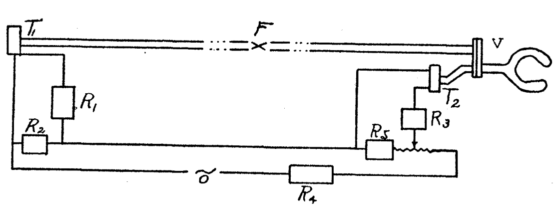

Fig. 5.

the transmission of the filters. T_1 and T_2 are similar telephone receivrse, V is a valve connecting a pair of stethoscope binaurals to either T_2 or the tube which connects to the filter F and T_1, the resistance boxes are marked by the R's, and the source of sinusoidal currents is an oscillator loaned by the Western Electric Company. The circuits of the latter consist of an oscillating circuit, an amplifier, an electrical low-frequency pass filter of two sections, a resistance R_4, and inductances and capacities by means of which it was possible to get fairly pure tones from 90 to 1,500 vibrations and tones up to 5,000 without the use of a filter. R_2 was small in comparison with R_1 and the latter about ten times the impedance of T_1 for a frequency of 1,000 cycles. R_5 was never greater than one tenth the impedance of T_2 and R_3. The brass tubing on either side of F was .557 cm. in internal diameter and 2.5 meters long. The method pursued was to adjust the values of the R's so that, without the acoustic filter present, the values of R_5 required to produce the same intensity at the stethoscope with either acoustic connection, were not in excess of the limit above stated. Each filter had its conducting tube of approximately the same diameter as the tubes from F_1 to T_1 and V. Settings of R_5 were recorded for the various frequencies with and without F, in the latter case a tube of the same length and internal diameter being substituted for F. The transmitted intensities were assumed proportional to the square of R_5. Thus the percentage transmission of F at a certain frequency would be the ratio between the squares of R_5 with F and with its substitute tube. Obviously, the tubes in the apparatus have resonance and this will greatly reduce the accuracy of the transmission values. For frequencies lower than 2,000, pieces of hair felt were inserted every 15 cm. or so to increase the viscosity and reduce resonance. Improved readings were then obtained. The curves herewith presented must be examined with the understanding that resonance does exist. Since our immediate object was merely to get an estimate of the filtering action rather than an accurate measure of transmission, no effort at further refinement was considered. Experiments showed that even with the felt present, variations of 10 per cent. to 20 per cent. apparent transmission with frequency might occur.

B. Low-Frequency Pass Filters.

The general construction of the low-frequency pass filters and the experimental transmission curve is shown in Fig. 6. The accompanying Table II. gives the dimensions, the computations and certain experimental facts concerning a number of low-frequency filters. In this Table l, r, and V refer to the length of section, internal radius and volume respectively. The subscript 1 refers to the conducting line or tube and the subscript 2 to the branches or in the case of the form here used, to the surrounding tube. The value f_m is the value of the frequency at which the transmission is 50 per cent. of the "unattenuated" transmission of the filter. Inasmuch as the beginning of attenuation cannot be sharp, this mean value is perhaps the fairest selection of an experimental value. The computations indicated by the letters, A, B, and C were made according to the formulæ so marked in the preceding theory. These formulæ when expressed in terms of the dimensions become, after

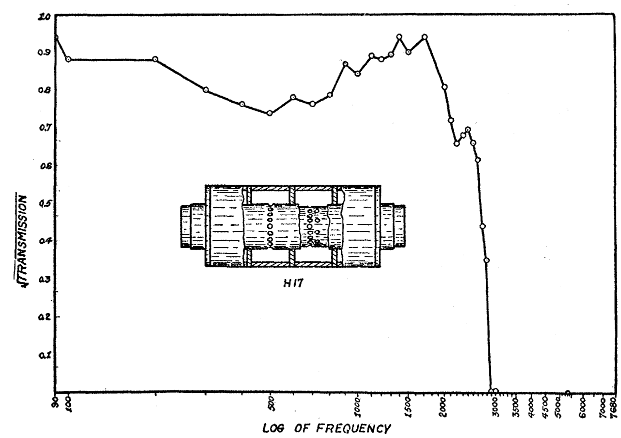

Fig. 6.

substitution in accord with (31), (33) and (34), respectively, (A'), (B') and (C') as follows:

$$f_1 = 0, \qquad f_2 = \frac{a}{\pi}\sqrt{\frac{S_1}{l_1 V_2}\left[\frac{1}{1 + 4\dfrac{l_1 S_1}{V_2}}\right]}, \qquad (A')$$

$$f_1 = 0, \qquad f_2 = \frac{a}{\pi}\sqrt{\frac{S_1}{l_1 V_2}\left[\frac{1}{1 + 4\dfrac{S_1}{c l_1}}\right]}, \qquad (B')$$

$$f_1 = 0, \qquad f_2 = \frac{a}{\pi}\sqrt{\frac{S_1}{l_1 V_2}}\,. \qquad (C')$$

It is evident that the formula (B') is to be preferred since it gives approximately the experimental values of f_m and also explains satisfactorily the variation of the frequency limit of attenuation with the conductivity of the orifices leading from the transmission conduit to the volume in the branch.

Attention should be directed to several other facts shown in the table. The percentage transmission in the "unattenuated" region given in column "T," seems to depend upon the ratio between r_2 and r_1, the

TABLE II.

No.	No. of Sections.	l_1 cm.	r_1 cm.	r_2 cm.	V_{2_3} cm.	f_m d.v.	$f_2(A')$ d.v.	$f_2(B')$ d.v.	$f_2(C')$ d.v.	No. of Apertures.	T %.	*Faf*	
												f	%.
2	3	2.66	.243	1.71	21.6	450	595	462	624	2	.20	5,250	708
3	5	5.0	.243	1.71	43.1	230	308	291	321	4	.36	3,100	933
13	4	4.0	1.42	2.57	44.4	920	1,140	880	2,075	6	.80	3,300	106
15	4	1.67	.75	1.42	5.94	2,200	2,670	2,710	4,620	16	.80	5,150	106
15	4	1.67	.75	1.42	5.94	1,700	2,670	2,120	4,620	8		2,900	38
16	4	1.67	.75	1.15	2.62	3,200	2,965	4,085	6,960	16		6,200	55
16	4	1.67	.75	1.15	2.62	2,500	2,965	3,200	6,960	8		4,000	29
17	4	1.67	.75	1.30	4.36	2,700	2,800	3,175	5,400	16	.65	5,200	66
17	4	1.67	.75	1.30	4.36	2,200	2,800	2,480	5,400	8		3,000	25
*R*1	3	1.58	1.19	1.82	7.1	2,700	3,080	2,840	6,885	8	.90	3,500	9.4
*R*2	4	1.58	1.19	1.82	7.1	2,750	3,080	2,840	6,885	8	.72	4,000	25
*R*3	4	1.32	1.19	1.82	5.87	3,350	3,700	3,170	8,300	8	.72	4,750	18.7
*R*4	4	1.45	1.19	1.82	6.48	3,025	3,370	3,010	7,520	8	.90	4,600	65
*R*5	6	2.58	1.19	1.82	11.82	2,000	1,890	2,095	4,180	8	.90	3,000	25
*R*6	6	1.58	1.19	1.82	7.1	2,700	3,080	2,700	6,885	4	.90	4,750	44
*R*6	6	1.58	1.19	1.82	7.1	2,075	3,080	2,100	6,885	4	.72	3,100	24

number of sections constant, the greater the ratio the less the transmission. In the last two columns labeled "*Faf*," the one under "f" refers to the first audible frequency above the cut-off, and under "%" the ratio between the range of inaudibility and the actual cut-off. Here we note that while filters in which r_2/r_1 is large have inferior transmission, yet they have a surprisingly great range of frequencies above the cut-off where the filter operates successfully. In filters 2 and 3 this range is at least over ten times the cut-off frequency. As the ratio of r_2/r_1 becomes smaller, this cut-off range decreases but the transmission in the "unattenuated" region improves. It is impossible to select, without specifications and trial, the most successful design for a given frequency cut-off, for success depends upon both the range of cut-off and upon the percentage of transmission in the unattenuated region desired. The remarkable feature is that the high attenuation in the region above the theoretical lower limit, f_2, is ever as great as ten times this limit. In fact, in the case of No. 2, that frequency having a wave-length of approximately the length of a section is highly attenuated. The performance is better than one would hope. Attention is directed to *R* 1, a tube about 6 cm. long and 2.4 cm. in diameter yet capable of having an attenuation producing inaudible transmission for a considerable range above the cut-off. Such a performance is a surprise to one accustomed to the difficulty in preventing the passage of sound through holes and channels.

C. *High-Frequency Pass Filters.*

We previously have mentioned three possible simple cases of a high-frequency pass filter but two of these seemed to consist merely of a straight tube with side branches each having an M_2 or orifices, only. The cross-section of such a filter and its transmission is shown in Fig. 7.

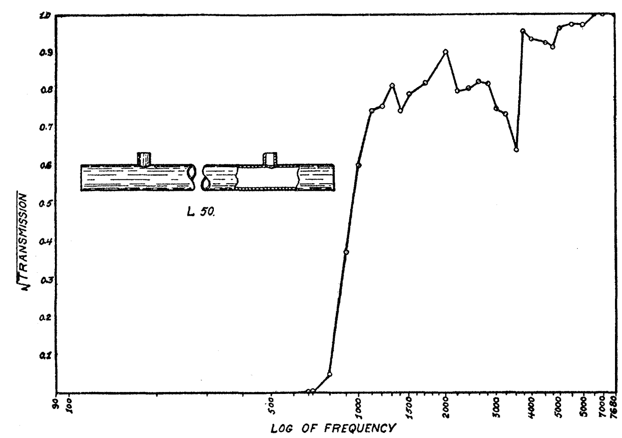

Fig. 7.

If we substitute in the formulæ, (E) and (F) in accord with (33) and (34), we obtain the following:

$$f_1 = \infty, \qquad f_2 = \frac{a}{2\pi}\sqrt{\frac{c}{4V_1}\left(1 + \frac{4S_1}{l_1 c}\right)}, \tag{E'}$$

$$f_1 = \infty, \qquad f_2 = \frac{a}{2\pi}\sqrt{\frac{c}{4V_1}}, \tag{F'}$$

wherein c is the conductivity of the orifice as defined in (32).

Data of several filters of differing dimensions are given in Table III. The last column found in Table II. is here omitted because these filters gave transmission up to the highest frequency tried—5,000–7,000 d.v. The experimental results are therefore strictly in accord with the limits of attenuation set by the theory from f_2 to zero. A reason is doubtless that the less the frequency the more nearly the experimental conditions meet those assumed in the theory.

The general conclusion is that the formula (E') is very satisfactory. It is to be noted that in (E) we have used C_1. This is the only case of the three where experiment has indicated that C_1 should be used. As stated in the discussion of the theory, the assumption of C_1 in parallel with M_1 is arbitrary. A justification may now be seen in the following fact. The branch lines, M_2, are short. Thus there is a much greater

TABLE III.

No.	No. of Sections.	l_1 cm.	r_1 cm.	l_2 cm.	r_2 cm.	c.	f_m.	$f_2(E')$.	$f_2(F')$.	T.
15 II....	8	2.5	.243	0.5	.115	.061	2,300	2,400	988	.75
19......	4	5.0	.243	0.5	.115	.061	1,500	1,300	699	.60
22......	8	2.5	.243	0.5	.152	.099	2,200	2,520	1,262	.50
1,2,3,4 ..	8	4.0	.483	1.22	.278	.146	1,500	1,490	610	.75
50......	12	5.0	.483	0.5	.139	.0845	920	1,170	415	.85
50......	6	10.0	.483	0.5	.139	.0845	810	620	293	.90
50......	4	15.0	.483	0.5	.139	.0845	525	436	240	.90
50......	3	20.0	.483	0.5	.139	.0845	450	344	207	.90

pressure gradient in M_2 than in the conduit or conducting line itself. The particle velocity in M_2 is therefore much greater than in the conduit. If the particle velocity in the line were actually at rest, the conduit would serve as a capacitance for M_2. Thus, the existence of an effective capacitance is not surprising.

Attention should be called to the fact that experiment showed that as M_2 was made longer or very narrow, introducing viscosity, the cut-off became less sharp.

D. Single-Band Filters.

We have already presented four pairs of formulæ for single-band filters. If we now have a filter of the construction shown in Fig. 8 and denote

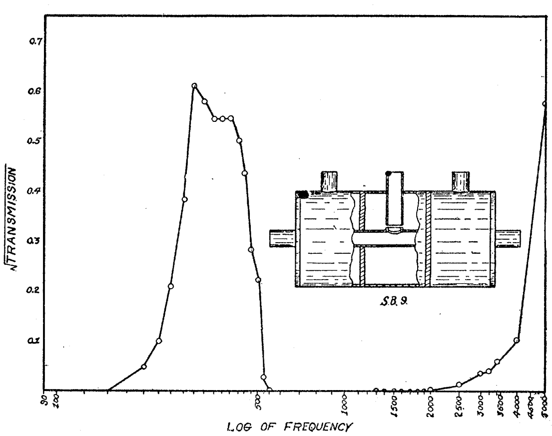

Fig. 8.

by l_2, S_2 the length and area of the side tube and by c the conductivity of the holes from the side branch into the chamber V_2, these four formulæ may be written after substitution according to (33) and (34) as follows:

$$f_1 = \frac{a}{2\pi}\sqrt{\frac{S_2}{2lV_2}}, \qquad f_2 = \frac{a}{2\pi}\sqrt{\frac{S_2}{l_2V_2}\left[\frac{1+4\frac{l_2S_1}{l_1S_2}}{1+4\frac{l_1S_1}{V_2}}\right]}, \qquad (H')$$

$$f_1 = \frac{a}{2\pi}\sqrt{\frac{S_2}{l_2V_2}}, \qquad f_2 = \frac{a}{2\pi}\sqrt{\frac{S_2}{l_2V_2}\left[\frac{1}{1+4\frac{l_1S_1}{V_2}}\right]}, \qquad (I')$$

$$f_1 = \frac{a}{2\pi}\sqrt{\frac{S_2}{l_2V_2}}, \qquad f_2 = \frac{a}{2\pi}\sqrt{\frac{S_2}{l_2V_2}\left(1+4\frac{l_2S_1}{l_1S_2}\right)}, \qquad (J')$$

$$f_1 = \frac{a}{2\pi}\sqrt{\frac{cS_2}{V_2(l_2c+S_2)}}, \qquad f_2 = \frac{a}{2\pi}\sqrt{\frac{S_2}{l_2V_2}\left[\frac{1+4\frac{l_2S_1}{l_1S_2}}{1+\frac{S_2}{l_2c}+\frac{4S_1}{cl_1}}\right]}. \qquad (K')$$

Fig. 8 also shows the transmission curve of one of the filters, *SB*9, with which the range of highly attenuated frequencies above f_2 is relatively small. The two-column "*Faf*" in Table IV. is similar to that in Table II. The reasons for the lack of attenuation in these high frequencies are doubtless the same as in the case of the low-frequency pass filters described in a previous section. For, as will be observed, the single-band filter may be looked upon as a combination of the two other types. The experimental results show the (K') formulæ to be the most satisfactory.

V. Discussion and Conclusions.

The high attenuation secured with but few sections was not anticipated. General experience in acoustics increases the remarkableness of the action of the filters. The agreement with the theory is, in view of the assumptions made, fairly satisfactory for it is possible to construct filters that meet specifications.

The physical action considered herein is clearly not dissipation but interference. With a source transmitting energy through a filter, the action of the latter is to prevent the emission of energy from the source in these frequencies for which there is attenuation. There is dissipation, or at least a decrease in transmission in the filters, extending over the unattenuated region and this seems to be the greater, in the case of the low-frequency-pass filter, the greater the ratio of r_2 and r_1.

The extent of the usefulness of the filter will be determined by experience. Its simplicity and cheapness of construction make it a serviceable

Table IV.

No.	No. of Sections.	l_1 cm.	r_1 cm.	l_2 cm.	r_2 cm.	V_2 cm.^{3}	c	Observed.		Computed.						T %.	Faf	
								f_1	f_2	$f_1(H'I'J')$	$f_2(H')$	$f_2(I')$	$f_2(J')$	$f_1(K')$	$f_2(K')$			%.
1	3	5.0	.243	2.33	.243	42.7	0.69	190	260	236	380	227	400	224	346	.12	3,000	900
2	3	2.66	.243	2.33	.243	21.19	0.69	275	375	304	680	322	710	318	575	.26	3,000	500
3	3	5.0	.243	2.20	.243	28.02	0.69	270	370	300	468	282	499	283	430	.23	2,200	389
4	3	2.66	.243	1.70	.243	14.81	0.69	430	520	470	830	441	882	436	707	.28	3,125	400
5	3	5.0	.243	3.35	.243	28.02	0.69	190	270	242	439	228	466	234	410	.28	2,100	425
6	3	2.66	.243	2.80	.243	14.81	0.69	350	500	366	782	344	835	349	680	.29	3,000	400
7	3	1.17	.243	1.20	.243	5.74	0.69	700	1,300	836	1,875	835	2,030	810	1,380	.54	3,200	129
9	3	2.66	.243	2.40	.243	22.7	.455	285	475	319	648	306	685	295	506	.37	1,300	106
10	4	1.77	1.12	0.5	.278	14.4	.58	600	1,250	1,407	2,255	960	3,590	1,080	1,530	.56	1,800	33

device. In the laboratory it can be used in the elimination of undesirable components in a sound wave, and for certain types of sound analysis. In the latter case its advantage over a resonating device is its selection of a band of adjustable width. Its possible fields of usefulness in practical instruments may be merely mentioned. In the rendition of records on the phonograph and in the wireless telephone many undesirable frequencies can be removed. In wireless telegraphy an acoustic filter makes possible the simultaneous reception of an indefinite number of messages with the same antenna. In connection with a megaphone on a loud-speaking device, the filter has also an application. The introduction of this new filtering phenomenon may in time affect the design of musical instruments. Although the future use of the acoustic filter cannot be foreseen, there is one fundamental fact to be recognized, namely, that an aerial wave is used in audition and that a modification of this wave by a strictly acoustical method wherein the air is the medium, gives an opportunity for the energy to flow directly from the filter to the ear without any transformation which would introduce undesirable modifications. These acoustic filters have a great acoustic interest and the investigation of them, both theoretical and experimental, is being continued.

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