NOTE ON MUSICAL DRUMS.

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SYNOPSIS.

Vibration of Symmetrically Loaded Circular Membranes.—In India there are musical drums which are loaded over a central zone in such a way as to give harmonic partials. The *theory* is presented for two cases, namely when the load varies inversely as the first power and as the second power of the distance from the center. It is found that in the second case the partials form a harmonic series, whereas in the first case they do not.

IN India there are many musical drums like kettledrums which are used in company with stringed instruments. They differ from kettledrums in two respects, (1) they are loaded over a central zone; (2) they elicit harmonic partials.[1] It is well known that the partials of an unloaded circular membrane do not form a harmonic series, and a few of them give a consonant chord. From a look at the Indian instruments it becomes evident at once that the load per unit area varies as we proceed outward from the center. The investigation of the nature of the vibration when the load varies inversely (1) as the first power, and (2) as the second power of the distance from the center, can be effected very simply, but the writer could not find any mention of it in the textbooks.

The equation of motion in the case of circular membrane is given by

$$\rho \frac{d^2\omega}{dt^2} = T_1 \left\{ \frac{d^2\omega}{dr^2} + \frac{1}{r}\frac{d\omega}{dr} + \frac{1}{r^2}\frac{d^2\omega}{d\theta^2} \right\},$$

where the symbols have their usual meaning. Assume $\omega \propto \cos(kct)$ and

$$\rho = \frac{\rho_0}{r}. \tag{1}$$

Put $T_1/\rho_0 = c^2$ and further assume $\omega = v \cos(n\theta)$, then v must satisfy

$$\frac{d^2v}{dr^2} + \frac{1}{r}\frac{dv}{dr} + \left(\frac{k^2}{r} - \frac{n^2}{r^2}\right) v = 0. \tag{2}$$

Put $x = 2k\sqrt{r}$, then (2) becomes

$$\frac{d^2v}{dx^2} + \frac{1}{x}\frac{dv}{dx} + \left(1 - \frac{4n^2}{x^2}\right) v = 0. \tag{3}$$

$$\therefore \quad v = AJ_{2n}(x) = AJ_{2n}(2k\sqrt{r}).$$

[1] Nature, Jan. 15 (1920), p. 500.

If at $r = a$ the membrane is fixed we must have $J_{2n}(2k\sqrt{a}) = 0$ giving real values of k. The nature of these roots is well known, they do not from a harmonic series. The effect of the variation in density is mainly upon the absolute pitch of the vibrating membrane.

$$\text{Case II} \quad \rho = \frac{\rho_0}{r^2}. \tag{4}$$

Let us consider the simple case of symmetrical vibrations only, so that (2) becomes

$$\frac{d^2v}{dr^2} + \frac{1}{r}\frac{dv}{dr} + \frac{k^2}{r^2}v = 0. \tag{5}$$

$$\therefore \quad v = A \cos (k \log r). \tag{6}$$

The boundary condition gives

$$\cos (k \log a) = 0.$$

$$\therefore \quad k \log a = (2n + 1)\frac{\pi}{2}. \tag{7}$$

The partials now form a series of odd harmonics, and the absolute pitch is also disturbed.

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