AN EXPERIMENTAL INVESTIGATION OF THE DISPERSION OF A LIMITED WAVE TRAIN.

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SYNOPSIS.

Dispersion by a Prism of a Limited Train of Hertzian Waves, 7.8 and 9 cm. Long.—According to the theory developed by Sommerfeld, Brillouin, and Colby, some energy should be propagated through the prism undeviated, the proportion decreasing as the angle of incidence is increased. This conclusion has now been experimentally confirmed. To insure a large number of short wave trains, a pliotron circuit adjusted to a frequency of 170,000 cycles was used to energize a modified Righi vibrator consisting of two steel balls suitably mounted close together. The waves so generated were sent as a parallel beam through an opening in a wall to a 30° pitch prism, and the energy as a function of the angle of deviation was determined by means of a molybdenite crystal detector, in series with a sensitive galvanometer, placed at the focus of a parabolic mirror. Various possible sources of error were eliminated or corrected for. The undeviated energy was found to decrease from 26 per cent. for 0° incidence to 8 per cent. for 35° incidence, in general agreement with the theory.

Index of refraction of pitch for Hertzian waves, 7.8 cm. long, was found to be 1.99.

INTRODUCTION.

IN the neighborhood of an absorption band the index of refraction of a dispersing medium may be less than unity and consequently the velocity with which a given wave crest in a monochromatic wave train is propagated may exceed its velocity in vacuum. This is true only after the medium has reached a steady state, *i.e.*, theoretically, for an infinite wave train. From this one might conclude that a signal could be sent with a velocity greater than the velocity of light in vacuum in contradiction to the principle of relativity. A signal, however, must differ from an infinite monochromatic wave train in that it has some peculiarity of form, a beginning or an end, by means of which it can be identified. Sommerfeld[1] was the first to attack this problem theoretically, taking as his signal the limited sine wave

$$f(t) = \sin\frac{2\pi t}{P}, \qquad \text{for } 0 < t < T,$$
$$f(t) = 0, \qquad \text{for } t < 0 \text{ and } t > T,$$

incident perpendicularly on the dispersing medium. He expresses the signal in the form of a Fourier integral which represents the sum of an infinite number of infinite wave trains. Having done this he applies

[1] Sommerfeld, Ann. d. Phys., 44, p. 177, 1914.

the ordinary dispersion theory and obtains an integral representing the disturbance, a distance x in the dispersing medium, as a function $f(t, x)$ of t and x, which he evaluates by Cauchy's method of integration in the complex plane. The result shows that in all cases, including the region of anomalous dispersion, the front of the wave train arrives at a point x in the medium in a time $t = x/c$ and consequently travels with vacuum velocity. The amount of energy which arrives at a given point with this velocity is very small and consists of waves which are very short compared with the incident radiation.

The work of Sommerfeld was extended by Brillouin,[1] who treated the complex integration of the integral $f(t, x)$ more exhaustively. He defines the arrival of the signal as the moment when the amplitude of the disturbance reaches half its final value. Colby[2] investigated the case of oblique incidence, both with reference to form and direction of propagation of the first part of the disturbance. He showed that in this case also the wave front is propagated with vacuum velocity and is undeviated at the boundary. As time goes on the disturbance in the medium swings continuously toward the normal until the steady state is reached when it has the direction required by the ordinary law of refraction. The period of the first part of the disturbance is very short and the amplitude very small. Both quantities decrease with increasing obliquity, and increase as time goes on, the latter passing through a maximum and minimum before the steady state is reached.

Consider now a wave train incident on a prism (Fig. 1). According to the theory outlined above, the front of the wave train will pass through the prism undeviated and arrive at the point A on the arc ABC. The succeeding part of the disturbance will arrive at consecutive points the emerging beam after the steady along AB, OB being the direction of state has set in. The disturbance which arrives at points between A and B is a transient effect which precedes the establishment of the steady state. It is the object of this investigation to detect this effect experimentally.

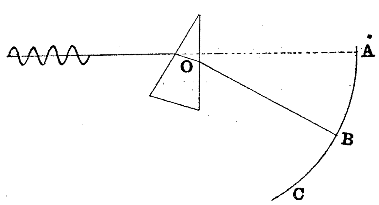

Fig. 1.

Since the effect we are investigating is very small and associated only with the beginning of a wave train, it is essential to use a source of radiation which emits a very large number of short wave trains per

[1] Brillouin, Ann. d. Phys., 44, p. 203, 1914.
[2] Colby, PHYS. REV., V., 3, 1915.

second. The limited sine wave assumed by Sommerfeld cannot be realized experimentally. The most convenient type of radiation to use is the highly damped wave emitted by a Righi vibrator. In this case most of the energy will be in the first period and hence the effect we are looking for will be a larger proportion of the total energy than would be the case if a longer wave train were used.

The Vibrator.

The source of radiation was a slightly modified form of Righi vibrator similar to the type frequently used in recent work[1] with short Hertzian waves. It consisted of two steel balls .95 cm. in diameter resting in sockets ground in a glass holder, or held between strips of wood. For an air gap and electric vector horizontal the holder shown partly in Fig. 2*A* was used. It was made as follows: A piece of glass tubing 35 cm. long and .95 cm. in diameter was bent in the shape of a wide *U*, having vertical sides 8 cm. long. The horizontal portion was then bent into a loop until the vertical sides almost touched. A glass tube sealed to the under side of the loop serves as a support. It is fastened to a horizontal slider which makes it possible to adjust the balls accurately at the focus of the parabolic reflector. The gap between the balls was adjusted by means of a thin piece of glass forced between the two nearly parallel vertical portions of the holder.

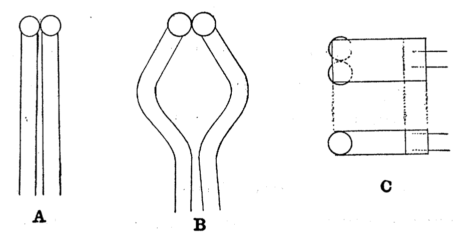

Fig. 2.
Different methods for holding vibrator.

When an oil gap was used the holder was similar to the above except that the vertical parts were bent out immediately below the balls, as shown in Fig. 2*B*. This permitted the oil to flow away freely and reduced to a minimum fouling of the gap due to deposits of carbon from the decomposing oil. To keep the balls firmly seated it was necessary to connect the holder to a filter pump. Paraffine lubricating oil was contained in a reservoir above the reflector whence it flowed through a glass tube and issued from a capillary nozzle a few millimeters above the gap. The spent oil was caught in a receptacle immediately below the reflector, and was used repeatedly, being filtered occasionally to remove the carbon.

[1] Webb and Woodman, Phys. Rev., XXIX., p. 89, 1909; Woodman and Webb, Phys. Rev., XXX., p. 561, 1910; Severinghaus and Nelms, Phys. Rev., I., p. 411, 1913; and others.

Some observations were made with the electric vector vertical. In this case the two balls were held between two thin strips of wood, 1.7 cm. by 2.8 cm., one end of which was glued to a piece .9 cm. square and 1.7 cm. long (Fig. 2*C*). A glass rod waxed to the latter served as a support.

The leads to the vibrator consisted of brass wire 2 mm. in diameter passing through ebonite bushings in the reflector immediately behind the vibrator. The ends were rounded off and a short piece bent over 45°.

The reflector was a sheet of silver-plated copper in the form of a paraboloid of revolution, having an aperture 50.8 cm. in diameter, a depth of 13.8 cm. and a focal length of 11.4 cm. It was mounted on a rigid stand with its axis horizontal and 155 cm. above the floor.

As pointed out above, it is necessary that the vibrator emit a large number of wave trains per second. To satisfy this condition the alternating potential used to excite the vibrator must have as high a frequency as possible. At first a 10-in. induction coil operating on 60 volts and a mercury jet interrupter giving about 900 interruptions per second were employed. The oil gap already described was used with this coil. It was necessary to regulate the current through the gap by means of a large water resistance in series with the secondary of the induction coil. When this was not done the gap readily fouled and the vibrator became very erratic.

The observations taken with the induction coil will be discussed later. They indicated the existence of the effect, but the galvanometer deflections for small deviations were only a few millimeters. It was therefore desirable to replace the induction coil by some device which would give a higher frequency. This was accomplished by means of the pliotron circuit shown in Fig. 3. The essential features of this circuit are a large inductance and a small capacity.[1] The inductances L_1, L_2 were about 5 millihenries each wound in four

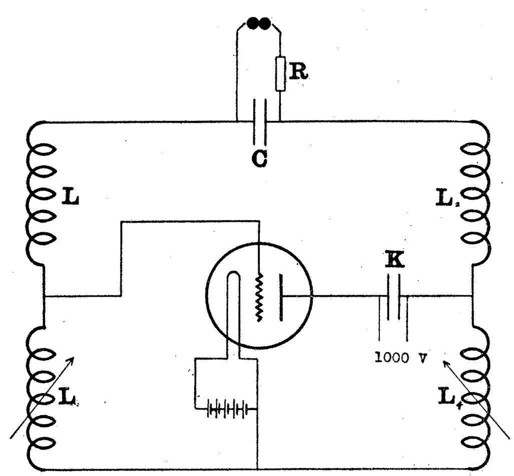

Fig. 3.
Pliotron circuit.

[1] W. C. White, Gen. Elec. Rev., Aug., 1917.

units. Each unit consisted of from 60 to 75 turns of No. 20 wire diameter and 20 cm. long. L_3 and L_4 were of wound on a wooden polygonal frame, the winding being 30 cm. in the variometer type, the settings being usually about .1 and .5 millihenry respectively. The condenser C consisted of two sheets of tin 45 cm. by 50 cm. supported on four glass rods passing through holes near each corner, and separated by pieces of glass tubing 10 cm. long fitting over the glass rods. The water resistance R was necessary to prevent the oscillatory discharge of the condenser C, through the circuit containing the vibrator. Two type P pliotrons were used in parallel. At the frequency employed (170,000) the energy emitted by a vibrator with an oil gap is no greater than with an air gap; since the sparks pass so rapidly that a bubble of vapor is always in the gap. Consequently the air gap described above was used exclusively. In this connection it is interesting to note that Bartenstein[1] and Lindmann[2] used a Tesla transformer to excite a vibrator partially or totally immersed in oil. It is doubtful whether the presence of the oil in this case increased the energy emitted by the vibrator except in so far as it increased the wave-length.

The adjustment of the circuit was somewhat critical. The tendency of the spark gap was to emit a musical note, indicating that the oscillations in the pliotron circuit died down after each spark and required a large number of cycles to be built up sufficiently for another spark to pass. With proper adjustment the spark made a noise resembling that due to a blast of air from a fine nozzle.

The Receiver.

The type of receiver most commonly used for quantitive work with electric waves has been that devised by Klemenčič.[3] It consists of two narrow copper wings joined together by a thermocouple which is connected to a sensitive galvanometer. For maximum sensitivity the galvanometer must be of the low resistance, moving magnet type which is frequently tedious to work with. Sjöström[4] has recently described a crystal detector which he claims is more sensitive than the thermocouple type, besides having the advantage of high resistance. The receiver used in the present work was a modified form of the type developed by Sjöström. A cross section of it is shown in Fig. 4.

Two brass rods b, b', 10 cm. long and .5 cm. in diameter, screw into brass cylinders c, c', which are held in an ebonite cylinder a. The hole

[1] Bartenstein, Ann. d. Phys., 29, p. 201, 1909.
[2] Lindmann, Ann. d. Phys., 38, p. 523, 1912.
[3] Klemenčič, Wied. Ann., 42, p. 416, 1891.
[4] Sjöström, Diss. Upsala, 1917.

in the cylinder *c* ends in a shoulder against which a thin flake of molybdenite *d* is pressed by the rod *b*. The rod *b′* ends in a blunt point which makes contact with the flake. A glass tube *e*, 40 cm. long, serves as a holder. The galvanometer leads are soldered to *c*, *c′*, closely twisted and passed through the glass tube. The main receiver was placed at the focus of a parabolic reflector having the same dimensions as that used with the vibrator and mounted on a stand the same height above the floor.

The resistance of such a receiver is high, of the order of several thousand ohms. This fact makes it possible to obtain high sensitivity using a d'Arsonval galvanometer. Two high sensitivity moving coil galvanometers made by the Leeds and Northrup Company were used. The one connected to the main receiver had a sensitivity of 2.5×10^{-10} ampere and a period of 8.5 seconds, while that connected to the check receiver had a sensitivity of 5×10^{-10} ampere and a period of 6 seconds.

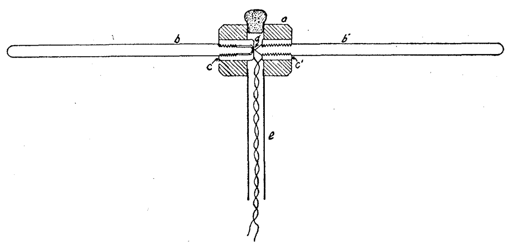

Fig. 4.
The receiver.

The Prism.

The dispersing medium was a pitch prism with a refracting angle of 30°. The faces were 75 cm. high and 85 cm. long. It was cast in a wooden box which was retained throughout the work. The sides of the box were made separately of 1-in. yellow pine boards, being held together by two bolts at each vertical edge.

Arrangement of the Apparatus and Method of Observation.

The final arrangement of the apparatus is shown in Fig. 5. The oscillating system *S* and the check receiver *C* were in one room, while the prism *P*, the main receiver *M* and the galvanometers *G* were in an adjacent room. The radiation passed through an aperture 50 cm. in diameter in the center of a metallic screen *A* about 270 cm. square covering the doorway and adjacent part of the wall. In the doorway, between this screen and the prism was a cylindrical screen *B*, 50 cm. in diameter and 100 cm. long. The object of the screening is to let no radiation enter the room in which the main receiver is located except that which passes through the prism.

The prism was mounted on a turntable which permitted a rotation of 40°, the axis of rotation intersecting the axis of the parabolic mirror at *S*. The mounting was adjusted so the angle of incidence could be varied from 0° to 40°, and the prism was placed as close to the screen *B* as the maximum angle of incidence permitted.

The distribution of energy in the spectrum produced by the prism was measured by moving the main receiver along the arc of a circle *D* having a radius of 200 cm., the axis of the receiving mirror always inter-

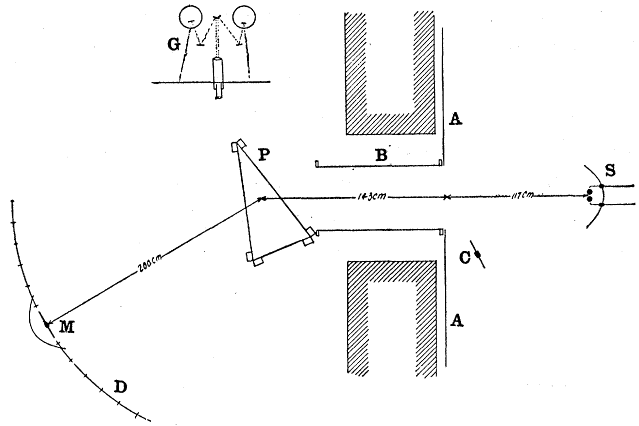

Fig. 5.

secting the axis of rotation of the prism. To take account of the variation of the energy emitted by the vibrator a check receiver *C* was employed. It was located about 100 cm. from the vibrator and just out of the beam of radiation passing through the aperture in the screen *A*.

The two galvanometers were located side by side on a wall bracket (*G*, Fig. 5). The same scale served for both, and by means of a system of mirrors the deflections of both were read simultaneously in the same telescope. The exciting source of the vibrator (induction coil or pliotron circuit) was controlled by strings passing through the wall. Thus all the observations could be made by one observer.

The energy emitted by the vibrator at any given instant was assumed to be proportional to the deflection of the galvanometer connected to *C*. The relative energy received at any point of the circle *D* was therefore proportional to the ratio of the deflection of *M* to the deflection of *C*. Several readings were taken for each setting of *M* and the average value of M/C used. With increasing age of the vibrator the ratio M/C did not always remain constant, but generally decreased more or less irregularly. Consequently, frequent readings were taken with the main receiver in some chosen position on the circle D. If M/C varied the other readings were corrected, *i.e.*, the energy received at the various points along D was expressed in terms of the energy received at the reference position.

The possible sources of error in the present work are the following:

1. Effect of the box surrounding the prism.
2. Radiation penetrating the walls beyond the screen A.
3. Radiation reflected from the walls, floor and ceiling of the room in which the observations were made.
4. The disturbing influence of the induction coil and pliotron circuit.

The boards which formed the faces of the prism were quite uniform and free from knots. No disturbance could be expected from them. The only course of disturbance in the box was scattering by the bolts and screws at the vertical edges. The nearest that any of the bolts approached to the geometrical edge of the beam, measured from the edge of the screen B and in a place containing the axis of B, was 15°. If we substitute $\theta = 15°$, $\lambda = 7.8$ cm., $R = 25$ cm. in the formula

$$\text{Sin } \theta = \frac{m\lambda}{\pi R}$$

for the diffraction pattern, we find $m/\pi = .83$, which corresponds to the second maximum.[1]

Now the intensity of the second maximum is .017 times the intensity of the central maximum. Furthermore, not more than two of the bolts ever approached as close as 15° at one time, and all the screws were farther removed than the bolts. From this it appears that no appreciable error could have been introduced by the box surrounding the prism. To investigate this point further some observations were made with the frame removed. In this condition the prism rapidly lost its shape, but enough readings could be taken to show that the frame caused little trouble compared with the total effect observed.

The disturbance due to radiation penetrating the wall between the two rooms could readily be determined by noting the deflection of M when the aperture in A was closed by a screen. This was made very small by making A sufficiently large.

Although the room in which the observations were made was large (9.5 m. long, 8.5 m. wide, and 4 m. high), there was nevertheless an

[1] Wood's Physics Optics, p. 237.

appreciable disturbance due to reflection from the walls, etc., which could not be eliminated. Its magnitude was determined by making observations with a screen 85 cm. square interposed between the prism and M, near the former and perpendicular to the axis of the receiving mirror. The correction thus obtained included also (2) and to a certain extent (1). It was applied to all the observations made. If the vibrator was erratic, as was frequently the case toward the end of a series of observations, the readings for any given position of M were taken alternately with and without the screen. The average value of M/C with the screen was subtracted from the average value of M/C without the screen and the result taken as the true relative energy for that position of M.

The disturbance due to the exciting source consists of two parts: radiation from the leads to the vibrator and induced currents in the galvanometer leads. To determine whether radiation from the leads caused any trouble the vibrator was removed from its holder and the leads brought together making a spark-gap in air a few millimeters long. If this gap was much less than 3 mm. it was difficult to get rid of radiation. In all the work the side gaps in air between the leads and the vibrator were never less than 3 mm. In the case of the induction coil the radiation effect was eliminated by making the leads short and enclosing them in red fiber tubing. This material is a very poor dielectric at high frequencies and consequently helped to damp out the oscillations in the leads. The radiation effect in the case of the pliotron circuit was due to the oscillatory discharge of the condenser C, Fig. 3. This was readily eliminated by making the water resistance R sufficiently large.

As Sjöström[1] has observed, some receivers seem to be more sensitive to induced currents in the galvanometer leads than others. Trouble due to this cause was entirely confined to the check receiver when the pliotron circuit was used. By properly choosing the receiver and closely twisting the galvanometer leads it could be eliminated or at worst made very small.

After the apparatus was finally adjusted no disturbance due to the exciting source could be detected in the main receiver, while the effect on the check receiver, if it existed at all, was small and fairly constant. Since we are only interested in the relative values the inaccuracy introduced by a small constant error in the deflections of the check galvanometer is negligible.

RESULTS.

The energy distribution under various conditions is shown graphically in Figs. 6 and 7. The abscissæ are deviations in degrees measured from

[1] Loc. cit., p. 26.

the direction of the incident beam. The ordinates are energy expressed in per cent. of the maximum. With this system of coördinates the various curves are qualitatively comparable. The curves in Fig. 6 were taken with a vibrator consisting of .95 cm. spheres with an oil gap, excited by an induction coil. They confirm the theory qualitatively in that they show that some energy passes through the prism undeviated and that this effect decreases with increasing angle of incidence.

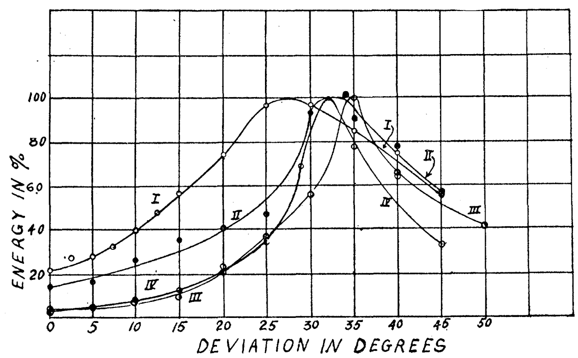

Fig. 6.

Spectral distribution of energy emitted by .95 cm. vibrator with oil gap.

I. Angle of incidence 0°
II. " " " 15°
III. " " " 24°
IV. " " " 32°

From curve IV., Fig. 6, it is seen that minimum deviation for the maximum in the energy curve occurs at 32° from which the index of refraction of pitch is found to be 1.99. This is much larger than the value 1.67 found by Hertz for waves about 60 cm. long.

Curves I. and II., Fig. 7, were obtained with a .95 cm. vibrator in air with the electric vector perpendicular to the refracting edge of the prism. They are much flatter than the corresponding ones for the oil gap and induction coil. This is due to the fact that the vibrator in air emits a more highly damped wave than in oil, so that a larger percentage of the total energy passes through the prism before the steady state is established. Figure 8 shows the variation of the energy which passes through the prism undeviated, with the angle of incidence. In this case the main receiver was kept fixed in the position of no deviation while the prism was rotated. The deflection of the main galvanometer for 0° incidence was 8.2 cm. and with the prism removed, 30.5 cm. With a screen interposed in the manner described the deflection was .2 cm.

Curve III., Fig. 7, shows the energy distribution for 32° incidence with a .95 cm. vibrator arranged so that the electric vector was parallel to the refracting edge. It differs from II. in that it has a more distinct maximum. The undeviated energy was less than 2 per cent. of the incident energy as compared with 10 per cent. when the electric vector was perpendicular to the refracting edge. This is partly due to the difference in orientation of the receiver in the two cases. With the electric vector horizontal, *i.e.*, perpendicular to the refracting edge, the receiver was also horizontal and subtended an angle of about 6° at the axis of rotation of the prism. It was 20 cm. long and consequently energy brought to a focus a distance of 10 cm. on either side of the geometrical focus of the mirror was caught by the receiver. When the receiver was vertical it caught little energy except that which was brought

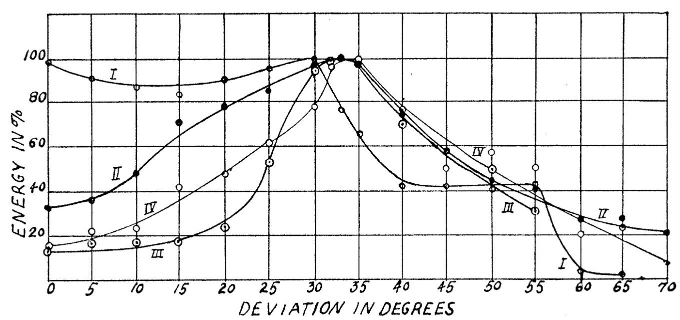

Fig. 7.

Distribution of energy emitted by vibrator with air gap.

I. Angle of incidence 0°, vibrator consisting of .95 cm. spheres.

II. Angle of incidence 32°, vibrator same as I.

III. Angle of incidence 32°, vibrator consisting of two .95 cm. spheres mounted with axis parallel to the refracting edge of the prism.

IV. Angle of incidence 32°, vibrator consisting of two 1.52 cm. spheres mounted with axis perpendicular to refracting edge of prism.

to a focus at the geometrical focus of the mirror. We should also expect a difference in the two cases in the direction observed since in the steady state the component of the electric vector in the plane of incidence is more readily transmitted than that perpendicular to the plane of incidence.

Curve IV., Fig. 7, shows the distribution of energy for 32° incidence using a 1.52 cm. vibrator in air mounted horizontally. The general shape of the curve is the same as in the corresponding case with the smaller vibrator. The deflections were very erratic beginning at 45° which accounts for the irregularities at 45° and 60°.

A few observations were made with a .61 cm. vibrator in air but the deflections were too small to be of much value. It does not seem feasible to use vibrators as small or smaller than this without some means of increasing the energy emitted. This might be accomplished by enclosing the vibrator in a glass bulb either exhausted to a very low pressure or filled with some gas at a pressure of several atmospheres. In the latter case the decreased mobility of the ions would tend to set an upper limit to the useful frequency of the exciting source.

Wave-length Measurements.

The wave-length of the radiation emitted by the vibrator was measured by the Boltzmann mirror interference method first used by Klemenčič and Czermak.[1] In this method the energy from the vibrator arrives at the receiver after reflection from a mirror which is divided into two halves, so that the line of separation is in the plane of incidence. By separating the two mirrors thus formed, interference between the two halves of the beam takes place at the receiver. Each mirror consisted of a sheet of plate glass 35 cm. by 70 cm. covered with tinfoil, one was fixed and the other mounted on the carriage of a dividing engine. The wave-length is

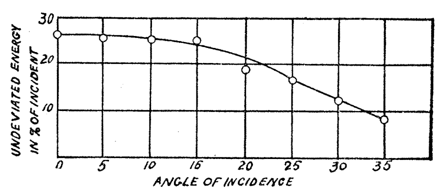

Fig. 8.

the distance between two successive maxima or two successive minima. For the .95 cm. spheres in air the wave-length was 7.8 cm. and for the 1.52 cm. spheres in air about 9 cm. In the latter case the maxima and minima of the interference curve were so flat that it was difficult to determine the wave-length accurately. These values indicate pronounced "mirror-action."[2]

The nature of this phenomenon is not very well understood. The size and shape of the collecting mirrors somehow influence the wave-length of the radiation. When the focal length is comparable to the wave-length the emitted beam has a more or less definite wave-length which is determined by the geometry of the mirror and not by the size of the vibrator. For the present purpose it is not essential that the vibrator emit its characteristic wave-length; all that it is desirable to know is what it actually does emit.

The theory indicates that the wave-length of the undeviated energy is very short and the intensity very weak compared with the incident

[1] Weid. Ann., 50, p. 177, 1893.

[2] Webb and Woodman, loc. cit., p. 100. These investigators found 6.7 cm. for the characteristic wave-length of a .95 cm. vibrator with an oil gap.

radiation. From this it appears that the effect observed is not due to strictly undeviated energy—*i.e.*, the disturbance called the "first forerunners" by Brillouin, but to energy that has undergone slight deviation. This view is strengthened by a consideration of the finite size of the receiver. An attempt was made to investigate the wave-length of the energy received in the neighborhood of zero deviation by means of the interferometer. No evidence of interference could be obtained. This was probably due to the fact that the energy reflected from the interferometer was not sufficiently monochromatic, since the wave-length depends upon the thickness of the medium traversed and consequently varies continuously from the refracting edge to the base.

The author takes pleasure in expressing his thanks to Dr. E. F. Nichols, who suggested the problem and method of solution, and to Professor L. Page, for advice concerning questions of theory. His thanks are also due Mr. G. C. Southworth, for advice on the construction and manipulation of the pliotron circuit.

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