

THE SCATTERING OF X-RAYS BY CRYSTALS.

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SYNOPSIS.

Scattering of X-rays by Crystals (Rocksalt and Calcite), Metal (Aluminum) and Amorphous Solid (Glass).—Both homogeneous rays (K_{α} of Mo obtained by preliminary reflection from a rocksalt crystal) and heterogeneous rays direct from a Mo or W target (70 to 110 kv.) were used, and *scattering curves* were obtained by measuring the ionization for various angles of scattering. On account of the small intensity of the scattered radiation, incident beams of $15'$ to 3° width and a scattered beam of 6° width were required even with a sensitive Compton electrometer. For angles from 50° to 150° the curves obtained for the crystals are of the same shape as those for Al and glass, each curve showing a minimum at about 100° ; but for angles below 50° the crystal curves fall below the others and each shows a maximum at a larger angle, between 15° and 30° , depending on the wave-length. In addition to the general scattering at all angles, there appear, for the crystalline substances, maxima corresponding to the Laue spots. When correction for the absorption of the crystal is made, for which a formula is given, the scattering in a particular direction is found not to depend on the orientation of the crystal. This indicates that the atoms are isotropic. As was to be expected the scattered radiation was softer than the primary. For the width of x-ray beams used the total energy scattered in all directions was an appreciable fraction (0.70 for homogeneous Mo K_{α} rays on calcite) of the energy reflected in a first order spectrum line. *Comparison with the Debye theory* shows fair agreement only in the case of rocksalt for angles greater than 60° . For calcite there is no agreement and the prediction that the scattering by amorphous substances should be quite different from that by crystals is not verified. Applying the Thomson formula, however, the number of electrons per molecule for each crystal has been calculated from the relative amount of energy scattered at 90° and comes out of the right order of magnitude.

IN his paper on the "Interference of Roentgen Rays and Heat Motion," Debye¹ states that the regular reflection of x-rays from a crystal surface is accompanied by diffusely scattered radiation which is the most intense at those angles at which the intensity of the regularly reflected radiation is least. He finds that, if the crystal is set at such angles as to reflect a particular x-ray spectrum line in different orders, and if the ionization chamber is set to receive the reflected beam in each order, the heat motions of the atoms should cause the intensity of the different orders to fall off as the number of the order increases. On the other hand, this thermal agitation causes the intensity of the diffusely scattered radiation to increase as the angle with the primary beam of x-rays increases. In another paper,² Debye discusses the effect of the

¹ P. Debye, Ann. der Phys., Band 43 (1914), pp. 49-95.

² P. Debye, Ann. der Phys., Band 46 (1915), pp. 809-823.

regular arrangement of the electrons in the atoms upon the x-ray scattering when the atoms themselves have the random arrangement characteristic of amorphous substances. Comparing these two papers, it appears that the general scattering by crystals should be quite different from that by amorphous substances. The present experimental investigation was undertaken primarily to see if the difference predicted by Debye really occurs.

EXPERIMENTAL METHODS.

If the primary x-rays strike a crystal face at a glancing angle θ , the regularly reflected rays occur at an angle 2θ with the primary rays. In the experiments described in this paper the ionization chamber instead of being set at an angle 2θ with the primary rays is set at some other angle ϕ . The intensity of the diffuse scattering in different directions is thus measured, and curves showing the variation of the intensity of the scattered radiation with the angle are obtained. These curves are then compared with similar curves obtained using amorphous substances.

In the first experiments, x-rays from the molybdenum target of a Coolidge tube, after reflection from rocksalt to isolate the K_α line, fell upon the crystal under investigation. The second crystal was now placed at the appropriate angle θ for the regular reflection of the molybdenum K_α line in either the first or second order (see Fig. 1). Keeping the

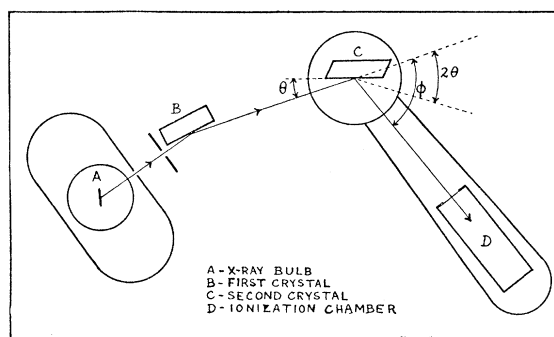


Fig. 1.

crystal in this position, readings were taken with the ionization chamber at various angles ϕ between 8° and 90° . The tube was operated by an x-ray transformer at about 70 kilovolts, and the current, 4 milliamperes, was controlled by a Victor current stabilizer. It was found that with this stabilizer it was not necessary to employ a balance method for measuring the x-rays.

The ionization chamber was filled with a mixture of methyl iodide vapor and air, and the ionization current produced in the ionization chamber measured by the deflection of a Compton electrometer operating at a sensibility of about 2500 millimeters per volt. With this two-crystal method the deflection due to the diffuse scattering of the x-rays from the second crystal was only a few millimeters per minute. As there was some difficulty in this method of eliminating small deflections due to stray scattering, a set of readings was taken with the second crystal in position and another set at the same angles with the second crystal removed. The reading with the second crystal removed was subtracted from the reading with the second crystal in position for each angle and the difference plotted against the angle of the ionization chamber. In this way the effect of stray scattering was eliminated.

In many of the experiments, especially those in which the scattering at large angles was measured, the first crystal of Fig. 1 was removed, and the x-rays fell directly upon the crystal under investigation. Though this single-crystal method employs heterogeneous x-rays, it was necessary in certain cases where the double-crystal method would have given an intensity too small for accurate measurement. With the single-crystal method, x-ray tubes with molybdenum and tungsten targets were used. In the case of the molybdenum tube the crystal, either rocksalt or calcite, was placed at an angle to reflect the molybdenum K_{α} line in the first order. With the tungsten tube the crystal was set to reflect only the general radiation, but was operated at different potentials, 70, 95 and 110 kilovolts.

The aluminum window of the ionization chamber which received the scattered x-rays from the crystal under investigation was 3 cm. high and 1.5 cm. wide. The distance of this window from the axis of the spectrometer was 14.3 cm., so that the angle subtended by the window at the face of the scattering crystal was 6° . Using the two-crystal method, when both crystals were set to reflect the molybdenum K_{α} line and the ionization chamber was placed so as to receive the regularly reflected beam from the second crystal, it was found that by rotating the second crystal through about $30'$ on either side of the maximum the intensity of the reflected line fell to half value, so that the angular width of the beam incident on the second crystal can be taken at approximately 1° . With the single crystal method, beams of angular widths 3° , $45'$ and $15'$ were used.

EXPERIMENTAL RESULTS.

Double-Crystal Method.—The scattering curves obtained with rocksalt as the second crystal when set to reflect the molybdenum K_{α} line in

the first and second orders are shown as Curves *a* and *b* respectively of Fig. 2. Curve *c* is a similar curve for calcite when set to reflect the line

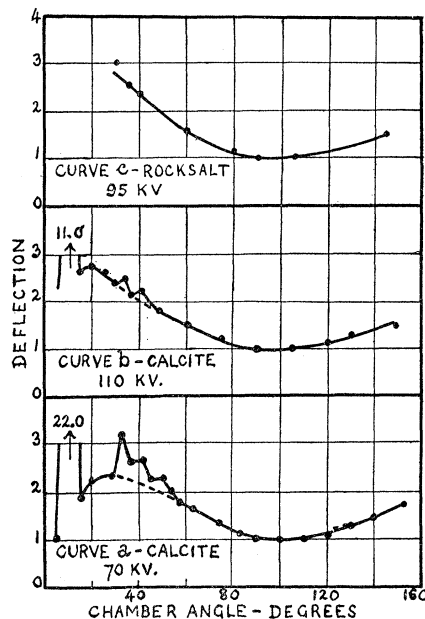


Fig. 2.

in the first order. Supplementary experiments showed that the beam incident upon the second crystal contained no appreciable x-rays of wave-length differing considerably from the K_{α} line. It will be noted that, even with the wide slits which were used, the intensity of the diffusely scattered radiation is only a fraction of 1 per cent. as great as the intensity of the first order reflection. In the case of calcite there is distinct evidence of a line reflected into the ionization chamber at 79° , though the crystal face is at a glancing angle of $6^{\circ} 45'$ with the incident beam.

It seemed possible that the intensity of the x-rays scattered by a crystal in a certain direction ϕ might be altered if the second crystal were turned out of the position in which it regularly reflected the molybdenum K_{α} line. It is known that the absorption coefficient of homogeneous x-rays in a crystal is increased when the rays are incident at the proper angle. Due to this increase of absorption coefficient the x-rays incident on the crystal will not penetrate so far into the crystal, and there should be less scattering. The chamber was accordingly set at an angle of 65° while the rocksalt crystal was set at various angles between $6^{\circ} 20'$ and $11^{\circ} 20'$. Fig. 3 shows the curve obtained. It will be seen that there is

no evidence of a minimum at $7^{\circ} 20'$, the angle at which the K_{α} line is reflected. The explanation of this is probably that the beam incident on the second crystal has a considerable angular width. It is only those rays which strike the crystal at the correct angle whose absorption coefficient is increased. With a wide beam such as used in these experi-

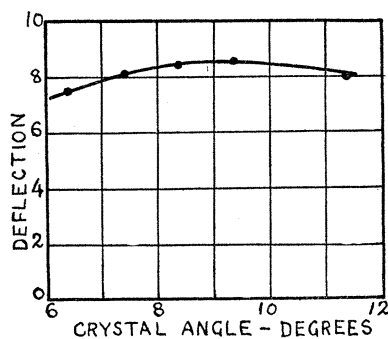


Fig. 3.

ments only a small fraction of the incident beam suffers this extra absorption.

The ratio of the intensity of the x-rays scattered at 90° to the energy of the beam of x-rays incident upon the second crystal is of considerable theoretical importance. The ratio is so small that it was necessary to obtain first the ratio of the intensity of the diffusely scattered x-rays to the energy of the regularly reflected rays when the crystal was set to reflect the molybdenum K_{α} line in the first order. The current through the x-ray tube was then greatly reduced, and the ratio of the deflection due to the regular reflection to that when the incident rays entered directly into the ionization chamber was obtained. The product of these ratios gives the ratio of the intensity of the x-rays scattered at 90° to the energy of the primary rays. The results are shown in the fourth column of Table I. Because of the small deflections given by the scattered x-rays these ratios are only approximate.

Single-Crystal Method.—When the direct radiation from the molybdenum tube fell upon a crystal of rocksalt, Curve *a*, Fig. 4, is obtained. In addition to the general scattering there appears a regularly reflected beam at 51° , though the crystal face makes an angle of $7^{\circ} 20'$ with the primary beam, the angular width of which was $45'$. The corresponding curve for calcite is shown in Curve *b*, Fig. 4. In this case the angular width of the primary beam was 3° . With the calcite crystal there are regularly reflected beams at 28° , 38° and 79° . The line at 79° was previously noticed in Curve *c*, Fig. 2. These curves for rocksalt and

calcite are drawn to have equal ordinates at 90° and are typical of all the curves obtained. There is scattering at all angles, but superimposed

TABLE I.

Intensity of X-Rays Scattered at 90° .

Crystal.	Nature of X-Rays.	Linear Absorption Coefficient in Crystal.	Intensity of Scattered Rays. Energy of Primary Rays.	Apparent Number of Electrons per Molecule.
Rocksalt	Homogeneous Mo- K_α Line	17.5	1.4×10^{-5}	16.3
Calcite	Homogeneous Mo- K_α Line	25.0	2.0×10^{-5}	47.6
Rocksalt	Heterogeneous Tungsten Tube	1.39	4.0×10^{-4}	29.7
Calcite	Heterogeneous Tungsten Tube	0.88	5.3×10^{-4}	34.2
Aluminum Sheet	Heterogeneous Molybdenum Tube	6.7	5.2×10^{-5}	8.5

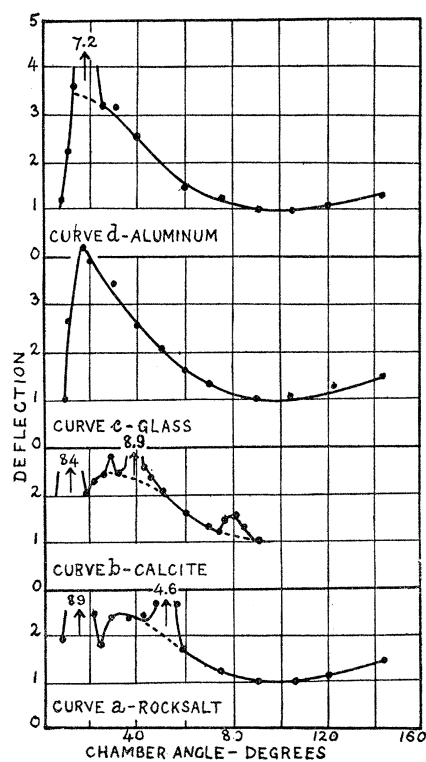


Fig. 4.

on the general scattering there is regular reflection at particular angles when the crystal is set at a certain angle.

An absorption experiment showed that the rays causing the large deflection at 38° with calcite are the molybdenum K_α rays. The humps at both 38° and 79° disappear if the orientation of the crystal is slightly changed, showing that these are undoubtedly regular reflections. There is however no such evidence of regular reflection at angles where no humps appear on the curve.

The Curves *c* and *d* of Fig. 4 were obtained when plates of glass and aluminum respectively were used in place of the crystal, the glancing angle for glass and aluminum being 8° . Otherwise these curves were obtained under the same conditions as the curves for the crystals. In addition to the general scattering, the curve for the aluminum shows evidence of regular reflection at 19° . This is due to the fact that aluminum is not amorphous but an agglomeration of minute crystals, and a powdered crystal effect is obtained.

A variation in the experiment with calcite was made by setting the chamber at an angle of 56° and turning the crystal from 5° to 56° . The curve so obtained is shown in Curve *b*, Fig. 5. Regularly reflected

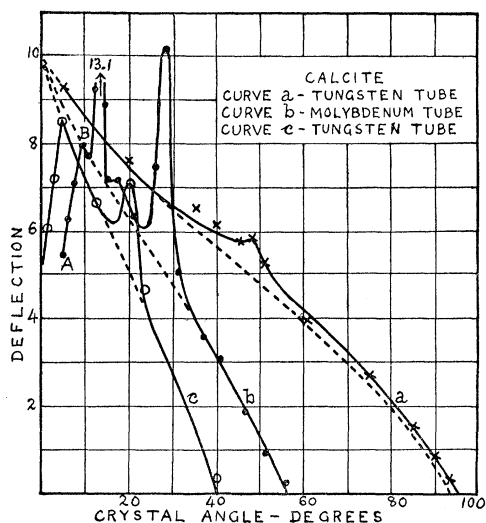


Fig. 5.

beams are obtained when the crystal is at 13° and 28° . The slope of the portion *AB* of the curve is due to the fact that at small angles of the crystal part of the primary x-rays pass by the crystal without striking it.

Using a tungsten tube, Curves *a*, *b* and *c* of Fig. 6 were obtained, the

crystal angle in each case being 5° . These curves are reduced to the same ordinate at 90° . Curve *a* is for calcite with the tube operated at a potential of 70 kilovolts, while the potential for Curve *b* was 110 kilovolts.

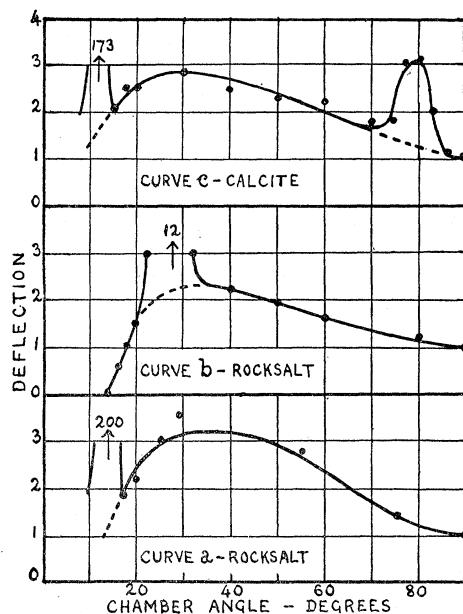


Fig. 6.

Regular reflection appears at 34° and 40° in each case, but with the greater potential the intensity of the general scattering is relatively much stronger. The angular width of the primary beam was $15'$. Curves *a* and *c* of Fig. 5 were obtained by setting the chamber at 94° and 40° respectively and turning the crystal.

The quality of the radiation diffusely scattered by calcite was tested by placing an aluminum absorbing screen alternatively in the path of the primary and of the scattered beam. It was found that the scattered rays were somewhat more readily absorbed than the primary rays which produced them. This result is in accord with the scattering measurements made by others on non-crystalline substances.¹

The ratio of the intensity of the x-rays scattered at 90° to the energy of the primary rays was measured by a method similar to that for the homogeneous x-rays. The results are shown in the fourth column of Table I. The calcite and rocksalt crystals were set at 5° while the aluminum plate was set at 8° .

¹ Cf. Sadler and Mesham, *Phil. Mag.*, Vol. 24 (1912), pp. 138-149; A. H. Compton, *Nature*, Vol. 108 (1921), p. 366; et al.

DISCUSSION OF RESULTS AND CALCULATIONS.

The scattering curves show that crystals scatter x-rays in a manner similar to amorphous substances, but that in addition to this general scattering there are regularly reflected beams in certain directions with the crystal set at a given angle. Not all of these humps are due to spectrum lines, although some of them are. For instance the hump at 38° in Curve *b*, Fig. 4 was shown to be due to the molybdenum K_α line. In other cases, however, reflection is obtained from some plane in the crystal which will reflect a certain wave-length λ of the general radiation falling upon the crystal according to the equation $n\lambda = 2d \sin \theta$.

It might be supposed that the intensity scattered by a crystal molecule in a certain direction ϕ with the primary x-rays might depend upon the orientation of the molecule with respect to the primary rays. In other words, if the ionization chamber is set at an angle ϕ and the crystal turned there might be a difference in the molecular scattering in the direction ϕ as the crystal is turned. Curves for this case have been obtained in Fig. 5. Those curves are of course for the scattered radiation which gets out of the crystal. Let us suppose that the linear scattering coefficient of the crystal for a direction ϕ is s_ϕ irrespective of the orientation of the crystal relative to the primary beam. It can be shown that, if the linear absorption coefficient of the primary x-rays in the crystal is k (the slight difference between k for the primary and for the scattered rays being neglected), the intensity of the scattered x-rays leaving a thick crystal in a direction ϕ is

$$(1) \quad \frac{s_\phi I_0}{k} \cdot \frac{\sin(\phi - \theta)}{\sin(\phi - \theta) + \sin \theta},$$

where I_0 is the energy of the primary x-rays and θ is the glancing angle of the primary rays upon the crystal face. If ϕ is kept constant, formula (1) may be written in the form

$$(2) \quad \text{constant} \times \frac{\sin(\phi - \theta)}{\sin(\phi - \theta) + \sin \theta}.$$

The broken curves in Fig. 5 are obtained by plotting values of (2) against θ for $\phi = 90^\circ$, 56° and 40° respectively, the appropriate value of the constant being taken in each case. It is seen that the experimental curves correspond with the theoretical curves excepting for the humps due to regular reflection. The result implies that the intensity of the rays scattered by a crystal molecule in a given direction ϕ does not depend upon the orientation of the crystal, and that therefore the atoms of the crystal are nearly isotropic.

The scattering curves should be corrected for absorption in the crystal in order to give the true molecular scattering in various directions. If each of the ordinates of each of the experimental scattering curves is divided by formula (2), the true scattering curves shown in Fig. 7 are

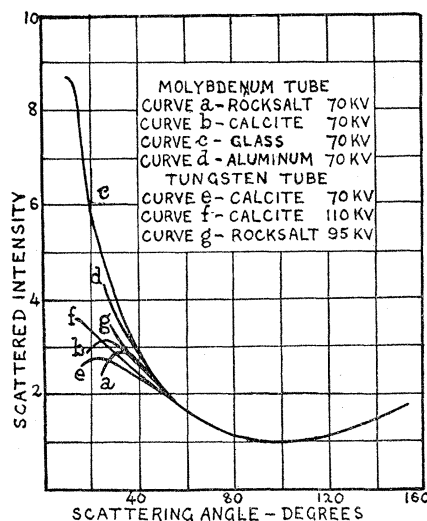


Fig. 7.

obtained, the humps due to regular reflection being omitted. These curves are drawn so as to have the same ordinate at 90° . Curves *a*, *b*, *c* and *d* are for heterogeneous x-rays from a molybdenum tube, while Curves *e*, *f* and *g* are for heterogeneous x-rays from a tungsten tube. It is seen that the curves are practically identical for the range $\phi = 50^\circ$ to $\phi = 150^\circ$. In this range the crystal curves (*a*, *b*, *e*, *f* and *g*) coincide with the curves for aluminum and glass (*c* and *d*). Below 50° the crystal curves all fall below the aluminum and glass curves. The crystal curves all reach a maximum between 15° and 30° . The curves for calcite and rocksalt, when the primary rays are homogeneous molybdenum K_α rays, are not shown but have the same general characteristics as the curves shown. Curves *e* and *f* are for the same calcite crystal and for the same tungsten tube, the only difference being that the potential for Curve *e* is 70 kilovolts while that for Curve *f* is 110 kilovolts. It appears as though the maximum is shifted to the smaller angles for the shorter wave-lengths.

It would be interesting to examine the shape of the scattering curves for small angles. It is known that the scattering curves for amorphous substances slope down towards zero for small angles, but the maximum occurs at much smaller angles than do the maxima for crystals in the

present experiments. For instance Curve *c* which is for glass does not show a maximum above 11° whereas calcite Curve *b* shows a maximum at 25° even though both curves are for a molybdenum tube operated at a potential of 70 kilovolts. The experimental difficulties when working at small angles necessitate the abandonment of this inquiry at this time.

According to Debye¹ the intensity of the x-rays scattered in a direction ϕ from a crystal should be equal to that from an amorphous substance multiplied by $(1 - e^{-M})$, where

$$M = \frac{0.569 \times 10^{-12}}{A\theta\lambda^2} \times (1 - \cos \phi) \times \frac{F(z)}{z}$$

when there is no zero-point energy and

$$M = \frac{0.569 \times 10^{-12}}{A\theta\lambda^2} \times (1 - \cos \phi) \times \left[\frac{F(z)}{z} + \frac{1}{4} \right]$$

when there is a zero-point energy. In these z is the ratio of the characteristic temperature θ of the crystal to its experimental temperature, A its atomic weight, λ the wave-length of the incident homogeneous x-rays, and $F(z)$ is a function of z which Debye evaluates. The value of the factor $(1 - e^{-M})$ has been found for both calcite and rocksalt when the incident wave-length is 0.717 \AA. U. , which is the wave-length of the molybdenum K_α line,² and the room temperature is 290° K. A. H. Compton³ gives the characteristic temperatures of calcite and rocksalt as 910° K. and 260° K. respectively. The atomic weight of calcite is taken as the average atomic weight, viz., 20. The value of $(1 - e^{-M})$ for a certain value of ϕ is divided by the value for $\phi = 90^\circ$. The curve obtained is shown as Curve *a*, Fig. 8. There are two possible

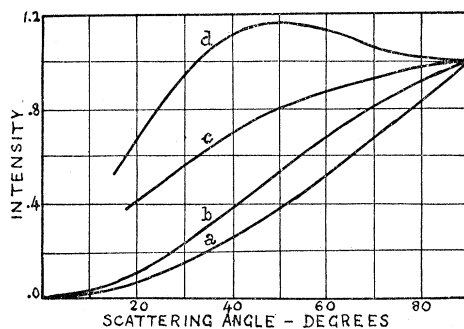


Fig. 8.

¹ Loc. cit.

² Kaye and Laby, *Physical and Chemical Constants*, 4th Ed., p. 99.

³ A. H. Compton, *PHYS. REV.*, Second Series, Vol. 9 (1917), p. 47.

curves, one for the case where there is no zero-point energy and the other for the case where there is a zero-point energy. The two curves, however, are very close together and the mean curve is the one shown. If each ordinate of Curve *c*, Fig. 7, which is for glass, is multiplied by the corresponding ordinate of Curve *a*, Fig. 8, the theoretical scattering curve for a crystal of calcite is obtained. This is shown as Curve *c*, Fig. 8. Curves *b* and *d*, Fig. 8, are the curves for rocksalt, the average atomic weight of rocksalt being taken as 29. It will be seen that Curve *d* which is the theoretical scattering curve for rocksalt coincides fairly well with the experimental curve for the range 90° to 60°, but below 60° there is a great discrepancy between the theoretical and experimental curves. In the case of calcite the curves do not coincide at any place.

According to J. J. Thomson¹ the intensity of x-rays falling on 1 cm.² at a distance *R* from the scattering substance is

$$(4) \quad \frac{Ne^4}{R^2m^2c^4} \cdot \frac{1 + \cos^2 \phi}{2} \cdot pI_0,$$

where *p* is the number of electrons per atom (or molecule), *N* is the number of atoms (or molecules) per unit volume and *I*₀ is the energy of the primary rays. This is also the formula obtained by Debye² for the case of an amorphous scattering substance when the wave-length of the x-rays is small compared with the radius of the atomic electron ring. If it is assumed that the absorption coefficient of the scattered x-rays is equal to that of the primary x-rays, then the total intensity of the rays entering the ionization chamber from a thick crystal when the crystal angle is θ and the chamber angle is 90° can be shown to be

$$(5) \quad I_{90} = \frac{SNe^4}{R^2m^2c^4} \frac{p}{2k(1 + \tan \theta)} I_0,$$

where *S* is the area of the window of the ionization chamber and *k* is the linear absorption coefficient of the x-rays in the crystal. Knowing the experimental value of *I*₉₀/*I*₀ and the value of *k*, *p* the number of electrons in the atom (or molecule) of the scattering crystal can be found. The linear absorption coefficients of molybdenum K_α rays in rocksalt and calcite were not found directly but were calculated from the known coefficient for aluminum using the law that the atomic absorption of an element varies as the fourth power of the atomic number of the element. The absorption coefficients for heterogeneous x-rays were measured directly. There is considerable doubt of the coefficients

¹ J. J. Thomson, *Conduction of Electricity through Gases*, 2d Ed., p. 325.

² Loc. cit.

for heterogeneous rays as they depend upon the thickness of crystal used. It is seen that the numbers of electrons per molecule are of the right order of magnitude, the true values for rocksalt and calcite being 28 and 50 respectively. According to Debye's theory of crystal scattering the factor $(1 - e^{-M})$ should enter into the intensity scattered. The absolute value of the factor for scattering at 90° from calcite for a wave-length of 0.717 \AA. U. (molybdenum K_α line) is 0.092 for no zero-point energy and 0.227 for zero-point energy, that is, the intensity scattered at 90° by a crystal of calcite should be either about one tenth or one fifth of that scattered by an amorphous substance of equal molecular weight. The fact that p comes out of the right order of magnitude and not one tenth or one fifth of the correct value shows that Debye's theory falls far short of explaining the experimental facts for calcite. For rocksalt the value of $(1 - e^{-M})$ for scattering at 90° and for a wave-length 0.717 \AA. U. is 0.73 for no zero-point energy and 0.82 for zero-point energy, the two experimental values for rocksalt being 0.56 and 1.06. Hence in the case of rocksalt the theory as far as the scattering at 90° is concerned is not opposed to the facts.

The discrepancy between Debye's theory of scattering and the observed results in the case of rocksalt is remarkable because in another direction Debye's theory has received confirmation. According to Debye the energy of a regularly reflected line should be proportional amongst other things to e^{-M} . This was tested by W. H. Bragg¹ in an experiment where he heated a crystal of rocksalt and observed the energy of the rhodium line as reflected by the crystal (1) when the crystal was at a temperature of 288° K. , and (2) when the crystal was at 643° K. The observed ratio of the energies was equal to the ratio as calculated from the theory. This might not have been so remarkable if the glancing angle in Bragg's experiment had been above 30° , making the chamber angle above 60° , as it is between 60° and 90° in my experiments that the scattering curve of rocksalt as found experimentally agrees to some extent with that indicated by the theory. However Bragg's experiments were done at glancing angles between 12° and $13^\circ 30'$ in the case of second order reflection and between $18^\circ 30'$ and 20° in the case of the third order reflection. In the former case the chamber angle is between 24° and 27° while in the latter case the chamber angle is between 37° and 40° . This is just the range where there is very great discrepancy between Debye's theory as applied to scattering from crystals and the experimental results of this present paper. For this reason it is a matter of great interest to test the temperature effect on the intensity of x-rays scattered by a crystal. This investigation however will be left for another paper.

¹ W. H. Bragg, *Phil. Mag.*, Vol. 27 (1914), pp. 881-899.

From the true scattering curves of Fig. 7 it is possible to calculate the ratio between the energy of the regularly reflected beam at $\phi = 2\theta$ to the total energy scattered. The energy scattered between the two cones of semi-angles $\phi - (\delta\phi/2)$ and $\phi + (\delta\phi/2)$ is given by

$$(3) \quad \frac{2\pi R^2 \sin \phi \delta\phi}{S} \cdot y,$$

where y is the ordinate of one of the curves in Fig. 7 for a particular value of ϕ , R is the distance of the window of the ionization chamber from the crystal, and S is the area of the window, and $\delta\phi$ is 6° . Values of (3) are calculated for each interval of 6° and these added for the range $\phi = \phi_1$, and $\phi = \phi_2$, thus giving the total intensity of true scattering between the two cones whose semi-angles are ϕ_1 and ϕ_2 . The results for calcite are shown in Table II. The ratio is calculated for homogeneous x-rays on the assumptions that the absorption coefficient in the crystal is the same when the crystal is in a reflecting position as when it is thrown out of this position, and also that the absorption coefficient of the scattered x-rays is equal to that of the primary x-rays. The ratio is therefore only an approximation. When the ratio is calculated for heterogeneous radiation consisting of the molybdenum K_α line plus general radiation a difficulty arises in that the absorption coefficient of the regularly reflected line is quite different from that of the scattered radiation. In the case of the x-rays coming from a tube with a molybdenum target the mass absorption coefficient of the general scattered radiation was 1.51 while that of the molybdenum K_α line was 5.17. A different value of k in formula (1) must be used for the line from that for the general scattered radiation.

TABLE II.

Proportion of X-Rays Regularly Reflected.

Nature of X-Rays.	ϕ_1 .	ϕ_2 .	Energy of Regularly Reflected X-Rays.
			Energy of Scattered Radiation
Homogeneous Mo K_α line.....	14°	93°	2.85
Heterogeneous Mo K_α line plus General Radiation.....	15°	153°	0.66

Calcite Crystal Set for First Order Reflection of Mo K_α Line.

There is also scattering of the homogeneous x-rays in the ranges 0° to 14° and 93° to 180° . The scattering throughout the whole range 0° to 180° is about double of that in the range 14° to 93° . The energy of the regularly reflected line is therefore about 1.43 times that of the total scattering. This is for the case where the angular width of the

primary beam is about 1° . For heterogeneous x-rays the proportion reflected is much less, being 0.66 of the amount scattered in the range 15° to 153° . Taking the whole range 0° to 180° , the energy reflected in the line is about one half of the total energy scattered. This is for an angular width of the primary beam equal to $45'$. For higher orders of reflection these ratios are much smaller and are smaller still for the case where the crystal is not set to reflect a line but general radiation. The total energy scattered from a crystal is less than that scattered from glass. Comparing calcite and glass using a molybdenum tube and interpolating Curves *b* and *c* of Fig. 7 for the angles 0° to 15° , the ratio of the total energy scattered by the crystal to that scattered by the glass for the range 0° to 150° is 0.85. It was thought that the regular reflection from a crystal might take place at the expense of the general scattering and that the difference between the energy scattered by glass and by a crystal might be equal to the energy of the regularly reflected radiation. This however is not the case.

SUMMARY.

The scattering of x-rays by the crystals calcite and rocksalt has been examined and the following results found:

1. The scattering curves for rocksalt and calcite are of the same shape as the curves for the amorphous substances glass and aluminum for the range 50° to 150° with the direction of the primary rays. The minimum of all the curves occurs at about 100° .
2. For scattering angles below 50° the crystal curves fall below those for the amorphous substances. The crystal curves reach a maximum between 15° and 30° , the position of the maximum being at smaller angles for smaller wave-lengths. The maximum in the case of amorphous substances occurs at much smaller angles than in the case of the crystals.
3. From the ratio of the intensity of the x-rays scattered at 90° to the energy of the primary rays the number of electrons per molecule of each crystal has been calculated by means of Thomson's formula. The numbers as calculated are of the right order of magnitude.
4. The x-rays scattered from crystals are softer than the primary x-rays, in this way resembling the x-rays scattered from amorphous substances.
5. A comparison between the experimental results and the theory of crystal scattering as developed by Debye has been made. It is shown that Debye's theory falls far short of explaining the observed facts. In the case of rocksalt there is fair agreement between the intensity scattered at 90° and that predicted by Debye, but at angles below 60° the difference between theory and experiment becomes great. For

calcite there is no agreement whatever between theory and experiment. This may be due to the fact that CaCO_3 is a more complicated molecule than NaCl .

6. In addition to the general scattering at all angles from a crystal, there appear Laue spots at particular angles.

7. The energy of x-rays scattered by a crystal in all directions is an appreciable fraction of the energy of a regularly reflected line.

8. The intensity of x-rays scattered in a certain direction is independent of the orientation of the crystal which suggests that the atoms are nearly isotropic.

9. The total scattering was not appreciably changed by turning the crystal from a position where a spectrum line was regularly reflected to a position where the line was not reflected.

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