THE EFFECT OF FIELD DIRECTION ON MAGNETO-RESISTANCE.

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SYNOPSIS.

Variation of Magneto-resistance Effect with Direction of the Magnetic Field.—(1) *Theoretical discussion.* The fact that in general the resistance of a conductor differs slightly according to whether the magnetic field is transverse or longitudinal, may be due to two factors. First, there may be a direct effect of the field on the motion of the electrons, as predicted by the electron theories of J. J. Thomson and others. But the author shows that if Townsend's method of developing the electron theory of conduction is adopted, the variation of resistance comes out zero. Second, the effect of the magnetic field may vary with the arrangement of crystals in the conductor, and therefore with the position of the conductor if the crystal structure is anisotropic. (2) *Experiments with cast bismuth, pressed graphite and rolled cadmium* are described in which a small bar or sheet of the material was placed in each of three mutually perpendicular positions in a field of 7,000 or 8,000 gausses, and resistance measurements were made for various positions of the rotatable Weiss magnet. For bismuth and cadmium the variations found are not symmetrical around the current direction and are evidently due chiefly to the crystal structure. For graphite the variations when the field was rotated in the plane of the sheet were within the experimental error, therefore both factors were inappreciable. These results are not conclusive but they suggest that the Townsend theory is correct. If so this magneto-resistance effect may be completely explained by assuming a change in the number and mean free period of the conducting electrons which depends not only on the magnetic field but on its direction with reference to the crystal axes.

When a metallic conductor of electricity is placed in a magnetic field there may be for any given conductor two factors which affect the magnitude of the resulting change of resistance. These two factors are the crystalline structure of the specimen and the angle between the magnetic field and the electric current. Experiments dealing with the latter of these two factors have concerned themselves chiefly with the two cases where the magnetic field is either transverse or longitudinal with respect to the electric current. Lenard [1] found that a longitudinal field produced a smaller resistance increase in a bismuth wire than a transverse field. The writer[2] has obtained a similar result in the case of tellurium, bismuth, lead sulphide, cadmium, zinc, gold, and graphite. Patterson[3] states that his experiments on copper appear

[1] Ann. d. Phys., 39, p. 619, 1890.

[2] Phys. Rev., 10, p. 366, 1917; Phil. Mag., 24, p. 813, 1912.

[3] Phil. Mag., 3, p. 643, 1902.

to indicate a slightly smaller effect for the longitudinal field. Barlow,[1] on the other hand, studying a plate of bismuth, found very little difference in the resistance change for the two directions of the field. The curves given by Barlow seem to indicate that a longitudinal field produces the greater increase of resistance.

The experiments cited above were performed for the most part with specimens of undetermined crystal structure, hence it is conceivable that the crystal structure of the specimens differed for the different directions of the magnetic field. In such a case we cannot say definitely that it is the orientation of the current with respect to the magnetic field which affects the magnitude of the resistance change; it might well be the lack of isotropy of the specimen which produces this effect.

The effect of crystal structure on magneto-resistance has been investigated for graphite by Roberts.[2] Roberts concludes that the resistance increase of this substance is independent of the direction of the electric current with respect to the field, depending only on the angle between the crystal axis and the magnetic field. De Haas[3] experimented with antimony and proved that for this metal the orientation of the crystal axis is of considerable importance. His experiments are not decisive in the matter but he draws the conclusion that the angle between the directions of the magnetic field and of the current is of no importance, at least to a first approximation.

Now if the conclusion of de Haas regarding this question is correct it is a matter of considerable importance in its bearing on the electron theory of metallic conduction. De Haas, considering the free electron theory, says that the influence of the magnetic field on the free paths of the electrons must be considered as negligible, and that such theories as that of J. J. Thomson[4] which try to calculate the phenomenon from the direct effect of the field on the free electrons cannot possibly give the right result. Now it is generally assumed in the literature of the subject that the electron theory of metals does afford an explanation of magneto-resistance, at least for transverse fields and non-ferromagnetic metals. A number of writers[5] following the general method of Sir J. J.

[1] Ann. d. Phys., 12, p. 921, 1903.
[2] Ann. d. Phys., 40, p. 467, 1913.
[3] Konink. Akad. Wetensch. Amsterdam, 16, p. 1110, 1914.
[4] Rapports presentes au Congres International de Physique, III., p. 138, 1900.
[5] E. P. Adams, PHYS. REV., 24, p. 428, 1907.
Gans, Ann. d. Phys., 20, p. 293, 1906.
Livens, Phil. Mag., 30, p. 526, 1915.
Heaps, PHYS. REV., 10, p. 366, 1917.
Richardson, Electron Theory of Matter, p. 439.
Righi, "I Fenomeni Elettro-atomici," p. 401.

Thomson have derived expressions for the increase of resistance of a metal when placed in a transverse magnetic field. These expressions involve the relation

$$\frac{dR}{R} = C\left(\frac{e}{u}\right)^2 \frac{l^2}{v^2} H^2, \tag{1}$$

where dR is the increase produced in the resistance R by the magnetic field H, e/m is the ratio of the charge to the mass of the electron, l is the mean free path of the electron, v is its velocity of agitation, and C is a constant depending on the type of theory adopted. Properly speaking, this expression for dR/R refers only to the increase of resistance produced when a longitudinal field is rotated into the transverse position and when the specimen is isotropic. It cannot be supposed, therefore, that calculations of mean free paths, etc., by the use of this formula will be of much value when experimental results are obtained from specimens of unknown crystalline structure or when the dR/R of the above formula is taken to represent the entire effect of a transverse field as was the case in Patterson's calculations.

If the conclusions of de Haas are correct, *i.e.*, that there is no intrinsic difference between the effects of a transverse and a longitudinal field, then we should expect the theoretical expression for dR/R of equation (1) to come out equal to zero. As a matter of fact, one form of the theory gives this result, as may be shown in the following manner.

Let us adopt the ordinary assumptions of the free electron theory of metals, assuming collisions between electrons and molecules to be like those between hard elastic spheres. With these conditions J. S. Townsend has developed an expression for the velocity with which a group of free electrons drifts through an aggregation of molecules under the combined influence of electric and magnetic fields.[1] In addition to assuming elastic collisions Townsend neglects persistence of velocities, assumes that the free periods of electrons may vary from zero to infinity, and lets the number of free periods, out of a total number N, comprised in the time interval between t and $t + dt$ be equal to $(N/T)e^{-t/T}dt$, where T is the mean free period.

If the magnetic field H acts along the z axis and the electric fields X and Y act along the x and y axes, respectively, Townsend's theory gives for the drift velocities U and V along the respective x and y axes:

$$U = \frac{e}{m} T \frac{(X - Y\omega T)}{1 + \omega^2 T^2}, \tag{2}$$

$$V = \frac{e}{m} T \frac{(Y + X\omega T)}{1 + \omega^2 T^2}, \tag{3}$$

[1] Electricity in Gases, p. 100.

where $\omega = He/m$. In Townsend's development Y is assumed zero. It is to be noted that the deduction of the above formulæ does not involve approximations like those made in the references cited above, where other methods are used.

If we consider a plate of metal lying in the x, y plane and carrying a current along the x direction the condition that there shall be no current along the y direction requires V to be zero. There must, therefore, exist an electric field given by $Y = -X\omega T$ in the metal to prevent the electron current along y. It is the potential difference due to this field which is observed as the Hall effect.

If we substitute this value of Y in equation (2) we get

$$U = X\frac{e}{m}T. \tag{4}$$

Hence the drift velocity along x under these conditions is not affected by H so long as T remains unchanged. The current density along x is given by

$$I = neU = \frac{ne^2}{m}XT, \tag{5}$$

where n is the number of free electrons per unit volume. Thus if n, T, and X are not functions of the magnetic field, the current I is independent of H and we have

$$\frac{dR}{R} = 0. \tag{6}$$

We may now consider a form of conductor in which $Y = 0$. Let the current enter at the center of a flat circular plate and leave it at its periphery, and let the plane of the plate be normal to H. This is the arrangement used by Corbino.[1] Under these conditions the magnetic field causes a circular current to flow in the plate, and since there is no banking up of electrons to produce the Hall e.m.f. we may assume that Y, taken as perpendicular to a radius, is zero. In this case the radial current may be represented by

$$I_r = neU = \frac{ne^2}{m}X\frac{T}{1 + \omega^2T^2}. \tag{7}$$

Hence

$$\frac{I_0 - I_r}{I_r} = \frac{dR}{R} = \omega^2T^2, \tag{8}$$

where I_0 is the current when H is zero. Thus with the Corbino arrangement the specific resistance of the metal will be found greater than in

[1] Phys. Zeits., 12, pp. 561, 842, 1911.

the case where the Hall e.m.f. is allowed to develop. Under the ordinary conditions of measuring magneto-resistance, however, the Hall e.m.f. is always present, so it is not permissible to assume $Y = 0$ in developing the theory, as has been done by many previous writers.

It thus appears according to Townsend's theory that there should ordinarily be no change of resistance produced in a metal by the direct action of the magnetic field on the motion of the electrons between collisions. Townsend's theory is therefore in agreement with the conclusions of de Haas.

The theory of magneto-resistance has been worked out by Gans and by Livens.[1] These writers follow the method of H. A. Lorentz in assuming a slight departure from Maxwell's law of distribution of velocities among the electrons. The effect of the Hall e.m.f. is not neglected. The formula obtained by Gans for small magnetic fields is

$$\frac{dR}{R} = \frac{12 - 3\pi}{8}\left(\frac{e}{m}\right)^2 \frac{l^2}{v^2} H^2. \tag{9}$$

This formula assumes collisions to be like those between elastic spheres and here also only the difference between the effect of a transverse and a longitudinal field is contemplated. Thus the theory of Gans does not agree with the conclusions of de Haas. Livens assumes molecules to act as centers of force and gets a more general formula. He points out that if the potential energy of an electron repelled by a molecule at a distance r is given by $m/2 \cdot (\mu/r)^s$ then when $s = 4$ the value of dR/R should be zero.[2] It appears therefore that if de Haas is correct in his conclusions regarding experiment—*i.e.*, that the difference between the longitudinal and transverse magneto-resistance effects is zero—then the theory of Livens demands a special type of field around a molecule. If on the other hand we adopt Townsend's method of handling the problem then the molecules and electrons may act like solid elastic spheres and we still get agreement with the conclusions of de Haas.

It seemed to the writer that further experiments were necessary in order to establish a theory, hence the effect of the direction of the magnetic field on the resistance of bismuth, graphite, and cadmium has been investigated. The magnetic field was furnished by a large Weiss electromagnet capable of being rotated about a vertical axis. The angular position of the magnet could be read from a scale at the base of the instrument. The pole-pieces were 10 cm. in diameter and were set so that the faces were 3.03 cm. apart. The specimen to be examined was

[1] L.c.

[2] The formula of Livens reduces to that of Gans if $s = \infty$.

supported between the pole-pieces so that the electromagnet could be rotated without disturbing it. This specimen was inserted in one arm of a Wheatstone bridge, balancing being accomplished by a shunt arrangement somewhat after the fashion described by Crandall.[1] A Leeds and Northrup moving coil galvanometer of resistance 13 ohms and sensitivity 11 mm. per microvolt was used. Since the current through the electromagnet had a value of 8 or 10 amperes, and since no special precautions were taken to maintain constancy of temperature, the resistance of the specimen was found to change very slowly during the time required to obtain a set of observations. A correction for this temperature effect was made by periodically repeating a standard resistance measurement of the specimen and assuming a linear resistance change during the interim between these periodic measurements. As a matter of fact when a set of observations was taken with reasonable rapidity no very great error was introduced by temperature effects. The bridge current was made as small as was convenient—of the order of 0.01 ampere—and was kept flowing during the whole time of taking a set of observations.

The process of making observations consisted in balancing the bridge with the specimen between the poles of the magnet but with no exciting current flowing. The magnetizing current was then set up and a new balance of the bridge obtained. With this magnetizing current kept constant the bridge was balanced for different angular positions of the electromagnet and values of $\delta R/R$—which is here the total increase of resistance of the specimen divided by the resistance in zero field—were calculated for these various positions of the magnet. In calculating $\delta R/R$ the resistance of the copper wires leading to the specimen was carefully allowed for, though it was unnecessary to consider the effect of the field on the resistance of these leads since copper shows very small magneto-resistance effects. When the specimen was removed from between the poles it was found that revolving the magnet did not affect the bridge balance. Spurious effects, such as the influence of the magnet on the zero position of the galvanometer can thus be considered as negligible.

The bismuth used in this investigation was rated by Merck as about 98 per cent. pure. A specimen was made by cutting a thin bar of rectangular cross-section from a thin plate of cast bismuth. The dimensions of this bar were roughly $1.1 \times 0.15 \times 0.05$ cm., the shortest dimension being perpendicular to the plane of the original large plate of metal. Experimental results for this bar are given in Fig. 1, where the

[1] PHYS. REV., 2, p. 343, 1913.

lenght of the radius vector from 0 to any point of a curve is set equal to $\delta R/R$ and the angular position of this radius vector is determined by the orientation of the magnet. Results are plotted only for positions of the magnet comprised between 0° and 180°, as duplication results in the case of larger angles. The field strength was 7,000 gausses and the temperature was that of the room—about 23° C.

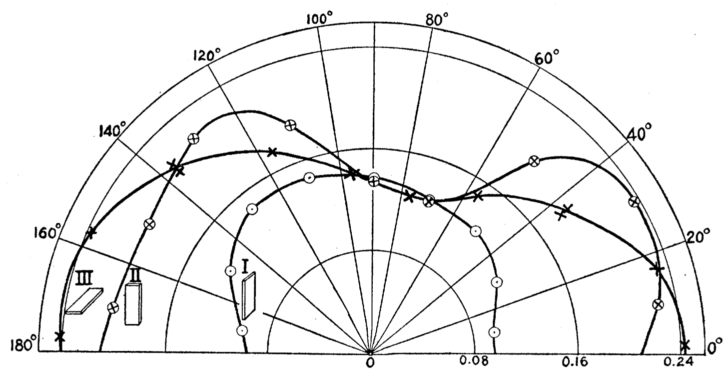

Fig. 1.

Three curves were obtained for this specimen, corresponding to three different settings of the bar between the poles of the magnet. The three-dimensional diagrams of the bar placed beside the respective curves will make the arrangement clear. The magnetic field is always in a horizontal plane, the plane of the figure. In curve I. the length and thickness of the bar are in this horizontal plane, the breadth in a vertical plane. The magnetic field for this curve was thus always perpendicular to the breadth of the specimen. In curve II. the breadth and length of the bar were in a horizontal plane; the magnetic field was thus always perpendicular to the shortest dimension of the specimen. In curve III. the breadth and thickness of the specimen were in a horizontal plane; the magnetic field was here, therefore, always perpendicular to the length of the specimen and to the current. It will facilitate interpretation of the curves to note that the relative orientation of field and specimen may be determined for any point on the curve by imagining the diagram of the specimen to be shifted without rotation to the desired point of the curve.

The lack of symmetry of these curves is an indication of a complex crystalline structure in the bar. Curves I. and II. coincide, within the limits of error of the experiment, at the angle 90°. This coincidence is, of course, to be expected, for at this point the relative arrangement of specimen and field is identical for the two curves. Curve III. should coincide with curve II. at 0° and 180° for the same reason. Similarly curve III. should have the same value at 90° as curve I. at 0° and 180°. The fact that curve III. does not have these values ($\delta R/R$ being too great in each case) is very probably due to a slight error in measuring lead resistances, for in setting the specimen for III. after I. and II. had been obtained it was necessary to disconnect the old leads and put on new ones. The specimen itself had a very small resistance—about 0.05 ohm—while the resistance of the leads was 0.0442 ohm. The Wheatstone bridge, though sensitive to changes of resistance, was not very accurate for the measurement of absolute values of small resistances. Slight inaccuracies in measuring these small resistances would affect the calculated value of $\delta R/R$ but would be of no consequence in making comparisons along any one curve. Another cause of error might lie in the altered resistance of the soldered joint between lead wire and specimen. No measurement of this junction resistance could be easily made. A third cause of error lies in the difficulty of setting the specimens accurately in position.

Certain general conclusions may be drawn from the curves of Fig. 1. In curve I. the effect of a longitudinal field (measured by the length of the radius vector at 90°) is greater than the effect of a transverse field (measured by the length of the radius vector at 0°). For curve II. the converse is true. It appears obvious that crystalline structure plays a very important rôle in the phenomenon of magneto-resistance. Since curves I. and II. are not symmetrical with respect to the 90° position (the direction of current flow) we must conclude that crystalline structure is producing a distortion of the curves. It thus appears impossible in this bismuth specimen to separate the effect due to current direction—if there is such an effect—from the effect due to crystal structure. We may conclude, however, that crystalline structure is a very important factor because curve III. obtained with the current always transverse shows great variations as the field changes direction. The conflicting results of Barlow and Lenard cited above can now be explained as arising solely from the different arrangement of crystalline axes with respect to the field. The complicated nature of the results obtained with bismuth might have been foreseen from the work of E. van Everdingen[1] who found that when a bismuth crystal is placed in a magnetic field of arbitrary direction its resistance may be represented by an ellipsoid with three unequal axes. In general, then, a cast bismuth plate is not apt to possess a plane in which a magnetic field may alter its direction without encountering dissimilar crystalline conditions.

[1] Konink. Akad. Wetensch. Amsterdam, III., p. 407, 1901.

Some further experiments were performed on wire made by forcing molten bismuth into a vertical capillary tube. It might be expected that for all directions of a magnetic field perpendicular to the wire the same magneto-resistance effect would be noted. Such, however, proved not to be the case, a small but unmistakable dissymmetry being observed. When the specimen was placed horizontally and the magnet rotated from the transverse to the longitudinal position the latter position was found to give the smaller effect. The greatest increase of resistance was observed when the magnetic field made an angle of 72° with the length of the wire.

For the experiments on graphite the specimen was made from the ordinary powdered graphite, consisting of small crystalline particles, which is used for lubricating purposes. This powder was compressed by means of a hydraulic press into the form of a thin plate on the top of an ebonite block. The graphite was made to adhere to the ebonite by a thin coat of beeswax and resin which had been previously applied to the ebonite. Two brass screws had been set into the block with their heads flush with its surface, so that by cutting away part of the graphite plate with a razor blade a thin bar was obtained with its ends resting in close contact with the screw heads. Copper wires were soldered to the ends of the screws and used for connecting the specimen into the Wheatstone bridge. The dimensions of the bar were roughly 1.2 × 0.15 × 0.03 cm. and its resistance was 1.15 ohms at 26.5° C. This resistance increased in one week to 1.23 ohms. Probably this increase was due to gradual readjustment of strains in the specimen.

The curves of Fig. 2 represent results obtained with this graphite in a

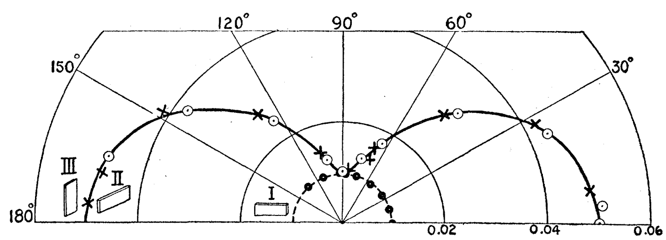

Fig. 2.

field of 8,000 gausses. The orientation of the specimen for any particular set of observations is represented diagrammatically as in the previous case with bismuth. It appears from curve I. which is practically a semicircle, that as long as the magnetic field is in the plane of the specimen we get the same change of resistance irrespective of the direction of the field. There is a slight deviation of the points from the curve. It is so slight, however, as not to show up in the graph and is probably caused by inaccurate setting of the specimen. The points for curve II. are marked with crosses, those for curve III. with circles. These two curves coincide to a fair degree of accuracy. The obvious interpretation of the curves is as follows.

In the plane of the specimen the arrangement of the small crystals is such that any direction in this plane is the same as any other direction as far as the average crystalline structure is concerned. If this is the case then the circular nature of curve I. implies that the direction of the electric current is of no importance as regards the magnitude of the resistance change produced by the magnetic field. Curves II. and III. indicate that the average crystalline structure in a direction perpendicular to the plate is different from that parallel to the plane of the plate, but the fact that these two curves coincide is again evidence that the change of resistance is independent of the relative directions of field and current.

The conclusions which Roberts[1] reached in his study of large graphite crystals are corroborated by the above experiments on a conglomerate. In the writer's previous work with graphite powder[1] it was concluded that $\delta R/R$ for a transverse field was greater than that for a longitudinal field by 8×10^{-4}. Differences of that order of magnitude were obtained in the various observations of curve I., but it seems probable that these differences were the result of experimental errors arising from temperature changes or inaccurate adjustment of the specimen. The longitudinal field of curve I. gave a value of $\delta R/R$ greater by 1.5×10^{-4} than the $\delta R/R$ of the transverse field. It is difficult to adjust such small specimens accurately with respect to the magnetic field and a small deviation from accuracy can produce quite large changes in $\delta R/R$. Possibly the writer's previous results can be explained as due to inaccurate adjustments.

The experiments on cadmium were performed on a specimen made as follows. A disk of the metal was sawed from a round bar. This disk was put through a rolling mill a number of times, and always in the same direction so that finally a long thin sheet of cadmium was secured. A section of this sheet was then cemented with wax flat upon a glass plate and cut with a razor into the form of a grid 0.9 cm. wide and 2.5 cm. long, containing 16 strips of cadmium connected in series and having a total resistance of 1.71 ohms. This grid was supported between the poles of the magnet in a horizontal position and examined for magneto-resistance in a field of 7,600 gausses. A fairly large bridge current had to be used with this metal in order to get sufficient sensitivity, so that troublesome

[1] L.c.

temperature effects were introduced. However, by taking in rapid succession a number of observations for the transverse and longitudinal positions of the magnet it was found that $\delta R/R$ for the transverse field was greater by 2.1×10^{-5} than for the longitudinal field. The specimen was then placed so that the strips of the grid were vertical, the field being always transverse to the current. A series of observations was taken with the field alternately perpendicular and parallel to the plane of the grid. It was found that $\delta R/R$ was greater by 5.2×10^{-5} for the perpendicular field than for the field parallel with the surface of the grid. Since the current was here always transverse to the field it is to be concluded that differences of crystalline structure are responsible for the difference of 5.2×10^{-5}. As regards the difference of 2.1×10^{-5} this might be due either to differences of current direction or to the anisotropic character of the strips.

In previous work by the writer on cadmium strips made by hammering no difference was detected in $\delta R/R$ for a transverse field whether normal or parallel with the surface of the strips, while a longitudinal field was found to produce a smaller effect than the transverse field. If this specimen made by hammering was really isotropic as regards magnetic effects then it appears that the current direction is a factor in determining the resistance change of cadmium. It cannot be said with certainty, however, that such an isotropic character was produced by this method. On the other hand, rolling the specimen does introduce an anisotropic character. Since the drawing of metal into the form of wires is more analogous to the rolling method than to the hammering method we might expect that cadmium wires would exhibit magneto-resistance effects for a longitudinal field which are different from those for a transverse field solely because of the anisotropic nature of the wires. We cannot say, therefore, that experiments which have been performed on cadmium have either proved or disproved the theory of magneto-resistance outlined above. Tests of other metals such as zinc, gold, copper, etc., could not be made because of insufficient sensitivity of the apparatus.

The general conclusions to be drawn from the above experiments are as follows. Influences of crystal structure are so great in the metals bismuth and cadmium that any effect arising from the direction of the current cannot be differentiated from effects arising from crystal structure. In the case of graphite, if the direction of the electric current with respect to the magnetic field is of any importance at all in changing the magnitude of magneto-resistance effects then any such resulting change is very small compared with the effect of crystal structure on magneto-resistance. The experiments with graphite may be considered as supporting the correctness of equation (6). If equations such as (9) hold true it must be supposed that l/v is very small, at least in graphite. On the whole, one must conclude that calculations such as those of Patterson, in which electron concentrations and mean free periods of electrons in different metals are determined from measurements of magneto-resistance, are by no means to be relied upon. The effect of crystal structure is too vital a factor to neglect. Furthermore the theory as developed above by Townsend's method indicates that there should ordinarily be no effect of a magnetic field on resistance (provided this effect is limited to direct action on the electrons' motion) and it appears that there is some experimental evidence in support of this theory. If we adopt the Corbino arrangement, however, and use equation (8) it is possible to calculate the mean free period of the electrons in the metal. Essentially this method has been used by the writer in a previous paper,[1] and it is perhaps surprising that the results obtained agree as well as they do with the results calculated by Patterson from J. J. Thomson's equation.

Suppose now that we consider equation (6) as essentially correct, that is, suppose a magnetic field does not alter the resistance of a metal by virtue of any direct action on the motion of an electron between collisions. Then in order to explain the change of resistance observed experimentally we shall have to consider the factors of equation (5) and determine which are functions of the magnetic field. It is conceivable that the electric field X in the region where the electrons move may be modified by the action of the magnetic field on the molecules of the metal. If polarization electrons play any part in modifying the internal field[2] it is not impossible that a magnetic field, say by changing the orientation of molecular magnets, should affect this field produced by the polarization electrons. However, since there appears to be no relationship between the direction of the magnetic field with respect to X and the resistance increase which this field produces in the metal, it seems safe to say that X is not dependent on H. The average distance between molecules is probably changed by a magnetic field[3] so that the free period T will depend on H. It has been suggested by a number of writers[4] that n should be affected by a magnetic field. If we imagine in the metal a system of electrons moving in open and closed orbits—

[1] PHYS. REV., 12, p. 340, 1918.

[2] See Richardson, Phil. Mag., 23, p. 614, 1912.

[3] See E. P. Adams, l.c.

[4] E. van Everdingen, Konink. Akad. Wetensch. Amsterdam, III., p. 177, 1900.
La Rosa, N. Cim., 18, p. 39, 1919.
Heaps, l.c.

as suggested by Richardson[1]—it is not difficult to understand why a magnetic field might alter equilibrium conditions in such a system and thus change n.

If we let n_0, T_0, and I_0 be the respective values of n, T, and I when the magnetic field is zero then

$$I_0 = n_0 \frac{e^2}{m} X\, T_0. \tag{10}$$

Combining this equation with (5) gives

$$\frac{I_0 - I}{I} = \frac{\delta R}{R} = \frac{n_0 T_0}{nT} - 1. \tag{11}$$

This equation, in the light of Townsend's theory, is probably the most general expression for magneto-resistance. The factor given by equation (1) does not appear. It is to be noted that both n and T may depend upon the direction of the field in a metallic crystal as well as upon the magnitude of the field.

The experimental work described in this paper serves to emphasize the importance of crystal structure in relation to magneto-resistance. The question might well be asked as to whether crystalline structure is not essential to the production of the phenomenon. Becker and Curtiss[2] have found that bismuth films made by cathode sputtering, and therefore presumably amorphous, show no magneto-resistance effects until after they have been subjected to a heating process which crystallizes the metal. As far as the writer is aware, a magnetic field has never been found to change the resistance of a strictly non-crystalline substance,[3] so it is not impossible that an orderly arrangement of the molecules is essential for the manifestation of the phenomenon. The settling of this question, however, requires more extensive experimental data than is available at present.

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[1] Electron Theory of Matter, p. 463.

[2] Phys. Rev., 15, p. 457, 1920. See also Richtmyer and Curtiss, Phys. Rev., 15, p. 467, 1920.

[3] Patterson found an effect with liquid mercury but later work (see Zahn, Jahrbuch der Radioaktivität und Elektronik, 5, p. 197, 1908) indicates that the observed increase of resistance is to be ascribed to electrodynamic actions.