THE EFFECT OF DAMPING ON THE WIDTH OF X-RAY SPECTRUM LINES.

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SYNOPSIS.

Width of X-ray Spectrum Lines Due to Damping.—(1) Assuming exponential damping, the equation for the ether vibrations, $y = e^{-kt} \sin (pt + A)$, is developed as a Fourier integral and thus a *theoretical relation of the width to the damping factor* is found. If we define the width of the line as the range in wave-length throughout which the intensity is greater than half the maximum intensity of the line, then $x\lambda/\lambda = dp/p = 2k/p$, very closely.

(2) *Theoretical Variation with Wave-length.* Assuming in addition that k has the value given by the classical theory of a single radiating electron, $d\lambda = 2k\lambda/p$ $= 4\pi e^2/3mc^2 = 1.2 \times 10^{-4}A$, *i.e.*, $d\lambda$ is independent of wave-length.

(3) *Comparison with Observation.*—A. H. Compton has found the widths of the W lines, $\lambda\lambda$ 1.242 and 1.279A, to be about $9 \times 10^{-4}A$, nearly eight times the above value. The discrepancy indicates that either the theory is incorrect, or the measurement is wrong, or that damping is not the chief cause of the observed width.

IN this issue of the PHYSICAL REVIEW, Professor A. H. Compton describes experiments in which he has measured the width of x-ray spectrum lines. The lines used were the tungsten lines $\lambda = 1.242$ and $\lambda = 1.279$ Å.U. After correcting for the angular aperture of the slits, the angular faults of the crystal grating, and the finite resolving power of the grating, Professor Compton finds that the width due to the non-homogeneity of the x-rays is given by $(\delta\lambda/\lambda) > 0.0007 \pm .00014$, or a width of the order of, but greater than, 0.5 minute of arc in the first order spectrum as reflected from calcite. It is the purpose of this paper to examine into a possible explanation of this width on the assumption that the wave motion sent out by the oscillator emitting x-rays can be represented by an equation of the type $y = e^{-kt} \sin [p(t - x/c) + A]$. This damped harmonic motion can be represented by a Fourier's integral. We shall assume that the oscillations begin at time $t = 0$, so that $y = F(t) = 0$ for $0 > t > -\infty$, but that $y = F(t) = e^{-kt} \sin (pt + A)$ for $\infty \geqq t \geqq 0$.

The formula for the expression of $F(t)$ as a Fourier's integral is well known[1] and we obtain

[1] Cf. Byerly, Fourier Series and Spherical Harmonics, p. 54.

$$e^{-kt} \sin (pt + A) = \frac{1}{\pi} \int_0^\infty \frac{d\omega \cdot \sqrt{C^2 + D^2} \cdot \cos (\omega t - \tan^{-1} D/C)}{(p^2 - \omega^2 + k^2)^2 + 4k^2\omega^2},$$

where

$$C = p(p^2 - \omega^2 + k^2) \cos A + k(p^2 + \omega^2 + k^2) \sin A,$$

and

$$D = 2kp\omega \cos A - \omega(p^2 - \omega^2 - k^2) \sin A.$$

If now I_ω represents the intensity of the radiation of frequencies between

$$\frac{\omega - (\delta\omega/2)}{2\pi} \text{ and } \frac{\omega + (\delta\omega/2)}{2\pi},$$

we have

$$(1) \qquad \frac{I_\omega}{I_p} = \frac{\omega^2}{p^2} \times \frac{(p \cos A + k \sin A)^2 + \omega^2 \sin^2 A}{p^2 + 2kp \cos A \sin A + k^2 \sin^2 A} \times \frac{k^2(4p^2 + k^2)}{(p^2 - \omega^2 + k^2)^2 + 4k^2\omega^2},$$

where I_p is the intensity when $\omega = p$. In Professor Compton's experiments the intensities were plotted against the wave-lengths and the difference between the two values of λ for which the intensity was one half of that of the center of the line was taken as $\delta\lambda$. We shall therefore take $I_\omega/I_p = \frac{1}{2}$, and

$$\frac{\omega - p}{p} = \frac{1}{2} \frac{\delta\lambda}{\lambda} = .00035 = a.$$

Since ω is very nearly equal to p and since therefore $p^2 \cos^2 A + \omega^2 \sin^2 A$ is very nearly equal to p^2, equation (1) reduces to the approximate form

$$(2) \qquad \frac{I_\omega}{I_p} = b = \frac{k^2(4p^2 + k^2)}{(p^2 - \omega^2 + k^2)^2 + 4k^2\omega^2},$$

which is independent of the phase angle A. Solving this equation for k and putting $\omega = p(1 + a)$ where a is small, we obtain approximately

$$(3) \qquad k = \frac{2\pi a c}{\lambda} \sqrt{\frac{b}{1 - b}},$$

where $p = 2\pi c/\lambda$ and c is the velocity of light in vacuo. If b is taken as $\frac{1}{2}$, $k = pa = \omega - p$.

In Professor Compton's experiments $a = .00035$, $b = \frac{1}{2}$, and $\lambda = 1.242$ Å.U. for one of the lines observed. These values give $k = 5.3 \times 10^{15}$ sec^{-1}. The amplitude of the vibration $y = e^{-kt} \sin (pt + A)$ is therefore damped to $1/e$ of its initial value in 1.89×10^{-16} secs. In this time the oscillator makes 457 vibrations and the energy of the wave train given

out by the oscillator therefore becomes damped to $1/e$ of its value in 229 wave-lengths.

Professor Compton gives the damping factor due to radiation for a single electron on the classical electrical theory as being equal to

$$(4) \qquad \frac{4\pi^2}{3} \cdot \frac{e^2}{m\lambda^2 c} \cdot$$

For $\lambda = 1.242$ A.U. this gives $k = 7.2 \times 10^{14}$ sec^{-1}, the amplitude being damped to $1/e$ of its value in 3,360 vibrations. Assuming that the damping factor of the emitted x-rays is equal to that of the oscillator, we have from (3) that a, or $\delta\lambda/2\lambda$, is equal to

$$(5) \qquad \frac{k\lambda}{2\pi c} \sqrt{\frac{1-b}{b}} = .000047.$$

This corresponds to a width of 4 seconds of arc in the first order spectrum as reflected from calcite, the experimental value being greater than 30 seconds of arc.

The inconsistency between the damping factor as calculated by means of Fourier's integral from the observed width and that as calculated from formula (4) may be due to, (a) the incorrectness of the formula (4) as applied to the x-rays emitted by the oscillator, (b) the inaccuracies of the experiments measuring the width, (c) the possibility that the spectrum lines are made up of a finite number of lines close together, and (d) the possibility that the oscillation at a point in the ether due to the emitted x-rays cannot be represented by an equation of the form $y = e^{-kt} \sin (pt + A)$. It is hoped in the near future to make a re-determination of the width experimentally so as to eliminate (b).

The damping of the oscillator which emits any electromagnetic radiation must tend to broaden the spectrum line observed. This is true for light waves as well as for x-rays. From formula (4) it is seen that for light waves

$$a = \frac{\delta\lambda}{2\lambda} = 1 \times 10^{-8}$$

approximately. The broadening of the spectrum lines for ordinary light due to damping is therefore too small to be observed. In the x-ray case, however, an appreciable part of the width (about 13 per cent. according to Professor Compton's experiments) is due to damping. It is interesting to note that from the formulas (4) and (5) $a = \delta\lambda/2\lambda$ varies as $1/\lambda$, so that $\delta\lambda$ is constant for all values of λ throughout the x-ray

and light spectra.[1]   This value of $\delta\lambda$ due to damping equals 1.2×10^{-4} Å.U. approximately.

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[1] The writer is informed that Mr. C. G. Darwin, of Cambridge University, England, has also noted this interesting fact.