OMETER HAVING FLUXMETER CHARACTERISTICS.

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Synopsis.

Deflection of an ideal fluxmeter in which the coil has great damping control but no torsional control, and has negligible inductance compared with its resistance, is shown to be directly proportional to the current and to the time during which it flows, for the range through which the magnetic torque is proportional to the current.

Measurement of time intervals with such a fluxmeter. In practice it would be necessary to calibrate the instrument, but the relation between deflection and time, for a constant current, should be approximately linear. A great advantage is that the range may be varied at will by proper control of the current.

SEVERAL years ago the writer published[1] the results of an investigation into the possibility of using a moving-coil galvanometer for the measurement of time intervals of the same order of magnitude as the half-period of the undamped galvanometer coil. The results of his experiments showed that such intervals, up to 6 seconds, could be measured with a galvanometer of 6.5 seconds' half-period, and that the probable error of a single observation did not exceed 0.5 per cent. In the work cited, results were obtained for two conditions of damping, viz., slightly damped and critically damped motion of the coil.

In the course of a more recent investigation, there was occasion to analyze the motion of a completely damped coil, *i.e.*, a coil without torsional control, as in the ideal fluxmeter, when the same conditions were imposed as have been outlined in the previous paper. The method of analysis may be briefly summarized as follows:

1. Set up the equation of motion of the coil, assuming zero torsional control.

2. From this equation, determine the angular position, θ_τ, and angular velocity, ω_τ, of the coil, initially at rest, at an instant τ seconds after a steady current has been started flowing in the coil.

3. Assuming that at the instant τ the current is interrupted, substitute the values θ_τ and ω_τ as initial conditions into the equation of motion of the coil, which, beginning at the instant τ, experiences no torque other than damping.

[1] Physical Review, VIII., 195, 1916.

4. Solve the resulting equation for the maximum angle of displacement, θ_m, in terms of τ.

Having obtained an expression involving the variables θ_m and τ—the other quantities being constant and determinable—the expression in the earlier work was made the basis for time measurement by observation of the angle θ_m corresponding to the unknown time interval, τ, during which a steady current passed through the galvanometer. In the present work we arrive at an expression which, on account of its simplicity, is interesting *per se*, and should be even better suited to time measurement than those applying to other conditions of damping, provided a galvanometer with characteristics approaching those of the ideal fluxmeter is used.

In the derivation, the major steps of which are reproduced below, the following symbols are employed, which are the same as those used in the writer's previous papers on galvanometers.[1] τ = time interval to be measured; θ = angular displacement of coil at time t; ω = angular velocity of coil at time t; θ_m = angle of maximum deflection; I_0 = moment of inertia of coil; $2f$ = proportionality constant between damping moment and angular velocity of coil; M = electromagnetic moment of coil; q^2 = elastic torque constant of suspensions; i = steady current impressed on circuit. Other designations are introduced as required. The principal assumptions to be made are that within the angle θ_m the magnetic field shall be uniform and radial, with resulting constant value for M; and that the inductive part of the reactance of the coil circuit shall be very small compared with the pure resistance, so that current growth and decay in this circuit are practically instantaneous.

Step 1.—The equation of motion, since the elastic torque constant $q^2 = 0$, is

$$\ddot{\theta} + \frac{2f}{I_0}\dot{\theta} = \frac{Mi}{I_0}\cdot \qquad (1)$$

Step 2.—When $t = \tau$, this equation leads directly to

$$\theta_\tau = \frac{Mi}{2f}\left[\tau + \frac{I_0}{2f}\left(\epsilon^{-2f\tau/I_0} - 1\right)\right], \qquad (2)$$

and

$$\omega_\tau = \dot{\theta}]_{t=\tau} = \frac{Mi}{2f}[1 - \epsilon^{-2f\tau/I_0}]. \qquad (3)$$

Step 3.—At the instant τ, the current is interrupted, and the resultant orque on the coil vanishes. The general equation of motion then becomes

$$\ddot{\theta} + \frac{2f}{I_0}\dot{\theta} = 0, \qquad (4)$$

[1] PHYS. REV., V., 266, 1915; VII., 633 and 640, 1916; VIII., 195, 1916; XV., 12, 1920.

which has the general solution

$$\theta = c_2 - \frac{c_1}{2a}\epsilon^{-2at}. \tag{5}$$

where c_1 and c_2 are integration constants, and $a = f/I_0$. To evaluate these, we impose the conditions: $t = 0$, $\theta = \theta_\tau$, $\dot{\theta} = \omega_\tau$, and find

$$\theta = \theta_\tau + \frac{I_0}{2f}\omega_\tau(1 - \epsilon^{-2ft/I_0}). \tag{6}$$

Step 4.—From inspection of equation (6) we see that θ has its greatest value θ_m when t is infinite, and that this value is given by

$$\theta_m = \theta_\tau + \frac{I_0}{2f}\omega_\tau. \tag{7}$$

Into equation (7) we introduce the values of θ_τ and ω_τ given in (2) and (3) and, upon simplifying, find

$$\theta_m = \frac{Mi}{2f}\tau. \tag{8}$$

Equation (8) shows that under the assumed conditions of an ideal fluxmeter, when used as an instrument for time measurement, the angle of deflection is accurately proportional to the time interval during which the current i is sent through the coil.

Under practical conditions, since the elastic torque of a fluxmeter suspension is never quite zero, such an instrument should be calibrated. This can readily be done by some such method as has been previously described.[1] The time scale will not depart far from uniformity, and its range can be varied at will by proper control of the current.

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[1] Loc. cit., p. 204.